BAULKHAM HILLS HIGH SCHOOL

DECEMBER 2014

YEAR 12 TASK 1

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 50 minutes
- Write using black or blue pen
- Board-approved calculators may be used
- Show all necessary working in Section II
- Marks may be deducted for careless or badly arranged work

Total marks – 43
Exam consists of 5 pages.

This paper consists of TWO sections.

Section 1 – Page 2 (5 marks)
Questions 1-5

- Attempt Questions 1-5
- Allow about **5** minutes for this section

Section II – Pages 3-5 (38 marks)

- Attempt questions 6-8
- Allow about **45** minutes for this section

Topics Tested: Integration and Series with applications

Section I – Multiple choice questions (5 marks)
Use the multiple choice Answer Sheet for Question 1 – 5.

1. The first three terms of an arithmetic progression are 26, 23, 20. The sum S_n of the first n terms of the series is:

   (A) $S_n = \frac{n}{2}(55 - 3n)$ (B) $S_n = 29 - 3n$

   (C) $S_n = \frac{n}{2}(29 - 3n)$ (D) $S_n = 26 - 3n$

2. The definite integral of $\int_0^1 (5x^4 - x^2)dx =$

   (A) $\frac{1}{15}$ (B) $\frac{2}{3}$

   (C) 3 (D) $\frac{1}{2}$

3. The primitive function of $\sqrt{x} + 1$ is:

   (A) $\frac{3}{2}\sqrt{x^3} + x + C$ (B) $\frac{3}{2}\sqrt{x^3} + x^2 + C$

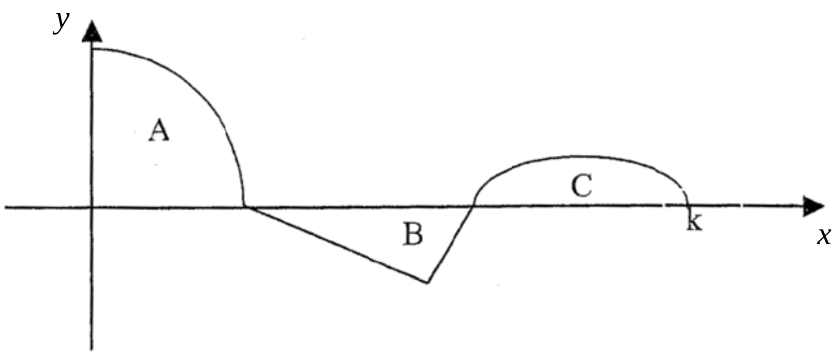
   (C) $\frac{2}{3}\sqrt{x^3} + x + C$ (D) $\frac{2}{3}\sqrt{x^5} + x + C$

4. The n^{th} term of the sequence 1, -2, 3, -4, 5, -6, 7, -8,….. is:

   (A) $(-1)^{n-1}(2n-1)$ (B) $(-1)^{n+1}n$

   (C) $(-1)^{n+1}(2n-1)$ (D) $(-1)^{n+1}(n+1)$

5. The graph shows $y = f(x)$ for $0 \le x \le k$.



   The value of $\int_0^k f(x)\,dx = 8.5$ units.
   If area A = 4 units^2 and area B = 3 units^2, then area C is:

   (A) 1.5 (B) 7.5

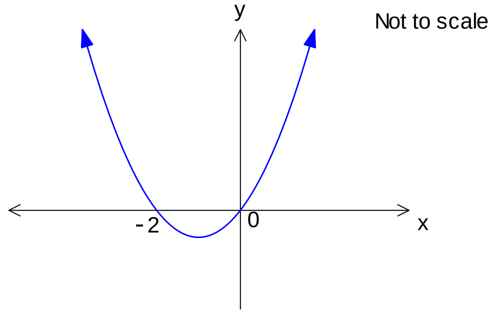
   (C) 5.5 (D) 4.5

Section II – Extended response questions (38 marks)

Question 6	**(12 marks) - Start a new page**	
a)	Find the 40^{th} term of the arithmetic sequence 2, 6, 10....	**2**
b)	Find $\int (3x-2)^5 dx$	**2**
c)	Evaluate $\sum_{k=1}^{8} (4k-1)$	**2**
d)	Evaluate $\int_2^3 \frac{x^2+5}{x^2} dx$	**2**
e)	Find the value(s) of x for which $(x-1), (x+3), (5x+3)$ form a geometric sequence.	**2**
f)	Find $g(x)$, given $g'(x) = 3x^2 - 4$ and $g(1) = 4$.	**2**

Question 7 (13 marks) - Start a new page

a) Find the area enclosed between the curve $y = x(x+2)$ and the x axis. **2**



b) (i) Copy the table into your answer booklet and complete for the function $y = \frac{x-1}{x}$ **1**

x	1	2	3	4	5
y			$\frac{2}{3}$		

(ii) Hence use Simpson's Rule with five function values to find the approximate value of $\int_1^5 \frac{x-1}{x}\,dx$ to three decimal places. **2**

c) The sum of the second and the fifth term of an arithmetic sequence is 32 whilst the sum of the third and the eighth term is 48.

(i) Find the first term and the common difference. **2**

(ii) Find the sum of the first 30 terms. **2**

d) The sum of the first n terms of a sequence is given by $S_n = 75n - 2n^2$.

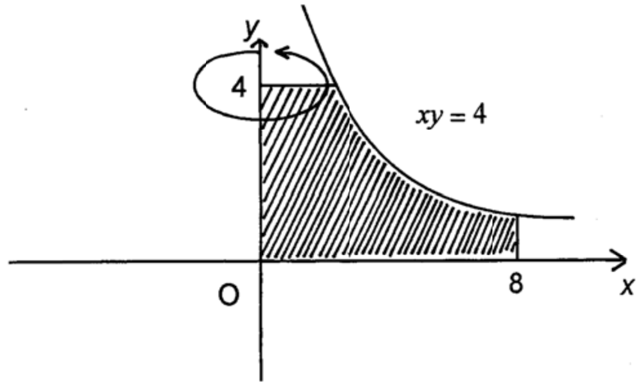
Find

(i) The 2^{th} term. **2**

(ii) The n^{th} term. **2**

Question 8 (13 marks) - Start a new page

a)



The area enclosed by the curve $xy = 4$, the x and y axes and the lines $y = 4$ and $x = 8$, is rotated about the y axis. **3**

By considering this area as two regions, find the volume of the solid.

b) Jessica has decided that she needs to set up a superannuation fund for her retirement. Her financial advisor told Jessica that she needs \$700,000 in the fund when she retires in 25 years' time.

To achieve this, she decides to make equal payments of \$$P$ at the beginning of each year. The fund pays an interest rate of 8% pa compounded annually.

Let A_n be the account balance at the end of *n* years, before she makes her payment for the following year.

(i) Show that her account balance after 3 years (before making the 4^{th} payment) is given by: **2**

$$A_3 = P(1.08)((1.08)^2 + (1.08) + 1)$$

(ii) Show that her account balance when she retires after 25 years is given by: **2**

$$A_{25} = 13.5P \times (1.08^{25} - 1).$$

(iii) Hence find the amount Jessica will need to pay each year to satisfy her retirement requirements. **2**

c) Consider the geometric series

$$1 + (\sqrt{\alpha} - 2) + (\sqrt{\alpha} - 2)^2 + (\sqrt{\alpha} - 2)^3 + \ldots$$

(i) Find the largest positive integral value of α for the series to have a limiting sum. **2**

(ii) Find the limiting sum for this series when $\alpha = 8$. **2**
Express your answer as a surd with a rational denominator.

END OF EXAM

Y12 Advanced Maths

Section I – Multiple choice questions (5 marks)
Use the multiple choice Answer Sheet for Question 1 – 5.

1. The first three terms of an arithmetic progression are 26, 23, 20. The sum S_n of the first n terms of the series is:

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2. The definite integral of $\int_0^1 (5x^4 - x^2)dx$ =

   (A) $\frac{1}{15}$ (B) $\frac{2}{3}$

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3. The primitive function of $\sqrt{x} + 1$ is:

   (A) $\frac{3}{2}\sqrt{x^3} + x + C$ (B) $\frac{3}{2}\sqrt{x^3} + x^2 + C$

   (C) $\frac{2}{3}\sqrt{x^3} + x + C$ (D) $\frac{2}{3}\sqrt{x^5} + x + C$

4. The n^{th} term of the sequence 1, -2, 3, -4, 5, -6, 7, -8,….. is:

   (A) $(-1)^{n-1}(2n-1)$ (B) $(-1)^{n+1}n$
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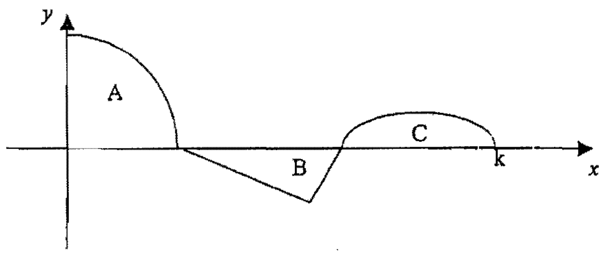
5. The graph shows $y = f(x)$ for $0 \le x \le k$.



   The value of $\int_0^k f(x)\,dx = 8.5$ units.
   If area A = 4 units^2 and area B = 3 units^2, then area C is:

   (A) 1.5 (B) 7.5
) 5.5 (D) 4.5

2

Q6. a) $a = 2$ $d = 6-2 = 4$ ①
both a and d.

$$T_{40} = a + (n-1)d$$
$$= 2 + 39\times 4$$
$$= 158 \quad ①$$

b) $$\int (3x-2)^5 dx$$
$$= \frac{(3x-2)^{5+1}}{5+1} \times \frac{1}{3} + C$$
①
$$= \frac{(3x-2)^6}{18} \; ① \; + C$$

c) $$S = \sum_{k=1}^{8} (4k-1) = (4\times1-1) + (4\times2-1) + \dots (4\times8-$$ ①

$a=3,\ d=4,\ n=8$

$S = 136$ ①

d) $$\int_2^3 \frac{x^2+5}{x^2}\,dx = \int_2^3 1 + \frac{5}{x^2}\,dx$$
$$= \left[x - \frac{5}{x}\right]_2^3 \; ① \quad = \left(3 - \frac{5}{3}\right) - \left(2 - \frac{5}{2}\right)$$
$$= \frac{11}{6} \; ①$$

e) $\frac{x+3}{x-1} = \frac{5x+3}{x+3}$

$x^2+6x+9 = 5x^2+3x-5x-3$

$4x^2-8x-12=0$

$\therefore x^2-2x-3=0$ (1)

$(x-3)(x+1)=0$

$x=3$ or $x=-1$

(1)

f) $\int (3x^2-4)\,dx = x^3-4x+C$ (1)

$g(1) = 1^3-4\times1+C = 4$

$\therefore C=7.$

$g(x) = x^3-4x+7$ (1)

Q7 a) $y = x(x+2) = x^2+2x$

$$A = \left| \int_{-2}^{0} (x^2+2x)\,dx \right|$$

$$= \left| \left[\frac{x^3}{3} + x^2 \right]_{-2}^{0} \right| \quad (1)$$

$$= \left| 0 - \left(\frac{(-2)^3}{3} + (-2)^2 \right) \right|$$

$$= \left| +\frac{8}{3} - 4 \right| = \frac{4}{3} \ u^2. \quad (1)$$

b) i)

x	1	2	3	4	5
y	0	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{4}{5}$

(1)

ii)

$$\int_1^5 \frac{x-1}{x}\,dx \doteqdot \int_1^3 \frac{x-1}{x}\,dx + \int_3^5 \frac{x-1}{x}\,dx$$

$$= \frac{3-1}{6}\left[f(1) + 4f\left(\frac{1+3}{2}\right) + f(3) \right] \quad (1)$$

$$+ \frac{5-3}{6}\left[f(3) + 4f\left(\frac{3+5}{2}\right) + f(5) \right]$$

$$= \frac{1}{3}\left[0 + 4\times\frac{1}{2} + \frac{2}{3} \right] + \frac{1}{3}\left[\frac{2}{3} + 4\times\frac{3}{4}\right.$$

$= \frac{1}{3}\left(2+\frac{2}{3}\right) + \frac{1}{3}\left(\frac{2}{3}+3+\frac{4}{5}\right)$

$= \frac{1}{3} \times \left[\frac{8}{3} + \frac{10+45+12}{15}\right]$

$= \frac{1}{3}\left[\frac{8}{3} + \frac{67}{15}\right]$

$= \frac{1}{3} \times \frac{107}{15}$

$= \frac{107}{45}$ ①

$= 2.378$ (3 decs)

c) i) $T_2 + T_5 = 32$

$T_3 + T_8 = 48$

$(a+d) + (a+4d) = 2a+5d = 32$

$(a+2d) + (a+7d) = 2a+9d = 48$

Eqns (2) − (1) $\quad 4d = 16$

$\therefore d = 4$ ①

$2a + 5d = 32$

$2a + 5 \times 4 = 32$

$\therefore a = 6$ ①

ii) $S_{30} = \frac{30}{2}\left(2 \times 6 + (30-1)4\right)$ ①

$= 1920$ ①

d) $S_n = 75n - 2n^2$

i) $S_1 = 75 \times 1 - 2 \times 1^2 = 73.$

$S_2 = 75 \times 2 - 2 \times 2^2 = 150 - 8 = 142.$ ①

$T_2 = S_2 - S_1 = 142 - 73 = 69$ ①

ii) $S_{n-1} = 75(n-1) - 2(n-1)^2$

$= 79n - 2n^2 - 77$ ①

$T_n = S_n - S_{n-1}$

$= 75n - 2n^2 - (79n - 2n^2 - 77)$

$= -4n + 77$ ①

(Q8) $x = 8,\ y = \frac{1}{2}$. $xy = 4$, $x = \frac{4}{y}$, $x^2 = \frac{16}{y^2} = 16y^{-2}$

a)

$$V = \pi r^2 h + \pi \int_{\frac{1}{2}}^{4} x^2\,dy$$

$$= \pi \times 8^2 \times \frac{1}{2} + \pi \int_{\frac{1}{2}}^{4} 16y^{-2}\,dy$$

$$= 32\pi + \pi \int_{\frac{1}{2}}^{4} \frac{16}{y^2}\,dy \quad (1)\ (1)$$

$$= 32\pi + \pi \left[-16y^{-1}\right]_{\frac{1}{2}}^{4}$$

$$= 32\pi + \pi\left[-4 - (-32)\right] \quad 28\pi$$

$$= 60\pi \text{ units} \quad (1)$$

or 188.50 units (2 decs)

If rotating about x-axis (Incorrect).

$y = 4,\ xy = 4$, $x = \frac{4}{4} = 1$

$xy = 4$, $y = \frac{4}{x}$, $y^2 = \frac{16}{x^2}$

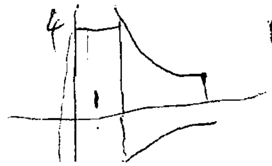
2 marks

If all correct

$16/4$

$$V = \pi r^2 h + \pi \int_1^8 y^2\,dx = \pi \times 4^2 \times 1 + \pi \int_1^8 \frac{16}{x^2}\,dx$$

$r = 4,\ h = 1$



$$= 16\pi + \pi\left[-\frac{16}{x}\right]_1^8$$

$$30\pi u^3 = 16\pi + \pi\left[-\frac{16}{8} + \frac{16}{1}\right] = 16\pi + 14\pi$$

b) $r = 8\%$

i) Beginning of Y1.

Balance $B = P$

Interest $I = P \times 8\%$

End of Y1

A_1 Balance $B = P + P\times 8\% = 1.08P$. (1)

Beginning of Y2

Balance $B = P + 1.08P$

End of Y2:

Interest $I = (P + 1.08P) \times 8\%$

Balance $B = (P + 1.08P) + (P + 1.08P)\times 8\%$

$= (P + 1.08P)(1 + 0.08)$

$= (P + 1.08P)(1.08)$

$= P\times 1.08 + P\times(1.08)^2$

$A_2 = P\times 1.08(1.08 + 1)$ (1)

Similarly end of Y3

$A_3 = P\times 1.08(1.08^2 + 1.08 + 1)$

ii) Similarly after 25^{th} Year

$A_{25} = P\times 1.08(1.08^{24} + 1.08^{23} + \dots + 1.08 + 1)$ (1)

$= P\times 1.08\left(\frac{1.08^{25} - 1}{1.08 - 1}\right)$ (1)

GP $a = 1$, $r = 1.08$, $n = 25$

$= P\times 1.08 \times \frac{(1.08^{25} - 1)}{0.08} = 13.5P(1.08^{25} - 1)$

1) $P \times 1.08 \times \dfrac{(1.08^{25}-1)}{0.08} = 700000$ (1)

$P = \dfrac{700000 \times 0.08}{1.08(1.08^{25}-1)}$ (1)

$= \$8865.88$ per Year

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c) $1+(\sqrt{\alpha}-2)+(\sqrt{\alpha}-2)^2+(\sqrt{\alpha}-2)^3+\ldots$ $\alpha \geqslant 0$

i) $r = \dfrac{T_2}{T_1} = \dfrac{\sqrt{\alpha}-2}{1} = \sqrt{\alpha}-2$ GP

$\dfrac{T_3}{T_2} = \dfrac{(\sqrt{\alpha}-2)^2}{(\sqrt{\alpha}-2)} = \sqrt{\alpha}-2$

It has limiting sum when $|\sqrt{\alpha}-2| < 1$ (1)

or $-1 < \sqrt{\alpha}-2 < 1$

$-1+2 < \sqrt{\alpha} < 1+2$

$1 < \sqrt{\alpha} < 3$

or $1^2 < \alpha < 3^2$

$1 < \alpha < 9$ (1)

$\therefore$ The largest positive integer of α is $\alpha = 8$

ii) $S_\infty = \dfrac{a}{1-r} = \dfrac{1}{1-(\sqrt{8}-2)}$ (1)

$= \dfrac{1}{1-\sqrt{8}+2}$

$= \dfrac{1}{3-\sqrt{8}}$

$= \dfrac{1\,(3+\sqrt{8})}{(3-\sqrt{8})(3+\sqrt{8})}$

$= \dfrac{3+\sqrt{8}}{3^2-8} = 3+\sqrt{8}$ (1)

$= 3+2\sqrt{2}$