BAULKHAM HILLS HIGH SCHOOL

MATHEMATICS ADVANCED ASSESSMENT

December 2011

Time allowed: 50 minutes
plus 5 minutes reading time

STUDENT NUMBER :____________________

TEACHER'S NAME:____________________

QUESTION	MARK
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
TOTAL	
PERCENTAGE	

<u>Topics Tested:</u> Integration and Series

BAULKHAM HILLS HS PERSEVERE

Advanced Mathematics

December 2011

Time: 50 minutes + 5 minutes reading time

DIRECTIONS

- Full working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Use black or blue pen only (*not pencils*) to write your solutions.
- No liquid paper is to be used. If a correction is to be made, one line is to be ruled through the incorrect answer.
- Write your teacher's name and your name on the cover sheet provided
- At the end of the exam, staple your answers in order behind the cover sheet provided, and your questions on the back
- Approved Maths aids and calculators may be used

1. Find the primitive function for $3x^2 - \frac{1}{\sqrt{x}}$ — 2

2. Evaluate $\int_1^4 (3x-5)^2 dx$ — 2

3. Given that the limiting sum of $15 + 15x + 15x^2$ .... is 45, find x — 2

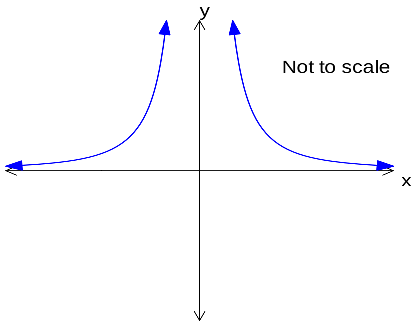
4. The sum of the first 4 terms of an arithmetic progression is 12, and the 15th term is 75. Find the first term and the common difference. — 3

5. The first three terms of an arithmetic series are -2+3+8+....

(i) Find the 60th term. — 2

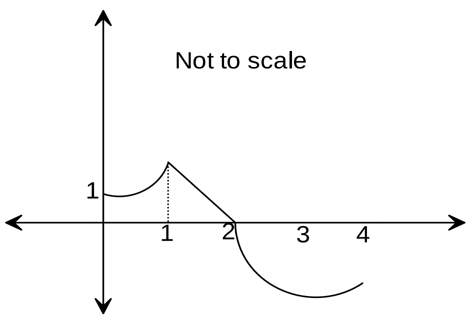
(ii) Hence or otherwise, find the sum of the first 60 terms of the series. — 2

6. Given the graph of $y = \frac{1}{x^2}$ below — 2



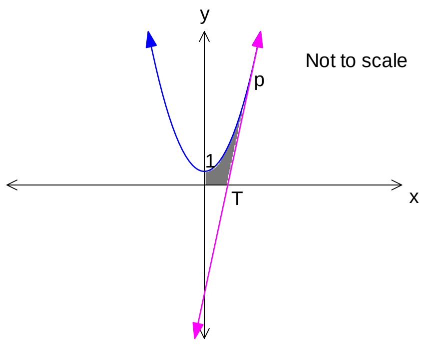
Find the area bounded by this graph, the x-axis and ordinates x= 1 and x =4.

7. Given the graph of $y = f(x)$ below, find the approximate area bounded by the curve, the x-axis and the lines $x = 0, x = 4$, using Simpson's Rule with five function values. — 3



x	0	1	2	3	4
$f(x)$	1	2	0	-4	-3

8. The diagram shows the graph of the parabola $y = x^2 + 1$. A tangent is drawn to the parabola at the point P(3, 10)

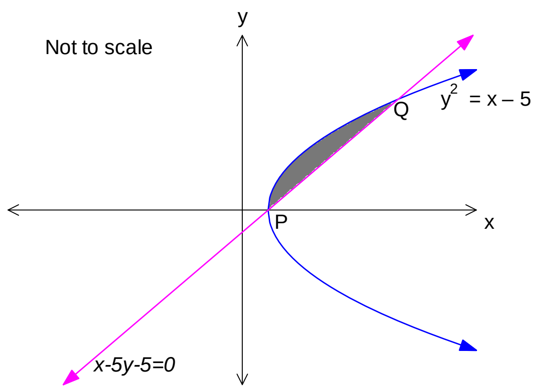


(i) Show that the equation of the tangent is $y = 6x - 8$. 2

(ii) T is the x-intercept of the tangent. Find the coordinates of T. 1

(iii) Find the area of the shaded region. 3

9.



The diagram shows the graph of the curves $f(x): y^2 = x - 5$ and the line $g(x): x - 5y - 5 = 0$.

(i) Show that the points of intersection between curves $f(x)$ and $g(x)$ are P(5, 0) and Q(30, 5). 2

(ii) Find the volume of the solid generated by rotating the shaded region around the x-axis. 3

10. Bianca would like to save \$60000 for a deposit on her first home. She decided to deposit at the start of each month her net monthly salary of \$3000 in a bank account that pays interest at a rate of 6% per annum compounded monthly.
Bianca intends to withdraw $\$E$ at the end of each month from this account for living expenses, immediately after the interest has been paid.
Let A_n be the amount in the account after the n-th withdrawal

i) Show that the amount of the money in the account following the second withdrawal of $\$E$ is given by 2

$$A_2 = \$3000(1.005^2 + 1.005) - E(1.005 + 1)$$

ii) Show that the A_n is given by 2

$$A_n = (3015 - E)\frac{1.005^n - 1}{0.005}$$

iii) Calculate the value of E if Bianca is to reach her goal after 4 years. 2

~ END OF EXAM ~

BS Q.1 – 7 → 18 marks
PS Q. 8, 9, 10 → 17 marks   out of 35

Answers   2 unit Ass. task Dec. 2011

① $\int 3x^2 - \frac{1}{\sqrt{x}}\,dx = x^3 - \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$

$= x^3 - 2x^{\frac{1}{2}} + c$ (1) (1); if missing c: (−1)

② $\int_1^4 (3x-5)^2\,dx = \left[\frac{(3x-5)^3}{9}\right]_1^4$ (1)

$= \left[\frac{7^3}{9} - \frac{(-2)^3}{9}\right] = 39$ (1)

③ $15 + 15x + 15x^2 + \dots = 45$

$\therefore 45 = \frac{15}{1-x}$ (1)

$\therefore x = \frac{2}{3}$ (1)

④ $S_4 = \frac{4}{2}(2a + 3d)$

$\therefore 12 = 2(2a + 3d)$

$T_{15} = a + 14d$

$\therefore 75 = a + 14d$ (1)

$\therefore d = \frac{144}{25}$ (1) $a = \frac{-141}{25}$ (1)

⑤ $-2 + 3 + 8 + \dots$ A.P

(1) $a = -2$ $d = 5$

i) $T_{60} = a + 59d = -2 + 59 \times 5$

$= 293$ (1)

ii) $S_{60} = \frac{60}{2}(-2 + 293)$ (1)

$= 8730$ (1)

⑥ $y = \frac{1}{x^2}$

$A = \int_1^4 \frac{1}{x^2}\,dx = \left[\frac{x^{-1}}{-1}\right]_1^4$ (1)

$= \left[\frac{-1}{4} - \left(\frac{-1}{1}\right)\right] = \frac{3}{4}$ (1)

⑦ $A \doteqdot \frac{h}{3}\left[1 + 4 \times 2 + 0\right] + \left|\frac{1}{3}\left[0 + 4 \times -4 + -3\right]\right|$ (1) (1)

$h = 1$

$\therefore A = \frac{1}{3}\left[1 + 8 + 0\right] + \left|\frac{1}{3}\left[0 + -16 - 3\right]\right|$

$= 3 + \left|-\frac{19}{3}\right| = \frac{28}{3}$ (1)

if no abs. value for negative

$\therefore A \doteqdot \frac{1}{3}\left[1 + 4(2 + -4) + 2 \times 0 + -3\right]$

$= \frac{-10}{3}$ ∴ 2/3

8) i) $y = x^2 + 1$ (6)

$\frac{dy}{dx} = 2x$

$m = 2 \times 3 = 6$ (1)

$\therefore$ tangent at $P(3, 10)$ $\therefore y - 10 = 6(x - 3)$ (1)

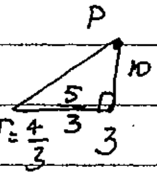
$\therefore y = 6x - 18 + 10$

$y = 6x - 8$ $\therefore$ shown

ii) $y_T = 0 \therefore 0 = 6x - 8 \therefore x = \frac{4}{3}$

$\therefore T\left(\frac{4}{3}, 0\right)$ (1)

iii) Area $= \int_0^3 x^2 + 1\, dx -$ (1)



$= \left[\frac{x^3}{3} + x\right]_0^3 - \frac{1}{2} \times \frac{5}{3} \times 10$ (1)

$= (9 + 3) - 0 - 5 \times \frac{5}{3} = \frac{11}{3}$ (1)

9) i) $y^2 = x - 5 \therefore x = y^2 + 5$ (5)

$x - 5y - 5 = 0 \therefore x = 5y + 5$

$\therefore y^2 + 5 = 5y + 5$

$y^2 - 5y = 0$ (1)

$y(y - 5) = 0$

$\therefore y = 0$ or $y = 5$ (1)

$\therefore x = 5 \times 0 + 5 \quad x = 5 \times 5 + 5$

$\therefore P(5, 0),\ Q(30, 5)$

ii) $V = \pi \int_5^{30} f^2(x)\, dx - \pi \int_5^{30} g^2(x)\, dx$

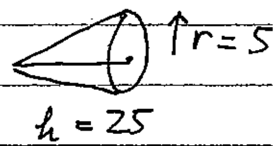
$\therefore V = \pi \int_5^{30} (x - 5)\, dx - \pi \int_5^{30} \frac{(x-5)^2}{5^2}\, dx$ (1)

$= \pi \left[\frac{x^2}{2} - 5x\right]_5^{30} - \pi \left[\frac{(x-5)^3}{25 \times 3}\right]_5^{30}$ (1)

$= \frac{625\pi}{6}$ or 327.25 (1)

OR

$V = \pi \int_5^{30} (x - 5)\, dx -$ (1)



$= \pi \left[\frac{x^2}{2} - 5x\right]_5^{30} - \frac{1}{3} \times 25 \times 5^2 \pi$ (1)

$= \frac{625}{2}\pi - \frac{625}{3}\pi = \frac{625\pi}{6}$ (1)

Question 10 (6)

i) $A_1 = 3000\left(1 + \frac{6 \div 12}{100}\right) - E = 3000 \times 1.005 - E$ (1)

$A_2 = (A_1 + 3000) \times 1.005 - E$

$= (3000 \times 1.005 - E + 3000) \times 1.005 - E$ (1)

$= 3000 \times 1.005^2 - E \times 1.005 + 3000 \times 1.005 - E$

$= 3000(1.005^2 + 1.005) - E(1.005 + 1)$ ∴ shown

ii) $A_n = 3000\underbrace{(1.005^n + 1.005^{n-1} + .. + 1.005)}_{} - E\underbrace{(1.005^{n-1} + .. + 1)}_{}$ (1)

G.P $a = 1.005$ $r = 1.005$ $n = n$ G.P $a = 1$ $r = 1.005$ $n = n$

$A_n = 3000 \times 1.005 \frac{1.005^n - 1}{1.005 - 1} - E \times 1 \times \frac{1.005^n - 1}{1.005 - 1}$ (1)

$\therefore A_n = \frac{1.005^n - 1}{0.005}\left[3000 \times 1.005 - E\right]$

$\therefore A_n = \frac{1.005^n - 1}{0.005}\left[3015 - E\right]$ ∴ shown

iii) $A_n = 60000$ $n = 4 \times 12 = 48$

$\therefore 60000 = \frac{1.005^{48} - 1}{0.005}\left[3015 - E\right]$ (1)

$\frac{60000 \times 0.005}{1.005^{48} - 1} - 3015 = -E$

(1)

$\therefore E = \$1905.898 = \$1905.90$

~~BS → 1 – 7 → 18 marks~~

(PS) Q.8, 9, 10 → 12 marks