BAULKHAM HILLS HIGH SCHOOL

MATHEMATICS ADVANCED ASSESSMENT

December 2012

Time allowed: 50 minutes + 5 minute reading time

STUDENT'S NAME:____________________

TEACHER'S NAME:____________________

General Instructions

- Write using black or blue pen
- Board-approved calculators may be used
- All necessary working should be shown in every question in section II
- Marks may be deducted for careless or badly arranged work
- Attempt all questions

Topics Tested: Integration, Series, Locus and Maximum, Minimum problems

Section I - **4 marks**
Use the multiple choice answer sheet for question 1-4

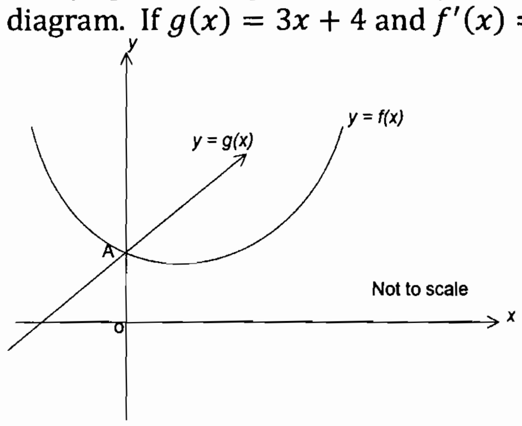
1. The n^{th} term of the sequence -1, 4, -9, 16, ... is

(A) 32 (B) $-2n$ (C) n^2 (D) $(-1)^n n^2$

2. Focus (1, -3), directrix is x=5. From the information given, the equation of the parabola is

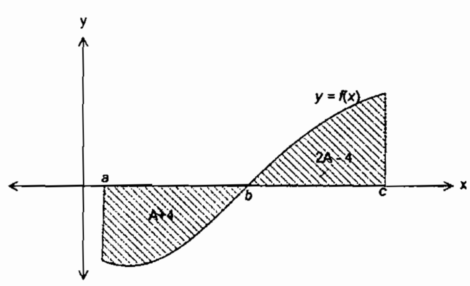
(A) $(y+3)^2 = 8(x-3)$ (B) $(y-3)^2 = -8(x-3)$ (C) $(y+3)^2 = -8(x-3)$
(D) $(y-3)^2 = 8(x-3)$

3. The graph of $y = g(x)$ and $y = f(x)$ intersect at the point A on the y-axis, as shown in the diagram. If $g(x) = 3x + 4$ and $f'(x) = 2x - 3$, find $f(x)$.



(A) $x^2 - 3x + 7$ (B) $3x^2 + 4x + 4$ (C) $x^2 - 3x + 4$ (D) $x^2 + 4x - 7$

4. In the diagram shown below, the area between the curve $y = f(x)$, the x-axis, and the line $x = a$ is equal to (A + 4) $units^2$. The area between the curve $y = f(x)$, the x- axis, and the line $x = c$ is equal to (2A – 4)$units^2$. Use this information to evaluate $\int_a^c f(x)dx$.



(A) $A + B$ (B) $4A - 8$ (C) $2A + 8$ (D) $A - 8$

Section II

1. Find the primitive function for

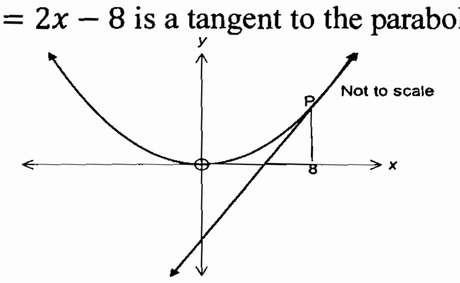
(i) $\frac{3x^3+7x}{x^2}$ 2

(ii) $\frac{1}{\sqrt{4x-3}}$ 2

2. *A* is the point (1, 5) and *B* (6, -2). The point *P*(*x, y*) moves so that *PA is* perpendicular to *PB*. Find the equation of the locus of *P*. 2

3. If $p, q, p+q$ is in an arithmetic progression and $p, q, 20$ is in geometric progression, where $p > 0, q > 0$, find the values of p and q. 2

4. The line $y = 2x - 8$ is a tangent to the parabola $x^2 = 4ay$ at the point P where $x = 8$



(i) Show that $a = 2$. 2

(ii) Write down the coordinates of the focus and equation of directrix. 2

5. Given the series $2^{x-3} + 2^{2x-4} + 2^{3x-5}$ ……

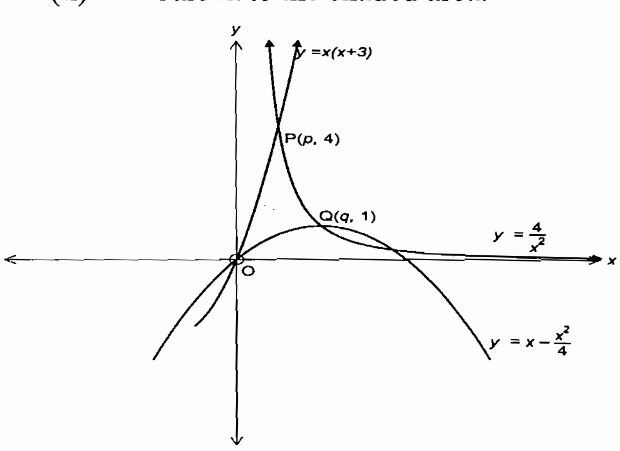
(i) Find an expression for T_{20}. 2

(ii) For what values of x does the series have a limiting sum? 2

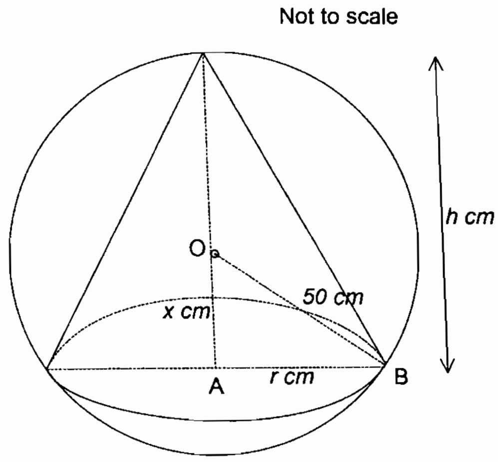
6. The origin, O and the point P and Q are the vertices of the shaded area in the diagram. The sides lie on curves with equations $y = x(x+3)$, $y = x - \frac{x^2}{4}$ and $y = \frac{4}{x^2}$.

(i) P and Q have the coordinates (p, 4) and (q, 1). Find the value of p and q. 2

(ii) Calculate the shaded area. 3



7.	The diagram shows a cone of base radius r cm and height h cm inscribed in a sphere of radius 50 cm. The centre of the sphere is O and $\angle OAB = 90^\circ$. Let OA $= x$ cm.	



(i) Show that $r = \sqrt{2500 - x^2}$. **1**

(ii) Hence show that the volume, V cm^3, of the cone is given by: **2**

$$V = \frac{\pi}{3}(2500 - x^2)(50 + x)$$

(iii) Find the radius of the largest cone which can be inscribed in the sphere.(give your answer to nearest *mm.)* **3**

8.	Mona is retiring next week and her Superannuation Fund contains \$1 200 000. The Fund is earning 6% p.a. compound interest, compounded monthly. Mona wishes to withdraw a regular amount of \$8 000 per month to live in her retirement.	

(i) Show that after 1 month she will have an amount A_1 in her account where $A_1 = 1\,200\,000(1.005) - 8\,000$ **1**

(ii) Show that after 3 months the amount in her account A_3 is given by $A_3 = 1\,200\,000(1.005)^3 - 8\,000\{(1.005)^2 + (1.005) + 1\}$ **2**

(iii) By finding a similar expression for the amount remaining after n months, find how many years the money will last. **3**

~ END OF EXAM ~

Solutions Yr 11 Dec. Task

Section I

M.C (1) D (2) C (3) C (4) D

Section II

Q1 (i) $\int \frac{3x^3+7x^2}{x^2}\,dx$

$= \int (3x+7)\,dx$ — (1)

$= \frac{3x^2}{2} + 7x + c$ — (1)

(ii) $\int \frac{1}{(4x-3)^{1/2}}\,dx$

$= \int (4x-3)^{-1/2}\,dx$

$= \frac{(4x-3)^{1/2}}{2\cancel{4}\times\frac{1}{\cancel{2}}} + c$ — (1) (1) [ignore c]

OR $= \frac{1}{2}\times\sqrt{4x-3} + C$

Q2 $m_{AP} = \frac{y-5}{x-1}$, $m_{BP} = \frac{y+2}{x-6}$ } (1)

$\therefore m_{AP}\times m_{BP} = -1$

$\frac{y-5}{x-1}\times\frac{y+2}{x-6} = -1$ } (1)

$y^2+2y-5y-10 = -(x^2-6x-x+6)$

$x^2-7x+y^2-3y-4=0$

Q3 $p, q, p+q$ in A.P

$q-p = p+q-q$

$\boxed{q=2p}$ — (1)

$p, q, 20$ in G.P

$\frac{q}{p} = \frac{20}{q} \Rightarrow \boxed{q^2 = 20p}$

$\therefore (2p)^2 = 20p$

$4p^2 - 20p = 0$

$4p(p-5)=0 \Rightarrow p = 0, 5$ but $p>0$

$\therefore p = 5$, $q = 10$ } (1)

Q4 $m_T = 2$ — (A)

Also $x^2 = 4ay$

$y = \frac{x^2}{4a}$

$\frac{dy}{dx} = \frac{2x}{4a} = \frac{x}{2a}$

$m_T = \left.\frac{dy}{dx}\right]_{x=8} = \frac{8}{2a} = \frac{4}{a}$ } (1) — B

From A & B

$2 = \frac{4}{a}$ — (1)

$\therefore \boxed{a=2}$ Shown.

(ii) $F(0,2)$ (1), eq. of directrix $y=-2$ (1)

Q5 (i) G.P with $r = \frac{2^{2x-4}}{2^{x-3}}$

$= 2^{2x-4-x+3}$

$\boxed{= 2^{x-1}}$ — (1)

$T_{20} = ar^{19}$

$= 2^{x-3}\times(2^{x-1})^{19}$

$= 2^{x-3}\times 2^{19x-19}$

$= 2^{20x-22}$ } (1)

i) $-1 < r < 1$

Since $r = 2^{x-1}$, $2^{x-1} > 0$ ①

OR $-1 < 2^{x-1} < 1$ ①

$0 < 2^{x-1} < 1$

$2^{x-1} < 2^0$

$x - 1 < 0$

$x < 1$ ①

16 $y = \frac{4}{x^2}$

$4 = \frac{4}{p^2} \Rightarrow p^2 = 1$ ①

$p = 1 \rightarrow$ in the first quad.

$P(1,4)$

$1 = \frac{4}{q^2} \Rightarrow q = 2$

$q = (2,1)$ ①

$$A = \int_0^1 x(x+3) - \left(x - \frac{x^2}{4}\right)\Big] dx + \int_1^2 \left(\frac{4}{x^2} - x + \frac{x^2}{4}\right) dx$$

$$= \int_0^1 \left(\frac{5x^2}{4} + 2x\right) dx + \int_1^2 \left(\frac{4}{x^2} - x + \frac{x^2}{4}\right) dx$$

$$= \left[\frac{5}{12}x^3 + \frac{2x^2}{2}\right]_0^1 + \left[\frac{-4}{x} - \frac{x^2}{2} + \frac{x^3}{12}\right]_1^2$$

$$= \left(\frac{5}{12} + 1\right) + \left[\left(\frac{-4}{2} - \frac{4}{2} + \frac{8}{12}\right) - \left(\frac{-4}{1} - \frac{1}{2} + \frac{1}{12}\right)\right]$$

$$= \frac{17}{12} + \left[-4 + \frac{8}{12} - \left(\frac{-48 - 6 + 1}{12}\right)\right]$$

$$= \frac{17}{12} + \left[\frac{-40}{12} - \left(\frac{-53}{12}\right)\right]$$

$$= \frac{17}{12} + \left[\frac{13}{12}\right] = \frac{30}{12} = \frac{5}{2} = 2.5 \text{ sq. units}$$

③

07 (i) In rt angled Δ OAB

$50^2 = r^2 + x^2$

$r = \pm\sqrt{2500 - x^2}$

as $r > 0$ $\therefore r = \sqrt{2500 - x^2}$ ①

(ii) $V = \frac{1}{3}\pi r^2 h$

$\because h = OC + OA = 50 + x$ ①

$= \frac{1}{3}\pi(2500 - x^2)(50 + x)$

(iii) $\frac{dv}{dx} = \frac{1}{3}\pi\left[(2500 - x^2) + (50 + x)x - 2\right]$

$= \frac{1}{3}\pi\left[2500 - x^2 - 2x^2 - 100x\right]$ ①

$= \frac{1}{3}\pi\left[2500 - 3x^2 - 100x\right]$

for max. & min $\frac{dv}{dx} = 0$

$2500 - 3x^2 - 100x = 0$

$x = \frac{-100 \pm \sqrt{(100)^2 + 4 \times 3 \times 2500}}{6}$

$= \frac{-100 \pm 200}{6}$

$= -\frac{300}{6}$ or $\frac{100}{6}$

But x can't be negative

$\therefore x = \frac{100}{6} = 50/3$ ①

$\frac{d^2v}{dx^2} = \pi/3\,(-100 - 6x)$

$= \pi/3\,(-100 - 100) < 0$

$\therefore$ max.

$\therefore r = 47.14.$ ①

08 6% p.a = 0.5% p.m

(i) $A_1 = 1200000(1.005) - 8000$ } — (1)

(ii) $A_2 = A_1 \times 1.005 - 8000$

$= [1200000(1.005) - 8000]1.005 - 8000$

$= 1200000(1.005)^2 - 8000(1 + 1.005)$

$A_3 = A_2 \times 1.005 - 8000$

$= [1200000(1.005)^2 - 8000(1 + 1.005)]1.005 - 8000$

$= 1200000(1.005)^3 - 8000(1 + 1.005 + 1.005^2)$

— 2 marks with working.

(iii) $A_n = 1200000(1.005)^n - 8000(1 + 1.005 + \cdots 1.005^{n-1})$

↓

G.P

$a = 1, r = 1.005$

Also $A_n = 0$.

(1) — $0 = 1200000(1.005)^n - \frac{8000(1.005^n - 1)}{1.005 - 1}$ — (1)

$1200000(1.005)^n = \frac{8000(1.005^n - 1)}{0.005}$

$5 \times 12(1.005)^n = 80[(1.005)^n - 1]$

$(1.005)^n[60 - 80] = -80$

$-20(1.005)^n = -80$

$(1.005)^n = 4$ } — (1)

$n \ln(1.005) = \ln 4$

$n = \frac{\ln 4}{\ln(1.005)} = 277.95$ months.

$= 23$ years.