CARINGBAH HIGH SCHOOL

YEAR 12 (2011)

ASSESSMENT TASK # 1 (DEC 2010)

2 UNIT MATHEMATICS

* Time allowed – 1 hour
* Start each question on a new page
* Do not use liquid paper
* Show all necessary working
* Approved calculators permitted

Question 1 (10 marks) **Marks**

(a) Using the discriminant determine the number of roots of the quadratic equation $x^2 - 2x - 5 = 0$ (you do not have to find the roots) (1)

(b) What is the equation of the axis of symmetry of $y = x^2 - 6x$ (1)

(c) Find the derivative of the following

(i) $y = \frac{3}{x^2}$

(ii) $y = \sqrt{2x-5}$

(iii) $y = \frac{(x-1)}{(2x-3)}$

(iv) $f(x) = ax^2 + bx + c$ (6)

(d) Find the second derivative of $f(x) = (\frac{1}{2}x - 4)^4$ (2)

Question 2 (10 marks) **Marks**

(a) If $x^2 - 2Kx + 1 = 0$ has real roots find the value(s) of K (2)

(b) Solve for x the inequality $x^2 - 7x + 12 \leq 0$ (2)

(c) Find the values of A, B, C if $x^2 + x - 2 \equiv A(x-2)^2 + Bx + C$ (2)

(d) If $f(x) = x^2 + 5x + 3$ find any value of x for which $f^1(x) = 39$ (2)

(e) A person has a fever and his temperature is increasing at a decreasing rate. Draw a graph of this with temperature (T) on the vertical axis and time (t) on the horizontal axis. (2)

Question 3 (10 marks) **Marks**

(a) Write the first 3 terms of $T_n = (-1)^n \times \frac{1}{n}$ (1)

(b) Find the common difference in the arithmetic series (1)

$\sqrt{3} + \sqrt{12} + \sqrt{27} .....$

(c) If one root of $3x^2 - 8x + K = 0$ is 3 times the other root find K (3)

(d) If α, β are the roots of $2x^2 + x - 8$ find the value of

(i) $\alpha + \beta$

(ii) $\alpha\beta$

(iii) $\alpha^2 + \beta^2$ (4)

(e) For the arithmetic series $\{4 + 7 + 10 + ...\}$

Find the general term T_n in its simplest form (1)

Question 4 (10 marks) **Marks**

(a) For what value of n will one root of the equation

$(n-2)x^2 + (n+2)x + 2n + 1 = 0$

be the reciprocal of the other. (2)

(b) In an arithmetic series $T_{10} = 5$ and $T_{17} = 54$ Find

(i) the first term (1)

(ii) common difference (1)

(c) Find the seventh term of the geometric series 54 -18+ 6..... (2)

Question 4 cont'd **Marks**

(d) An author writes a manuscript so that on the first day he writes 54 pages, on the second day 36 pages and so on each succeeding day he writes $\frac{2}{3}$rds of the number of pages of the preceding day.

(i) How many pages does he write on the fifth day (1)

(ii) How many pages has he written in 5 days (1)

(iii) What is the maximum number of pages he will write (2)

Question 5 (10 marks) **Marks**

(a) For the curve with equation $y = 2 + 9x - 3x^2 - x^3$

(i) Find any stationary points (2)

(ii) Determine the nature of the stationary points (2)

(iii) Find any inflexion points (2)

(iv) Draw a neat $\frac{1}{2}$ page sketch of this curve including the above information and y intercept. (4)

END OF EXAM

Caringbah H/S Mathematics 2 unit Ass Task 1 H.S.C. (Dec 2010)

Question 1

(a) $b^2 - 4ac = \Delta$

$= (-2)^2 - 4(1)(-5)$

$= 24$

$\therefore \Delta > 0$

2 roots

(b) $3 = x$

(c) (i) $y' = -\frac{6}{x^3}$

(ii) $y' = \frac{1}{\sqrt{2x-5}}$

(iii) $y' = \frac{-1}{(2x-3)^2}$

(iv) $f'(x) = 2ax + b$

(d) $f''(x) = 3(\frac{1}{2}x - 4)^2$

Question 2.

(a) $x^2 - 2Kx + 1 = 0$

now $\Delta \geq 0$

$(-2K)^2 - 4(1)(1) \geq 0$

$4K^2 - 4 \geq 0$

$(K-1)(K+1) \geq 0$

$\therefore K \leq -1 \cup K \geq 1$

(b) $x^2 - 7x + 12 \leq 0$

$(x-3)(x-4) \leq 0$

$\therefore 3 \leq x \leq 4$

(c) $x^2 + x - 2 = A(x^2 - 4x + 4) + Bx + C$

$x^2 + x - 2 = Ax^2 + x(B - 4A) + 4A + C$

$\therefore A = 1, \quad B - 4A = 1, \quad 4A + C = -2$

$B = 5 \qquad C = -6$

(d) $f(x) = x^2 + 5x + 3$

$f'(x) = 2x + 5$

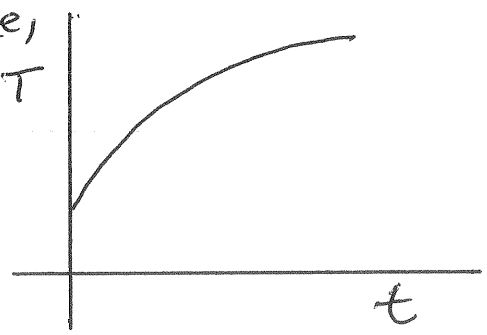
$\therefore 2x + 5 = 39$

$2x = 34$

$x = 17$

Question 2

(e)



Question 3.

(a) $T_1 = -1$

$T_2 = 1/2$

$T_3 = -1/3$

(b) $\sqrt{3}$

(c) $3x^2 - 8x + K = 0$

let roots be

$\alpha, 3\alpha$

$\therefore 4\alpha = 8/3$

$\alpha = 2/3$

$\therefore 3\alpha^2 = \frac{K}{3}$

$3(2/3)^2 = \frac{K}{3}$

$\therefore K = 4$

(d) $2x^2 + x - 8 = 0$

(i) $\alpha + \beta = -1/2$

(ii) $\alpha\beta = -4$

(iii) $\alpha^2 + \beta^2$

$= (\alpha + \beta)^2 - 2\alpha\beta$

$= 1/4 + 8$

$= 8\,1/4$

(e) $T_n = 4 + (n-1)3.$

$T_n = 3n + 1.$

Question 4

(a) Since $\alpha\beta = 1$

$\frac{2n+1}{n-2} = 1$

$2n + 1 = n - 2$

$n = -3$

(b) $5 = a + 9d \quad -①$

$54 = a + 16d \quad -②$

② − ①

$49 = 7d$

$d = 7$

$a = -58$

(c) $T_n = ar^{n-1}$

$T_7 = 54(-1/3)^6$

$T_7 = \frac{2}{27}$

(d) $54, 36, 24, \ldots$

(i) $T_5 = 54(\frac{2}{3})^4$

$= 10\,2/3$

(ii) $S_5 = \frac{54\left[1 - (\frac{2}{3})^5\right]}{1/3}$

$S_5 = 140\,2/3$

(iii) $S_\infty = \frac{a}{1-r}$

$= \frac{54}{1/3}$

$= 162$

Question 5

a) $y = 2 + 9x - 3x^2 - x^3$

$y' = 9 - 6x - 3x^2$

when $y' = 0$

(i) $9 - 6x - 3x^2 = 0$

$x^2 + 2x - 3 = 0$

$(x+3)(x-1) = 0$

$\begin{bmatrix} x = -3 \\ y = -25 \end{bmatrix}$ $\begin{bmatrix} x = 1 \\ y = 7 \end{bmatrix}$

(ii) $y'' = -6 - 6x$

when $x = -3$: $y'' > 0$: $\therefore$ Min T.P.

when $x = 1$: $y'' < 0$

(iii) Inflexion points occur when $y'' = 0$

$-6 - 6x = 0$

$6x = -6$

$x = -1$

$[-1, -9]$

(iv)