FINAL MARK

GIRRAWEEN HIGH SCHOOL
MATHEMATICS
YEAR 12 HSC Task 1 2011
ANSWERS COVER SHEET

Name: _____

QUESTION	MARK	H2	H3	H4	H5	H6	H7	H8	H9
PART A	/5								✓
PART B Q1	/19				✓				✓
Q2	/15				✓				✓
Q3	/9				✓				✓
Q4	/14		✓						✓
Q5	/13			✓	✓				✓
Q6	/15			✓	✓				✓
TOTAL	/90		/14	/28	/85				/90

H2   constructs arguments to prove and justify results.

H3   manipulates algebraic expressions involving logarithmic and exponential functions.

H4   expresses practical problems in mathematical terms based on simple given models.

H5   applies appropriate techniques from the study of calculus, geometry, probability, trigonometry and series to solve problems.

H6   uses the derivative to determine the features of the graph of a function.

H7   uses the features of a graph to deduce information about the derivative.

H8   uses techniques of integration to calculate areas and volumes.

H9   communicates using mathematical language, notation, diagrams and graphs.

MATHEMATICS

YEAR 12 HSC   **Task 1, 2011**

Time Allowed:90 minutes   **Name:**_____

Instructions: **Examiner: C. Howard**

- Attempt all questions
- Circle the best response for the questions in Part A
- Detach Part A and submit with your written answers for Part B
- Start each question in Part B on a new page
- All necessary working must be shown
- Marks may be deducted for careless or badly arranged work

PART A (5 marks)

For questions 1-5 circle the best response from the following:

Question 1: Two dice are rolled, the probability of rolling at least one 6 is:

A) $\dfrac{1}{6}$ B) $\dfrac{11}{36}$ C) $\dfrac{1}{3}$ D) None of these.

Question 2: $\displaystyle\sum_{n=1}^{3} n^2 =$

A) 9   B) 36   C) 14   D) 18

Question 3: For the parabola $x^2 = 4y$ the directrix is:

A) x axis   B) y axis   C) $x = -1$ D) $y = -1$

Question 4: For the Arithmetic Progression 1, 4, 7, 10,........ the 10th term is:

A) 28   B) 29   C) 30   D) 31

Question 5: For a random student the probability of passing Mathematics is 0.5 and Physics is 0.3. The probability of passing both is:

A) 0.8   B) 0.2   C) 0.6   D) 0.15

PART B

Question 1 (19 marks)

(a) For the Arithmetic Progression 91, 87, 83, 79, ... Find:

i) the value of a and d (2)

ii) the formula for the nth term (2)

iii) the 30th term (2)

iv) the least number of terms for the sum to be negative (3)

(b) The sum of an Arithmetic Progression is given by $S_n = n^2 + 4n$. Find:

i) the value of a and d (2)

ii) the formula for the nth term (2)

iii) the 30th term (2)

iv) the sum of the first 30 terms (2)

(c) Using the limiting sum convert $0.\overset{\cdot\cdot}{5\,7}$ to a fraction. (2)

Question 2 (15 marks)

(a) For the Geometric Progression 1024, -512, 256, -128, ... Find:

i) the common ratio (1)

ii) the 10th term (2)

iii) the sum of the first 10 terms (2)

iv) the limiting sum of the series (2)

(b) The sum of a Geometric Progression is given by $S_n = 3^n - 1$. Find:

i) the value of a and r (2)

ii) the 10th term (2)

iii) the sum of the first 10 terms (1)

(c) A ball drops from 1.2 metres, then bounces back up to $\dfrac{3}{5}$ of this height.

On the next bounce, it bounces up to $\dfrac{3}{5}$ of this height and so on. Through

what distance will the ball travel? (3)

Question 3 (9 marks)

(a) A die is rolled.

   i)   Find the probability of rolling a 6.   (1)

   ii)   Find the probability of not rolling a 6 ($\tilde{6}$).   (1)

   iii)   If 3 dice are rolled use a probability tree with only two outcomes

            per roll $\left(6 \ or \ \tilde{6}\right)$, list all 8 outcomes for the three rolls.   (3)

   iv)   Find the probability of rolling exactly one 6 in the three rolls.   (2)

   v)   Find the probability of rolling no 6's.   (2)

Question 4 (14 marks)

(a) A standard pack of playing cards contains 52 cards. Find the probability of
    Drawing at random:

   i)   a picture card (not including Ace's)   (1)

   ii)   a red card   (1)

   iii)   a club   (1)

   iv)   3 hearts in a row without replacement   (2)

   v)   A black card or a picture card   (2)

(b) It is known that of 130 students in Year 11 Girraween High School, 90 study
    Chemistry, 60 study Physics and 10 students do neither Chemistry or Physics.

   i)   Draw a Venn Diagram to illustrate this situation using C for Chemistry

            and P for Physics.   (3)

   ii)   Find the number of students who do both Chemistry and Physics   (2)

   iii)   What is the probability that a student chosen at random does not do

            Physics?   (2)

Question 5 (13 marks)

(a) Find the locus of the point $P(x, y)$ that moves so that:

   i)   it is equidistant from $A(2,3)$ and $B(-2,-1)$ (3)

   ii)   its distance from $A(6,3)$ is always twice its distance from the origin   (4)

   iii)   its distance from the x axis is equal to its distance from $B(4,2)$ (4)

   iv)   it is 3 units from the point $B(4,2)$ (2)

Question 6 (15 marks)

(a) For the parabola $(x-2)^2 = 4(y+4)$:

i)	Find the focal length	(1)
ii)	Find the coordinates of the vertex	(1)
iii)	Find the equation of the axis of symmetry	(1)
iv)	Find the equation of the directrix	(1)
v)	Find the coordinates of the focus	(1)
vi)	Find the x intercepts and y intercept of the parabola	(3)
vii)	Find the equation of the focal chord passing through the origin	(3)
viii)	Sketch the parabola showing the vertex, directrix, axis of symmetry, focus and the intercepts.	(4)

END OF PAPER.

PART A.

1. B 2. C 3. D 4. A 5 D

PART B.

Q1 a(i) $a = 91$ $d = -4$ ②

(ii) $T_n = 91 + (n-1)(-4)$
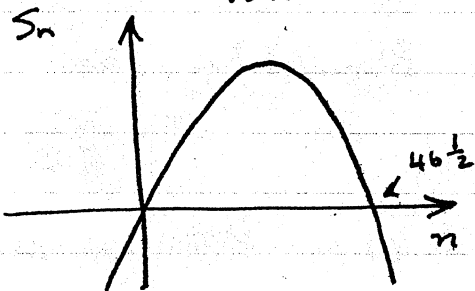$= 95 - 4n$ ②

(iii) $T_{30} = 95 - 4(30)$
$= -25$ ②

(iv) $S_n < 0$

$S_n = \frac{n}{2}[2a + (n-1)d]$
$= \frac{n}{2}[2(91) + (n-1)(-4)]$
$= \frac{n}{2}[186 - 4n]$

$= 93n - 2n^2$

∴ least # of term
47. ③

b) (i) $S_n = n^2 + 4n$
$S_1 = 5$
$S_2 = 12$
$S_3 = 21$
∴ $T_1 = 5$ $T_2 = 7$ $T_3 = 9$
$(T_3 = S_3 - S_2)$.

∴ $a = 5$ $d = 2$ ②

(ii) $T_n = 5 + (n-1)2$
$= 3 + 2n$ ②

(iii) $T_{30} = 3 + 2(30)$
$= 63$ ②

(iv) $S_{30} = \frac{30}{2}[2(5) + (29)2]$
$= 1020$ ②

(c) $0.5\dot{7} = 0.57 + 0.0057$
$+ 0.000057 + \ldots$
∴ $a = 0.57$ $r = 0.01$

$S_\infty = \frac{a}{1-r}$

$= \frac{0.57}{0.99} = \frac{57}{99}$

$= \frac{19}{33}$ ②

Q2 a (i) $r = -\frac{512}{1024}$

$= -\frac{1}{2}$ ①

(ii) $T_n = ar^{n-1}$
$T_{10} = 1024(-\frac{1}{2})^9$
$= -2.$ ②

(iii) $S_n = \frac{a(1-r^n)}{1-r}$

$S_{10} = \frac{1024(1 - (-\frac{1}{2})^{10})}{1 - (-\frac{1}{2})}$

$= 682$ ②

(iv) $S_\infty = \frac{a}{1-r}$

$= \frac{1024}{1 - (-\frac{1}{2})}$

$= 682.67$

b(i)

$$S_n = 3^n - 1$$

$$S_1 = 2 \qquad S_2 = 8$$

$$S_3 = 26$$

$$\therefore T_1 = 2 \quad T_2 = 6 \quad T_3 = 18$$

$$(T_3 = S_3 - S_2).$$

$$\therefore a = 2 \qquad r = 3. \quad ②$$

(ii)

$$T_{10} = 2(3)^9$$

$$= 39366 \qquad ②$$

(iii)

$$S_{10} = \frac{2(3^{10} - 1)}{3 - 1}$$

$$= 59048 \qquad ①$$

(c)

$$a = 1.2 \qquad r = \tfrac{3}{5}.$$

$$S_\infty = \frac{1.2}{1 - 3/5} = 3$$

Ball bounces up and down
so total distance
$$(3 \times 2) - 1.2 = 4.8m \quad ③$$

Q3 a(i) $\quad P(6) = \tfrac{1}{6} \qquad ①$

(ii) $\quad P(\tilde{6}) = 1 - \tfrac{1}{6}$

$$= \tfrac{5}{6} \qquad ①$$

(iii)

③

666		$\tfrac{1}{216}$
666̄		$\tfrac{5}{216}$
66̄6		$\tfrac{5}{216}$
66̄6̄		$\tfrac{25}{216}$
6̄66		$\tfrac{5}{216}$
6̄66̄		$\tfrac{25}{216}$
6̄6̄6		$\tfrac{25}{216}$
6̄6̄6̄		$\tfrac{125}{216}$
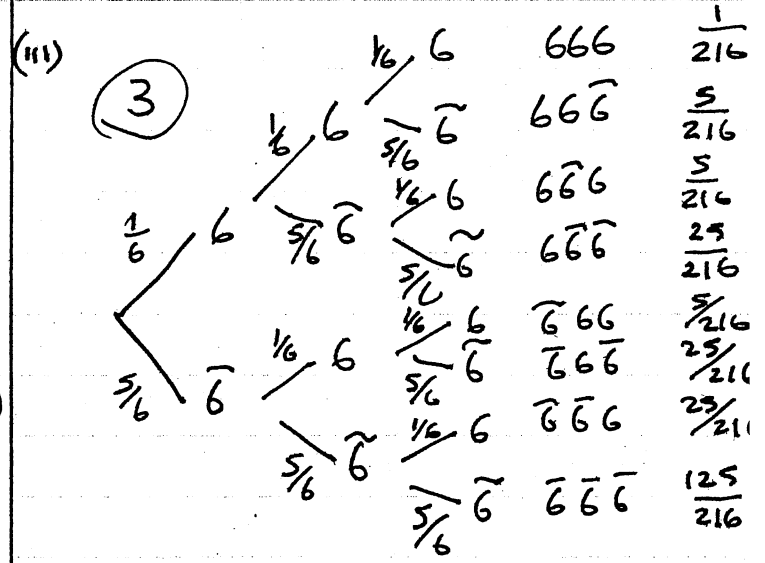
(iv) $\quad P(\text{exactly one } 6) = \overline{6}6\overline{6} + 6\overline{6}\,\overline{6} + \overline{6}\,\overline{6}6$

$$= \frac{25}{216} \times 3$$

$$= \frac{75}{216}.$$

(v) $\quad P(\text{no } 6's) = \overline{6}\,\overline{6}\,\overline{6}$

$$= \frac{125}{216}.$$

Q4 (a) (i) $\quad P(\text{PICTURE CARD}) = \dfrac{12}{52}$

$$= \tfrac{3}{13} \quad ①$$

(ii) $\quad P(\text{RED}) = \tfrac{1}{2} \quad ①$

(iii) $\quad P(\text{CLUB}) = \tfrac{1}{4} \quad ①$

(iv) $\quad P(3 \text{ hearts w/o replacement})$

$$\tfrac{1}{4} \times \tfrac{12}{51} \times \tfrac{11}{50} = \tfrac{11}{850} \quad ②$$

(v) $\quad P(\text{black or picture})$

$$= \tfrac{1}{2} + \tfrac{3}{13} - \tfrac{3}{26}$$

$$= \tfrac{8}{13} \qquad ②$$

b(i)

C   P
60 30 30   ③

10
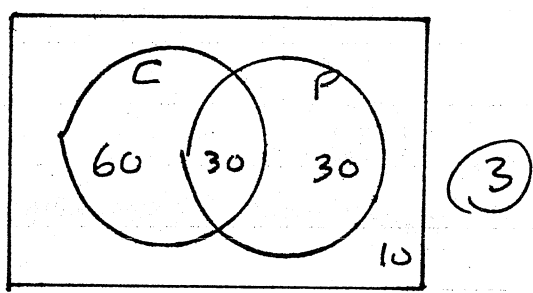
(ii)   30 STUDENTS   ②
{ 130 - (10+60+90) = -30 }
P∩C = 30

(iii) P = $\frac{70}{130}$ = $\frac{7}{13}$ ②

Q5 (a)(i) P(x,y) A(2,3) B(-2,-1)
PA = PB   (PA² = PB²)
$(x-2)^2 + (y-3)^2 = (x+2)^2 + (y+1)^2$
$x^2 - 4x + 4 + y^2 - 6y + 9$
$= x^2 + 4x + 4 + y^2 + 2y + 1$
$8x + 8y - 8 = 0$
$y = -x + 1$ ③

(ii) P(x,y) A(6,3) B(0,0)
$PA^2 = (2(PB))^2$
$(x-6)^2 + (y-3)^2 = 4(x^2 + y^2)$
④ $\boxed{3x^2 + 12x + 3y^2 + 6y - 45 = 0}$
$x^2 + 4x + y^2 + 2y - 9 = 0$
$(x+2)^2 + (y+1)^2 = 20$.

(iii) B(4,2) x axis A(x,0)
$PB^2 = PA^2$
④ $(x-4)^2 + (y-2)^2 = (x-x)^2 + (y-0)^2$
$x^2 - 8x + 16 + y^2 - 4y + 4 = y^2$
$\boxed{x^2 - 8x - 4y + 20 = 0}$

IN STANDARD FORM ..
$(x-4)^2 = 4(y-1)$
iv) B(4,2)
$PB^2 = 3^2$
$(x-4)^2 + (y-2)^2 = 9$ ②

Q6 $(x-2)^2 = 4(y+4)$
i) $4a = 4$ $a = 1$ ①
ii)   V(2,-4)   ①
iii) AXIS of SYMMETRY x=2 ①
iv) DIRECTRIX   y = -5   ①
v). Focus   (2,-3)   ①

vi) $(x-2)^2 = 4(y+4)$
when x = 0
$4 = 4y + 16$
$4y = -12$
$y = -3$
when y = 0   ③
$(x-2)^2 = 16$
$x - 2 = \pm 4$
$x = 6, -2$

$x = 2$

AXIS OF SYMMETRY

$S(2,-3)$

$V(2,-4)$

DIRECTRIX $Y = -5$

4