FINAL MARK

GIRRAWEEN HIGH SCHOOL
MATHEMATICS
YEAR 11 HSC TASK 1 2009
ANSWERS COVER SHEET

Name: _____

QUESTION	MARK	H2	H3	H4	H5	H6	H7	H8	H9
Q1	/12			✓	✓				✓
Q2	/14			✓	✓				✓
Q3	/14			✓	✓				✓
Q4	/17	✓		✓	✓				✓
Q5	/24				✓				✓
Q6	/22				✓				✓
Q7	/18				✓				✓
TOTAL	/121	/17		/57	/121				/121

HSC Outcomes **Mathematics**

H2 constructs arguments to prove and justify results.

H3 manipulates algebraic expressions involving logarithmic and exponential functions.

H4 expresses practical problems in mathematical terms based on simple given models.

H5 applies appropriate techniques from the study of calculus, geometry, probability, trigonometry and series to solve problems.

H6 uses the derivative to determine the features of the graph of a function.

H7 uses the features of a graph to deduce information about the derivative.

H8 uses techniques of integration to calculate areas and volumes.

H9 communicates using mathematical language, notation, diagrams and graphs.

GIRRAWEEN HIGH SHOOL
MATHEMATICS

YEAR 11 **HSC Task 1, 2009**

Time Allowed: 90 minutes

Instructions: Attempt all questions.
Start each question on a new page.
All necessary working must be shown.
Marks may be deducted for careless or badly arranged work.

Question 1 (12 marks)

a) A child's printing set contains five lots of each of the letters of the alphabet. What is the probability of reaching into a box containing all the letters and choosing:

i) the letter W? **1**

ii) a vowel? **1**

iii) a consonant? **2**

b) In a game, tiles in a bag are marked 1, 2 or 3. There are equal quantities of each. A tile is removed, noted and replaced.

i) Construct a Probability Tree diagram showing the outcomes for two successive draws. **3**

ii) Calculate the probability of drawing:
α) two 3's **1**
β) no 3's **2**
γ) at least one three **2**

Question 2 (14 marks)

a) Find the locus of the point $P(x, y)$ that moves so that:

i) it is equidistant from the points $A(-1, 6)$ and $B(3, 2)$. **3**

ii) its distance from $A(2, 4)$ is always twice its distance from $B(3, -2)$. **4**

iii) its distance from the y-axis is equal to its distance from the point $(2, 3)$. **5**

iv) it is 3 units from the point $(3, 6)$. **2**

Question 3 (14 marks)

a) Find the focus and directrix of:

i) $x^2 = 2y$ **3**

ii) $y^2 = -4x$ **3**

b) Find the equation of the parabola with its focus at (1, 4) and directrix at $y = 8$. **3**

c) A parabola has its vertex at the point (3, 1) and focus at the point (3, 3).

i) What is the focal length? **1**

ii) What is the equation of the directrix? **2**

iii) What is the equation of the parabola? **2**

Question 4 (17 marks)

a) Sketch the graph $y^2 = x$ marking its focus and directrix. **3**

b) Find the radius and coordinates for the centre of the circle
$x^2 + y^2 - 4x - 10y + 20 = 0$. **3**

c) Find the equations of the ***tangent*** and ***normal*** to the parabola $x^2 = -8y$ at point (4,-2). **6**

d) Points $P(8,8)$ and $Q\left(-2, \frac{1}{2}\right)$ are on the parabola $x^2 = 8y$.
Show that PQ is a focal chord. **5**

Question 5 (30 marks)

a) For the series 5+12+19+26+....., find

i) the common difference. **1**

ii) the 24^{th} term. **2**

iii) the sum of the first 24 terms. **2**

b) If the seventh term of an arithmetic sequence is 20 and the thirteenth term is 38, find the first term and the common difference. **4**

c) Find the number of terms in an arithmetic sequence with a=5, d=2 and the last term 43. **2**

d) For the series 12+6+3+....., find

i) the common ratio. **1**

ii) the 6^{th} term. **2**

iii) the sum of the first 6 terms. **2**

e) The third term of a geometric sequence is 8, the sixth term is -1, find

i) the common ratio. **4**

ii) the first term. **2**

iii) the eighth term. **2**

Question 6 (22 marks)

a) Find whether 2863 is a term of the sequence 5,8,11,.... **3**

b) The numbers 2, a, b are in arithmetic progression and a, b, 9 are in geometric progression. Find a and b. **6**

c) Express $0.5\dot{7}$ as a fraction, using a geometric series. **3**

d) Find the common ratio of a geometric series with a first term of 125 and a limiting sum of 100. **3**

e) If the sequence 4, -2.5, ... is geometric, find:

i) an expression for the nth term. **2**

ii) an expression for the sum of n terms and find its value when n=5. **3**

iii) the limit of S_n as $n \to \infty$. **2**

Question 7 (18 marks)

a) Evaluate:

i) $\sum_{n=1}^{5}(-5)^{n-1}$ **2**

ii) $\sum_{n=10}^{20}(15-n)$ **2**

b) In a class of 24 students 3 play no sport, 14 play cricket and 12 play tennis. If a student is chosen at random;

i) Draw a venn diagram to represent the data. **2**

ii) What is the probability that he or she plays tennis only? **1**

iii) What is the probability that he or she plays both tennis and cricket? **1**

iv) If two students are chosen at random, which is more likely: They both play cricket only or they both play tennis only. **3**

c) Cans of fruit in a supermarket display are stacked so that there are 4 cans in the top row, 6 in the next row, 8 in the next and so on. If there are 10 rows in the display, find:

i) the number of cans in the bottom row. **2**

ii) the total number of cans in the display. **2**

d) The lengths of the rungs of a ladder increase uniformly from 40cm in the top rung to 75cm in the bottom rung. If 13.8m of wood are used to make the rungs, how many rungs are there? **3**

4/11 HSC Task 1

Question 1

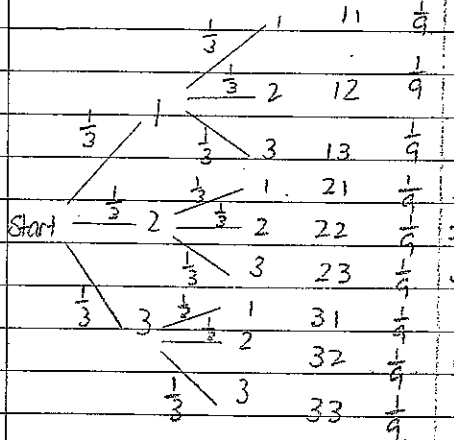
a) i) $P(W) = \frac{1}{26}$ (1)

ii) $P(\text{Vowel}) = \frac{5}{26}$ (1)

iii) $P(\text{consonant}) = 1 - P(\text{Vowel})$

$= \frac{21}{26}$ (2)

(3) bi)



ii α) $P(\text{two } 3\text{'s}) = \frac{1}{3} \times \frac{1}{3}$

$= \frac{1}{9}$ (1)

β) $P(\text{no } 3\text{'s}) = \frac{1}{9} + \frac{1}{9} + \frac{1}{9} + \frac{1}{9}$

$= \frac{4}{9}$ (2)

γ) $P(\text{at least one } 3)$

$= 1 - P(\text{no } 3\text{'s})$

$= 1 - \frac{4}{9} = \frac{5}{9}$ (2)

Question 2

i) $A(-1,6)$ $B(3,2)$ $P(x,y)$

$PA = PB$

$PA^2 = PB^2$

$(x+1)^2 + (y-6)^2 = (x-3)^2 + (y-2)^2$

$x^2+2x+1+y^2-12y+36 = x^2-6x+9+y^2-4y+4$

$8x - 8y + 24 = 0$

$\therefore x - y + 3 = 0$ (3)

ii) $A(2,4)$ $B(3,-2)$ $P(x,y)$

$PA = 2 \times PB$

$PA^2 = 4 \times PB^2$

$(x-2)^2 + (y-4)^2 = 4[(x-3)^2 + (y+2)^2]$

$x^2-4x+4+y^2-8y+16 = 4[x^2-6x+9+y^2+4y+4]$

$x^2-4x+4+y^2-8y+16 = 4x^2-24x+36+4y^2+16y+16$

$3x^2-20x+3y^2+24y+32 = 0$ (4)

iii) point $(2,3)$ y-axis $P(x,y)$

$A(2,3)$ $B(0,y)$. $P(x,y)$

$PA = PB$

$PA^2 = PB^2$

$(x-2)^2 + (y-3)^2 = (x-0)^2 + (y-y)^2$

$x^2-4x+4+y^2-6y+9 = x^2$

$y^2-6y-4x+13 = 0$

$y^2-6y+9 = 4x-4$

$(y-3)^2 = 4x-4$

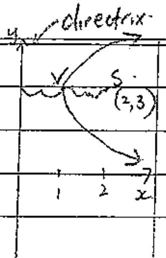
$(y-3)^2 = 4(x-1)$ (5)

OR

$(y-k)^2 = 4a(x-h)$

$(y-3)^2 = 4(1)(x-1)$

$(y-3)^2 = 4(x-1)$



iv) Point $(3,6)$. $P(x,y)$

$PA = 3$

$PA^2 = 9$ (2)

$(x-3)^2 + (y-6)^2 = 9$

Question 3

a i) $x^2 = 2y$

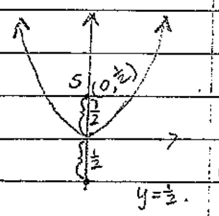
$x^2 = 4ay$

$4a = 2$

$a = \frac{1}{2}$

$\therefore S(0, \frac{1}{2})$

directrix: $y = -\frac{1}{2}$ (3)



ii) $y^2 = -4x$

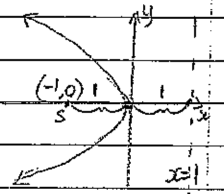
$y^2 = -4ax$

$4a = 4$

$a = 1$

$\therefore S(-1,0)$

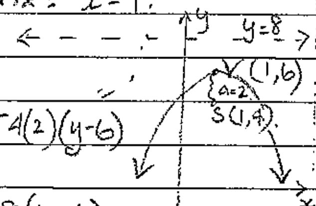
directrix: $x = 1$ (3)



b)

$(x-1)^2 = -4(2)(y-6)$

$(x-1)^2 = -8(y-6)$ (3)



c) $V(3,1)$ $S(3,3)$

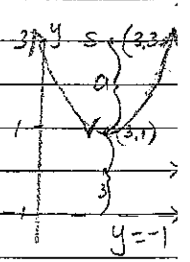
$a = 2$ (1)

ii) directrix

$y = -1$ (2)

iii) $(x-3)^2 = 4(2)(y-1)$

$(x-3)^2 = 8(y-1)$ (2)



Question 4

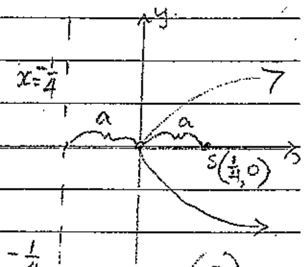
a) $y^2 = x$

$4a = 1$

$a = \frac{1}{4}$

$S(\frac{1}{4}, 0)$

directrix: $x = -\frac{1}{4}$ (3)



b) $x^2+y^2-4x-10y+20=0$

$x^2-4x+4+y^2-10y+25 = -20+4+25$

$(x-2)^2+(y-5)^2 = 9$

$\therefore$ Centre of the circle is $(2,5)$ and the radius $= 3$. (3)

c) $x^2 = -8y$ $(4,-2)$

$y = -\frac{x^2}{8}$

$y' = -\frac{2x}{8} = -\frac{x}{4}$

when $x = 4$

$y' = -\frac{4}{4} = -1$

$m_{tangent} = -1$ $m_{normal} = 1$

Eqn normal: $y+2 = 1(x-4)$.

$y+2 = x-4$.

$y = x-6$ or.

$x-y-6 = 0$.

Eqn tangent: $y+2 = -1(x-4)$.

$y+2 = -x+4$.

$y = 2-x$ OR.

$x+y-2 = 0$. (6)

d) $x^2 = 8y$.

$4a = 8$

$a = 2$.

$\therefore S(0,2)$

$m_{PQ} = \frac{\frac{1}{2}-8}{-2-8}$

$= \frac{-7\frac{1}{2}}{-10}$.

$= \frac{15}{20} = \frac{3}{4}$

Eq PQ: $y-8 = \frac{3}{4}(x-8)$.

$4y-32 = 3x-24$.

$3x-4y+8 = 0$.

When $S(0,2)$

$3(0)-4(2)+8 = 0$.

$0-8+8 = 0$. (5)

$0 = 0$.

$\therefore$ The chord PQ passes through the focus so it is a focal chord.

Questions.

ai) $12-5 = 7$. (1)

$\therefore d = 7$.

ii) $a = 5$ $d = 7$.

$T_{24} = 5+(24-1)7$.

$= 5+23(7)$. (2)

$= 166$.

iii) $S_n = \frac{n}{2}[2a+(n-1)d]$

$S_{24} = \frac{24}{2}[10+23(7)]$

$= 12(10+161)$ (2)

$= 2052$.

b) $T_7 = 20$ $T_{13} = 38$

$20 = a+6d$ $38 = a+12d$.

Solve simultaneously.

$20 = a+6d$

$38 = a+12d$

$18 = 6d$.

$d = 3$. (4)

Sub $d=3$ into.

$20 = a+6d$.

$20 = a+18$.

$\therefore a = 2$.

$\therefore$ First term is 2 common diff 3.

c) $a = 5$, $d = 2$, $L = 43$.

$T_n = 43$

$43 = 5+(n-1)2$.

$43 = 5+2n-2$.

$40 = 2n$.

$\therefore n = 20$. (2)

di) $r = \frac{6}{12} = \frac{1}{2}$. (1)

ii) $T_n = ar^{n-1}$. $a=12$ $r=\frac{1}{2}$

$T_6 = 12\left(\frac{1}{2}\right)^5$

$= \frac{3}{8}$ (2)

iii) $S_n = \frac{a(1-r^n)}{1-r}$.

$S_6 = \frac{12\left(1-\left(\frac{1}{2}\right)^6\right)}{\frac{1}{2}}$ (2)

$= \frac{189}{8} = 23\frac{5}{8}$.

e) $T_3 = 8$ $T_6 = -1$

i) $8 = ar^2$ $-1 = ar^5$.

Solving simultaneously.

$-1 = ar^5$

$8 = ar^2$.

$-\frac{1}{8} = r^3$.

(4)

$\therefore r = -\frac{1}{2}$

ii) $8 = a\left(-\frac{1}{2}\right)^2$

$8 = \frac{a}{4}$. (2)

$\therefore a = 32$.

iii) $T_8 = 32\left(-\frac{1}{2}\right)^7$.

$= 32 \times -\frac{1}{128}$ (2)

$= -\frac{1}{4}$.

Question 6

a) $a = 5$ $d = 3$.

$2863 = 5+(n-1)3$.

$2863 = 5+3n-3$

$2861 = 3n$ (3)

$\therefore n = 953.667$

$\therefore$ 2863 is not a term in this sequence.

b) AP GP.

$a-2 = b-a$ $\frac{b}{a} = \frac{9}{b}$.

$b = 2a-2$.

$\therefore \frac{2a-2}{a} = \frac{9}{2a-2}$

$(2a-2)^2 = 9a$

$4a^2-8a+4 = 9a$

$4a^2-17a+4 = 0$.

$4a^2-16a-a+4 = 0$

$4a(a-4)-(a-4) = 0$

$(4a-1)(a-4) = 0$

$a = \frac{1}{4}$ and $a = 4$

Sub $a = \frac{1}{4}$ $a = 4$.

$b = 2\left(\frac{1}{4}\right) - 2$ $b = 2(4) - 2$.

$b = -1\frac{1}{2}$ $b = 6$.

$\therefore a = \frac{1}{4}$ or $a = 4$

$b = -\frac{3}{2}$ $b = 6$ (6)

c)

$0.5\dot{7} = 0.57 + 0.0057 + 0.000057$.

$a = 0.57$

$r = 0.01$.

$S_\infty = \frac{0.57}{1 - 0.01}$

$= \frac{0.57}{0.99} = \frac{57}{99}$

$= \frac{19}{33}$ (3)

d) $a = 125$ $S_\infty = 100$.

$100 = \frac{125}{1 - r}$

$100(1 - r) = 125$

$100 - 100r = 125$ (3)

$-100r = 25$

$r = -\frac{1}{4}$.

$\therefore$ Common ratio $= -\frac{1}{4}$.

e) i) $a = 4$ $r = -0.625$

$T_n = 4(-0.625)^{n-1}$. (2)

ii) $S_n = \frac{4(1 - (-0.625)^n)}{1 + 0.625}$

$= \frac{4 - 4(-0.625)^n}{1.625}$

When $n = 5$.

$S_5 = \frac{4 - 4(-0.625)^5}{1.625}$ (3)

$= 2.7$ (2dp).

iii) $S_\infty = \frac{4}{1 + 0.625}$

(2)

$= 2.46$ (2dp).

Question 7.

a) i) $\sum_{n=1}^{5} (-5)^{n-1} = 1 - 5 + 25 - 125 + 625$

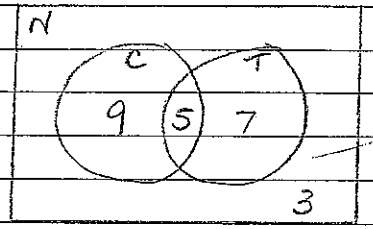
$= 521$. (2)

ii) $\sum_{n=10}^{20} (15 - n)$

$= 5 + 4 + 3 + 2 + 1 + 0 - 1 - 2 - 3 - 4 - 5$

$= 0$. (2)

b) i) (2)



ii) $P(\text{Tennis only}) = \frac{7}{24}$ (1)

iii) $P(\text{Both Tennis and Cricket}) = \frac{5}{24}$ (1)

iv) $P(\text{Both Cricket only}) = \frac{9}{24} \times \frac{8}{23}$.

$= \frac{72}{552}$.

$P(\text{Both Tennis only}) = \frac{7}{24} \times \frac{6}{23}$

$= \frac{42}{552}$. (3)

$\therefore$ It is more likely they both play cricket only.

c) i) $a = 4$ $d = 2$ $n = 10$

$T_{10} = 4 + (10 - 1)(2)$.

$= 4 + (9)(2)$ (2)

$= 22$.

$\therefore$ There are 22 cans in the bottom row.

ii) $S_{10} = \frac{10}{2}\left[2(4) + (10 - 1)(2)\right]$.

$= 5(8 + 18)$.

$= 5 \times 26$ (2)

$= 130$

$\therefore$ There are 130 cans in the display.

d) $a = 40$ $L = 75$

$S_n = 1380$

$1380 = \frac{n}{2}(40 + 75)$.

$2760 = 115n$.

$n = \frac{2760}{115}$. (2)

$n = 24$

$\therefore$ There are 24 rungs on the ladder.