Girraween High School

Mathematics

Year 11 **HSC Task 1, 2010**

Time Allowed : 90 minutes Examiner: Mr. Chris Howard

Instructions : Attempt all questions.

Start each question on a new sheet of paper .

Use only one side of paper (with no columns).

All necessary working **must** be shown.

Marks may be deducted for careless or badly arranged work.

16

Question 1. (15 Marks)

(a) Find the locus of the point $P(x,y)$ that moves so that

(i) It is equidistant from A(-1 , -2) and B(3 , 1) 3

(ii) It is equidistant from the point S(2 , 3) and the line $y = -2$ 3

(iii) Its distance from A(-1 , -2) is twice its distance from B(5 , 1) 3

(iv) Describe fully and sketch your locus in part (iii) 4

(b) Find the equation of the parabola with focus (0 , a) and directrix y = -a 3

(show all working)

Question 2 (16 Marks)

(a) Express the equation of the circle $x^2 - 4x + y^2 + 8y + 11 = 0$ in the form

$(x-l)^2 + (y-k)^2 = r^2$. Hence state the radius and centre . 4

(b) Sketch the graph of $y^2 = -4x$ clearly indicating the focus and the directrix. 4

(c) Find the equation of the curve which is always 3 units from (2 , -3) 2

(d) Find the equation/s of the curve which is always 3 units from $x = 2$ 2

(e) Find the vertex, focal length, focus and the directrix of the parabola

$(x-2)^2 = 8(y-1)$ 6

Question 3. (13 Marks)

For the parabola $y=\frac{1}{4}\left(x^2+6x+5\right)$ find

(i) The equation of the tangent at (3 , 8) **3**

(ii) The equation of the normal at (1 , 3) **3**

(iii) The coordinates of the vertex **2**

(iv) Find the coordinates of the focus **2**

(v) Find the equation of the focal chord passing through (1 , 3) **3**

Question 4 (16 Marks)

(a) For the arithmetic progression 2, 5, 8, 11, 14 .... Find

(i) a and d 2

(ii) The formula for the n^{th} term 2

(iii) The 20^{th} term 2

(iv) The sum of the first 20 terms 3

(b) The sum of an Arithmetic Progression is given by $S_n=n^2+2n$

(i) Find the first four terms of the A.P. 3

(ii) Find the sum of the first 20 terms 2

(iii) Find the 20^{th} term 2

Question 5 (16 Marks)

(a) For the geometric progression 1000, 900, 810, 729, ... Find

(i) The common ratio 1

(ii) The 10^{th} term 2

(iii) The sum of the first 10 terms 2

(iv) The limiting sum of the series. 2

(b) The 3^{rd} term of a G.P. is 12 the 5^{th} term is 24

(i) Find the two common ratios. 3

(ii) Find the first term 2

(iii) Find the value of the sum of the firdt 8 terms for both values of r leave your answer as a rationalised surd. 4

Question 6 (23 Marks)

(a) Evaluate

(i) $\sum_{n=1}^{10} 3n+1$ 3 (ii) $\sum_{n=3}^{10} 3(2^n)$ 3

(b) The numbers a, b, 18 are in Arithmetic Progression the numbers 24, b, a are in Geometric progression find all possible values for a and b 5

(c) P(x, y) is a point on the locus which is equidistant from the point S(3, 1) and the line $y = x$.

(i) Find an expression for the distance PS. 2

(ii) Using the perpendicular distance formula find an expression for the perpendicular distance from y = x to P 3

Hint $P_d = \dfrac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

(iii) Find the equation of the locus. 3

(iv) Find the points of intersection with the x axis 2

(v) Prove the locus never intersects the y-axis. 2

HSC ASS. TASK 1
DEC. 2010.

Q1(a) PA = PB

$\sqrt{(x+1)^2+(y+2)^2} = \sqrt{(x-3)^2+(y-1)^2}$

$(x+1)^2+(y+2)^2 = (x-3)^2+(y-1)^2$

$x^2+2x+1+y^2+4y+4 = x^2-6x+9+y^2-2y+1$

$8x+5 = -6y+10$

$8x+6y-5 = 0.$

OR $y = -4/3\,x + 5/6$ (3)

ii) PS = PM

$\sqrt{(x-2)^2+(y-3)^2} = y+2$

$(x-2)^2+(y-3)^2 = (y+2)^2$

$x^2-4x+4+y^2-6y+9 = y^2+4y+4$

$(x-2)^2 = 10y-5.$

$(x-2)^2 = 5(2y-1)$ (3)

iii) PA = 2PB

$\sqrt{(x+1)^2+(y+2)^2} = 2\sqrt{(x-5)^2+(y-1)^2}$

$x+1)^2+(y+2)^2 = 4\{(x-5)^2+(y-1)^2\}$

$^2+2x+1+y^2+4y+4$

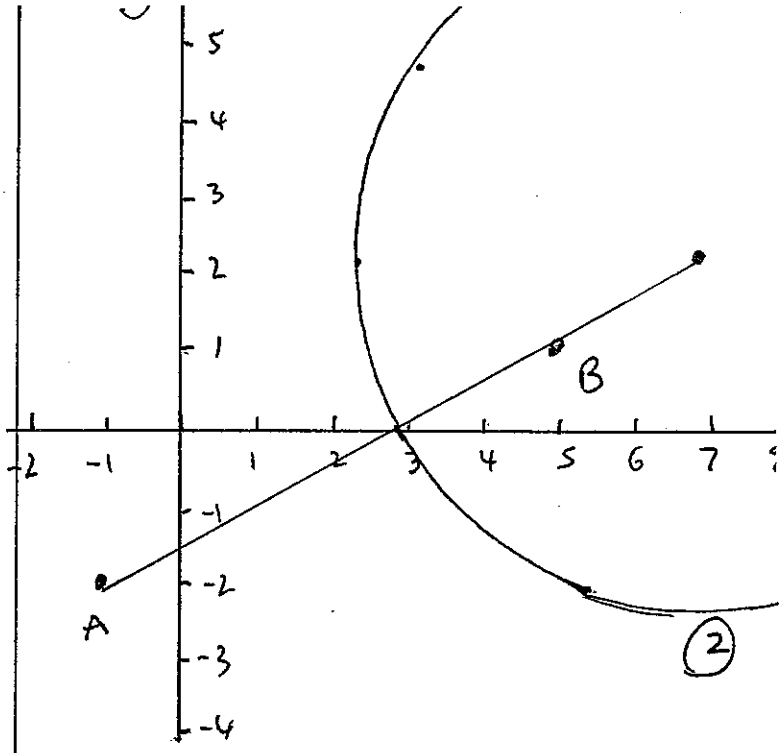
$= 4x^2-40x+100+4y^2-8y+4$

$0 = 3x^2-42x+99+3y^2-12y$

$0 = x^2-14x+y^2-4y+33$

$20 = (x^2-14x+49)+(y^2-4y+4)$

$20 = (x-7)^2+(y-2)^2$



(2)

CIRCLE CENTRE (7,2).
RADIUS ($\sqrt{20}$). (2)

(b) PS = PM

$\sqrt{x^2+(y-a)^2} = y+a$

$x^2+y^2-2ay+a^2 = y^2+2ay+a^2$

$x^2 = 4ay.$ (3)

Question 2.

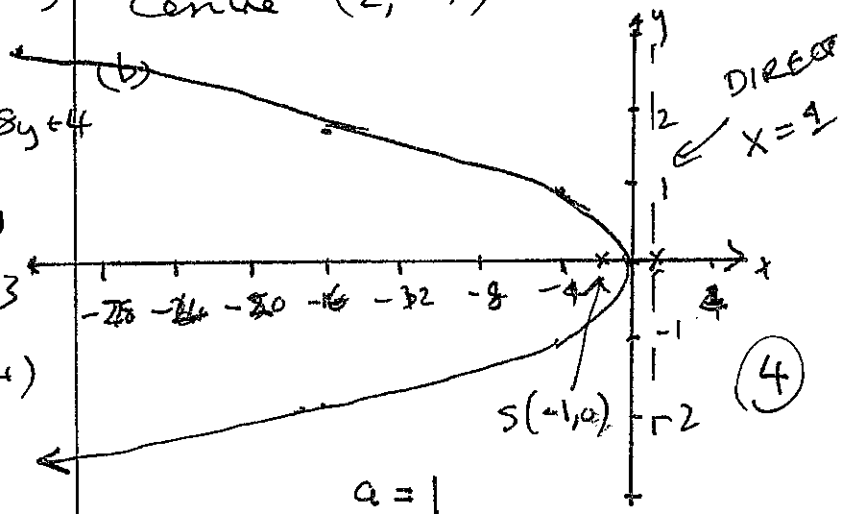
(a) $x^2-4x+y^2+8y = -11$

$x^2-4x+4+y^2+8y+16 = -11+20$

$(x-2)^2+(y+4)^2 = 9.$ (4)

Centre (2, −4) Radius 3.

(b)



(4)

a = 1

(c) $(x-2)^2+(y+3)^2 = 9$ (2)

(d) $x = 5$ or $x = -1$ (2)

(e) $(x-2)^2 = 8(y-1)$

VERTEX $(2,1)$ (1)

FOCAL LENGTH IS 2 (1)

FOCUS $(2,3)$ (2)

DIRECTRIX $y = -1$ (2)

Question 3

(i) $y = \frac{1}{4}(x^2+6x+5)$

$\frac{dy}{dx} = \frac{1}{4}(2x+6) = \frac{1}{2}(x+3)$

$\frac{dy}{dx}_{(x=1)} = \frac{1}{2}(3+3)$

$= 3.$

EQ^n OF TANGENT

$y - y_1 = m(x-x_1)$

$y - 8 = 3(x-3)$

$y - 8 = 3x - 9$

$\boxed{y = 3x - 1}$ (3)

(ii) GRADIENT OF TANGENT AT $(1,3)$

$\frac{dy}{dx}_{(x=1)} = \frac{1}{2}(1+3)$

$= 2.$

$\therefore$ GRADIENT OF NORMAL $= -1/2$

$(m_1 m_2 = -1)$

$y - 3 = -\frac{1}{2}(x-1)$

$y - 3 = -\frac{x}{2} + 1/2$

$y = -x/2 + 7/2$ (3)

(iii) Coordinate of Vertex

$\frac{dy}{dx} = 0$

$0 = \frac{1}{2}(x+3)$

$0 = x+3$

$x = -3.$

Point $V(-3,-1)$. (2)

(iv) $4y+4 = x^2+6x+9$

$4(y+1) = (x+3)^2$

FOCAL LENGTH $a = 1$

$\therefore$ FOCUS $(-3,0)$ (2)

(v) $S(-3,0)$ $P(1,3)$

$\therefore m_{SP} = \frac{3-0}{1+3} = 3/4$

$\therefore$ EQ^n SP

$y - 3 = \frac{3}{4}(x-1)$

$y - 3 = \frac{3}{4}x - 3/4$

$\boxed{y = \frac{3}{4}x + \frac{9}{4}}$ (3)

Q4

(a) (i) $a = 2$ $d = 3$ (2)

(ii) $T_n = a + (n-1)d$

$T_n = 2 + (n-1)3$. (2)

(iii) $T_{20} = 2 + (19)3$

$= 59$ (2)

(iv) $S_{20} = \frac{n}{2}(a+l)$

$= \frac{20}{2}(2+59)$

(2)

Question 4

(b) (i) $S_n = n^2 + 2n$

$S_1 = 3 = T_1$

$S_2 = 8 = T_1 + T_2$

$\therefore T_2 = 5.$

$S_3 = 15 \quad \therefore T_3 = 7$

$S_4 = 24 \quad \therefore T_4 = 9$

3, 5, 7, 9. (3)

(ii) $S_{20} = 20^2 + 2(20)$

$= 440$ (2)

(iii) $T_{20} = S_{20} - S_{19}$

$= 440 - 399$

$= 41$ (2)

Question 5.

(a) (i) $r = 0.9$ (1)

(ii) $T_{10} = a(r)^9$

$= 387.42$ (2)

(iii) $S_{10} = \frac{a(r^n - 1)}{r - 1}$

$S_{10} = \frac{1000(1 - 0.9^{10})}{0.1}$

$= 6513.22.$ (2)

(iv) $S_\infty = \frac{a}{1 - r}$

$= \frac{1000}{0.1}$

$= 10{,}000$ (2)

(b) (i) $T_3 = ar^2$

$ar^2 = 12$

$T_5 = ar^4$

$ar^4 = 24$

$\frac{24}{12} = \frac{ar^4}{ar^2}$ (3)

$r^2 = 2 \quad r = \pm\sqrt{2}$

(ii) $ar^2 = 12$

$2a = 12$

$a = 6$ (2)

(iii) $S_n = \frac{a(r^n - 1)}{r - 1}$

$S_8 = \frac{6(\sqrt{2}^8 - 1)}{\sqrt{2} - 1}$

$= \frac{6(15)}{\sqrt{2} - 1} \times \frac{\sqrt{2} + 1}{\sqrt{2} + 1}$

$= 90(\sqrt{2} + 1).$ (2)

OR $S_8 = \frac{6((-\sqrt{2})^8 - 1)}{-\sqrt{2} - 1}$

$= \frac{-90}{\sqrt{2} + 1} \times \frac{\sqrt{2} - 1}{\sqrt{2} - 1}$

$= 90(1 - \sqrt{2}).$ (2)

Question 6

(a) (i) $\sum_{n=1}^{10} 3n+1 = 4+7+10+13 \ldots 31$

A.P.

$S_n = \frac{n}{2}(a+l)$

$= \frac{10}{2}(4+31)$

$= 175.$ (3)

(ii) $\sum_{n=3}^{10} 3(2^n) = 24+72+216 + \ldots + 3072$

$S_8 = \frac{a(r^n-1)}{r-1}$

$= \frac{24(2^8-1)}{1}$

$= 6120$ (3)

(b) A.P.

$\therefore 18-b = b-a$

$18+a = 2b$ (A)

G.P.

$\frac{b}{24} = \frac{a}{b}$

$b^2 = 24a$ (B)

$\therefore b^2 = 24(2b-18)$

$b^2 - 48b + 432 = 0$

$(b-36)(b-12) = 0$

$\therefore b = 12, 36$

$\therefore a = 6, 54$ (5)

(c) (i) $PS = \sqrt{(x-3)^2 + (y-1)^2}$ (2)

(ii) $y = x \Rightarrow x - y + 0 = 0$.

$\therefore a = 1 \quad b = -1 \quad c = 0$

$P_d = \frac{|1x - 1y + 0|}{\sqrt{1^2 + (-1)^2}}$

$= \frac{|x-y|}{\sqrt{2}}$ (3)

(iii) $PS = P_d$

$\sqrt{(x-3)^2 + (y-1)^2} = \frac{|x-y|}{\sqrt{2}}$

$2\{(x-3)^2 + (y-1)^2\} = (x-y)^2$

$2x^2 - 12x + 18 + 2y^2 - 4y + 2 = x^2 - 2xy + 1$

$x^2 - 12x + y^2 - 4y + 2xy + 20 = 0.$ (3)

(iv) x axis $y = 0$

$x^2 - 12x + 20 = 0$

$(x-10)(x-2) = 0$

$x = 2, 10.$ (2)

(v) y axis $x = 0$

$y^2 - 4y + 20 = 0$

$y = \frac{4 \pm \sqrt{16-80}}{2}$ ← no solution

$\Delta < 0.$ (2)