FINAL MARK

GIRRAWEEN HIGH SCHOOL
MATHEMATICS
YEAR 12 HSC Task 1 2012
ANSWERS COVER SHEET

Name: _____

QUESTION	MARK	H2	H3	H4	H5	H6	H7	H8	H9
PART A	/5								✓
PART B Q1	/17				✓				✓
Q2	/14				✓				✓
Q3	/13				✓				✓
Q4	/20				✓				✓
Q5	/11				✓				✓
Q6	/12				✓				✓
TOTAL	/82				/77				/82

H2 constructs arguments to prove and justify results.

H3 manipulates algebraic expressions involving logarithmic and exponential functions.

H4 expresses practical problems in mathematical terms based on simple given models.

H5 applies appropriate techniques from the study of calculus, geometry, probability, trigonometry and series to solve problems.

H6 uses the derivative to determine the features of the graph of a function.

H7 uses the features of a graph to deduce information about the derivative.

H8 uses techniques of integration to calculate areas and volumes.

H9 communicates using mathematical language, notation, diagrams and graphs.

GIRRAWEEN HIGH SCHOOL

MATHEMATICS

YEAR 12 HSC **Task 1, 2012**

Time Allowed: 90 minutes **Name:**_____

Instructions: Examiner: C. McMillan

- Attempt all questions
- Circle the best response for the questions in Part A
- Detach Part A and submit with your written answers for Part B
- Start each question in Part B on a new page
- All necessary working must be shown
- Marks may be deducted for careless or badly arranged work

PART A (5 marks)

For questions 1-5 circle the best response from the following:

Question 1: The equation of a parabola with its vertex at (2,3) and directrix at $y=1$ is:

A) $(x-3)^2=8(y-2)$ B) $(x-2)^2=8(y-3)$

C) $(x-3)^2=-8(y-2)$ D) $(x-2)^2=-8(y-3)$

Question 2: $\sum_{n=2}^{5} n^2+n=$

A) 30 B) 42 C) 68 D) 72

Question 3: For the parabola $y^2=16x$ the focus is:

A) $(0,4)$ B) $(0,-4)$ C) $(4,0)$ D) $(-4,0)$

Question 4: For the Arithmetic Progression 4, 9, 14,..... the 14th term is:

A) 38 B) 49 C) 58 D) 69

Question 5: The limiting sum for the Geometric Progression 48, 36,27,... is:

A) 192 B) 129 C) 92 D) 12

PART B

Question 1 (17 marks)

(a) For the Arithmetic Sequence 14,10,6, ... Find:

i) the value of a and d (2)

ii) the formula for the n^{th} term (2)

iii) the 20^{th} term (2)

iv) the least number of terms for the sum to be negative (3)

(b) The first and last terms of an Arithmetic Progression with a common difference Of 11 are 23 and 1453 respectively.

i) How many terms are there in this series? (2)

ii) Find their sum. (2)

iii) Which is the first term greater than 714? (2)

(c) Using the limiting sum convert $0.\dot{4}\dot{3}$ to a fraction. (2)

Question 2 (14 marks)

(a) The fortieth term of an Arithmetic Series is 353 and the fourteenth term is 119. Find:

i) the common difference. (3)

ii) the first term. (2)

(b) Given that $8, x, -1$ are the first three terms of an Arithmetic Series, find:

i) the value of x. (2)

ii) the next term. (2)

(c) A restaurant is growing in popularity such that an additional 15 people eat there each week. The maximum capacity of the restaurant is 550 per week. If 25 customers attended the restaurant in the first week, find:

i) after how many weeks of operation will the restaurant have reached its full capacity? (2)

ii) the total gross takings for the year (if on average each customer spends $25). Assume the restaurant is open all year round. (3)

Question 3 (13 marks)

(a) For the Geometric Progression 3,-6,12,-24, ... Find:

i) the common ratio (1)

ii) the 8^{th} term (2)

iii) the sum of the first 8 terms (2)

(b) Find the limiting sum of the Geometric Progression

$$\frac{1}{3}+\frac{4}{27}+\frac{16}{243}+\ldots$$ (2)

(c) If p, q and 32 are the first three terms of a Geometric Series and q, 4, p are the first three terms of another Geometric Series, find p and q. (3)

(d) The first term of a Geometric Series is 4 and the eighth term is 8748. Find the twelfth term. (3)

Question 4 (20 marks)

(a) For the parabola $(x-3)^2=12(y+1)$:

i) Find the focal length (1)

ii) Find the coordinates of the vertex (1)

iii) Find the equation of the axis of symmetry (1)

iv) Find the equation of the directrix (1)

v) Find the coordinates of the focus (1)

vi) Find the x intercepts and y intercept of the parabola (4)

vii) Find the equation of the focal chord passing through the origin (2)

viii) Sketch the parabola showing the vertex, directrix, axis of symmetry, focus and the intercepts. (4)

(b) Find the coordinates of the vertex of the parabola $y^2-6y-9x-9=0$. (2)

(c) Find the equation of the tangent to the parabola $x^2=6y$ when $x=3$. (3)

Question 5 (11 marks)

(a) Find the locus of the point $P(x, y)$ that moves so that:

i) it is equidistant from $A(-5,0)$ and the line $x = -3$ (2)

ii) PA is always perpendicular to PB where $A(4,2)$ and $B(-1,2)$ (3)

iii) it is 2 units from the point $B(3,4)$ (2)

(b)

i) Show that the locus of a point $P(x, y)$ which moves so that its distance from the point $A(5,0)$ is always twice its distance from the point $B(2,0)$ is given by the circle $x^2 + y^2 - 2x - 3 = 0$. (2)

ii) Find the centre of the circle. (2)

Question 6 (12 marks)

(a) A plant when first observed has a height of 420mm. In the first week it grows 10cm and each succeeding week its growth is found to be 70% of that of the previous week. Find the height of the plant to the nearest millimetre at the end of 8 weeks. (2)

(b) Find the first term of a Geometric Series which has a common ratio of $\frac{2}{3}$ and a limiting sum of $\frac{3}{2}$. (2)

(c) Find the values of r for which the Geometric Progression

$$1 + (r+2) + (r+2)^2 + \ldots\ldots$$

has a limiting sum. (2)

(d) In the following series,

$$(1-x)^{\frac{1}{2}} + (1-x) + (1-x)^{\frac{3}{2}} + \ldots\ldots$$

i) For what values of x will a sum to infinity exist? (2)

ii) Find the value of x, if the series has a limiting sum of $2\sqrt{3} + 3$. (4)

END OF PAPER.

4/12 HSC TASK 1 2012.

Multiple Choice

1) B 2) C 3) C 4) D 5) A.

PART B.

Question 1

a) i) $a = 14 \quad d = -4$ (2)

ii) $T_n = 14 + (n-1)(-4)$
$= 14 - 4n + 4$
$= 18 - 4n$ (2)

iii) $T_{20} = 18 - 4(20)$
$= 18 - 80$
$= -62$ (2)

iv) $S_n = \frac{n}{2}[2a + (n-1)d]$.

$S_n = \frac{n}{2}[28 + (n-1)(-4)]$.

$= \frac{n}{2}[28 - 4n + 4]$.

$= 14n - 2n^2 + 2n$

$= 16n - 2n^2$

$S_n < 0$.

$16n - 2n^2 < 0$

$2n^2 - 16n > 0$

$2n(n-8) = 0$.

$n = 0 \quad n = 8$ (3)

$\therefore$ 8 terms.

b i) $a = 23 \quad d = 11 \quad L = 1453$.

$T_n = 23 + (n-1)(11)$
$= 23 + 11n - 11$
$= 12 + 11n$ (2)

$1453 = 12 + 11n$.

$1441 = 11n$.

$\therefore n = 131$

$\therefore$ There are 131 terms.

ii) $S_{131} = \frac{131}{2}(23 + 1453)$

$= \frac{131}{2} \times 1476$ (2)

$= 96678$.

iii) $T_n > 714$.

$12 + 11n > 714$

$11n > 702$ (2)

$n > 63.8\ldots$

$\therefore$ The 64th term.

c) $0.\dot{4}3 = 0.43 + 0.0043 + 0.000043 + \ldots$.

$a = 0.43 \quad r = 0.01$

$S_\infty = \frac{0.43}{1 - 0.01}$ (2)

$= \frac{0.43}{0.99}$

$= \frac{43}{99}$.

Question 2

a) i) $T_{40} = 353$ $T_{14} = 119$.

$T_{40} = a + 39d$ $T_{14} = a + 13d$.

$a + 13d = 119$ ①

$a + 39d = 353$. ②

③

$26d = 234$

$d = 9$

ii) $a + 13(9) = 119$

$a + 117 = 119$ ②

$a = 2$

b i) $x - 8 = -1 - x$

$2x = 7$ ②

$x = 3 \cdot 5$.

ii) $8, 3 \cdot 5, -1$ $d = -4 \cdot 5$ ②

∴ The next term is $-5 \cdot 5$.

c i) $a = 25$ $d = 15$. $T_n = 550$

$T_n = 25 + (n-1)15$.

$= 25 + 15n - 15$

$= 10 + 15n$

$T_n = 550$ ②

$550 = 10 + 15n$

$15n = 540$

$n = 36$

∴ After 36 weeks.

ii) $S_{36} = \frac{36}{2}[2(25) + 35(15)]$

$= 18(50 + 525)$

$= 18 \times 575$

$= 10350$.

③

52 weeks in the year.

$52 - 36 = 16$.

$16 \times 555 = 8800$

Total customers $10350 + 8800$

$= 19150$

Gross takings $= 19150 \times \$25$

$= \$478,750$.

Question 3

a i) $r = \frac{-6}{3} = -2$ ①

ii) $T_n = ar^{n-1}$ $a = 3$.

$T_8 = 3(-2)^7$ ②

$= -384$

iii) $S_n = \frac{a(r^n - 1)}{r - 1}$

$S_8 = \frac{3((-2)^8 - 1)}{-2 - 1}$ ②

$= \frac{765}{-3}$

$= -255$.

b) $r = \frac{4}{9}$ $\quad a = \frac{1}{3}$.

$$S_\infty = \frac{\frac{1}{3}}{1 - \frac{4}{9}} \quad (2)$$

$$= \frac{\frac{1}{3}}{\frac{5}{9}} = \frac{1}{3} \times \frac{9}{5}.$$

$$= \frac{9}{15} = \frac{3}{5}.$$

c) $p, q, 32$ $\qquad q, 4, p.$

$\frac{q}{p} = \frac{32}{q}$ $\qquad \frac{4}{q} = \frac{p}{4}.$

$q^2 = 32p$ $\qquad 16 = pq$

$q^2 = 32\left(\frac{16}{q}\right)$ $\qquad p = \frac{16}{q}.$

when $q = 8$.

$q^2 = \frac{512}{q}$ $\qquad p = \frac{16}{8}$

$q^3 = 512$

$\therefore q = 8$ $\qquad \therefore p = 2.$ (3)

i) $T_1 = 4$ $\qquad T_8 = 8748$

$T_1 = a$ $\qquad T_8 = ar^7$

$a = 4$ $\qquad ar^7 = 8748$

$4r^7 = 8748$

$r^7 = 2187$

$\therefore r = 3.$

$T_{12} = ar^{11}$

$= 4(3)^{11}$ (3)

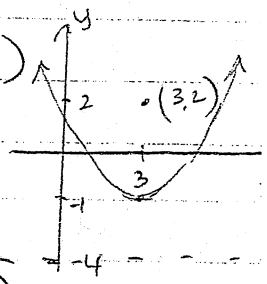
$= 708588.$

Question 4

a) $(x-3)^2 = 12(y+1)$

i) $4a = 12$ (1)

$a = 3.$



ii) Vertex $= (3, -1)$ (1)

iii) Axis of symmetry : $x = 3$. (1)

iv) Directrix : $y = -4$. (1)

v) Focus : $S = (3, 2)$ (1)

vi) x int when $y = 0$.

$(x-3)^2 = 12(0+1)$.

$x^2 - 6x + 9 = 12.$

$x^2 - 6x - 3 = 0.$

$a = 1 \quad b = -6 \quad c = -3.$

$$x = \frac{6 \pm \sqrt{(-6)^2 - 4(1)(-3)}}{2(1)}$$

$$= \frac{6 \pm \sqrt{36+12}}{2}$$

$$= \frac{6 \pm \sqrt{48}}{2}$$

$$= \frac{6 \pm 4\sqrt{3}}{2}$$

$$= 3 \pm 2\sqrt{3}.$$

$\therefore$ x int are $(3+2\sqrt{3}, 0)$, $(3-2\sqrt{3}, 0)$

Question 4 cont.

y int when $x = 0$.

$(0-3)^2 = 12(y+1)$.

$9 = 12y + 12$.

$12y = -3$.

$y = -\frac{1}{4}$.

(4)

$\therefore$ y int is at $(0, -\frac{1}{4})$

vii) S: (3,2) (0,0).

Eqn focal chord:

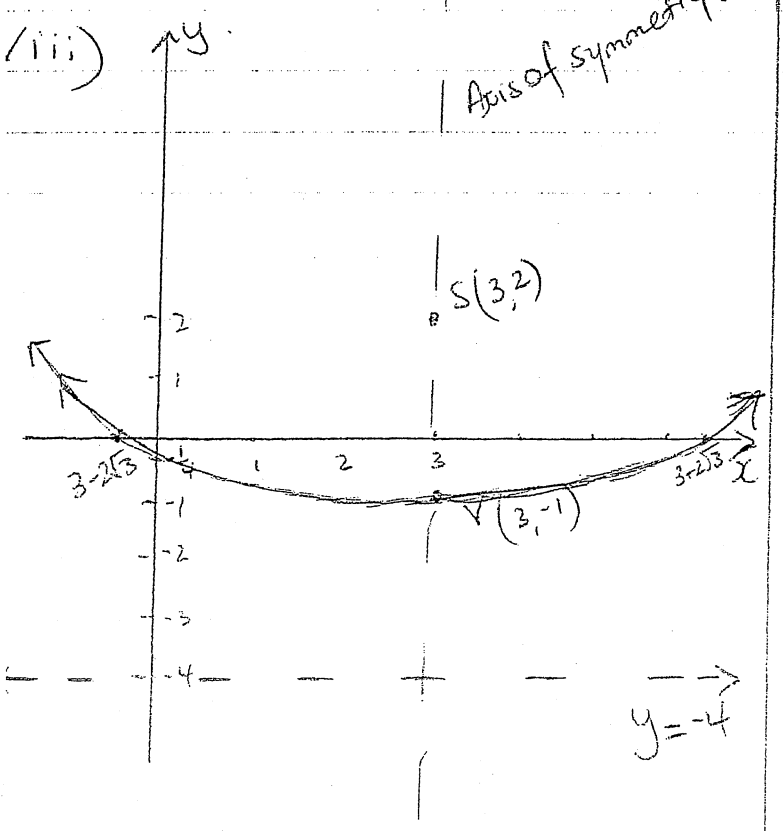
$y - 0 = \frac{2}{3}(x-0)$. (2)

$y = \frac{2}{3}x$.

OR.

$2x - 3y = 0$

(iii)



(4)

b) $y^2 - 6y - 9x - 9 = 0$.

$y^2 - 6y + 9 = 9x + 9 + 9$ (2)

$(y-3)^2 = 9(x+2)$.

Vertex: $(-2, 3)$.

c) $x^2 = 6y$ $x = 3$.

$y = \frac{x^2}{6}$.

$\frac{dy}{dx} = \frac{2x}{6} = \frac{x}{3}$.

When $x = 3$ $\frac{dy}{dx} = 1$.

$y = \frac{3^2}{6} = \frac{9}{6}$

$= \frac{3}{2}$.

Point $(3, \frac{3}{2})$ $m = 1$.

Eqn tagent:

$y - \frac{3}{2} = 1(x-3)$ (3)

$2y - 3 = 2(x-3)$

$2y - 3 = 2x - 6$

$2x - 2y - 3 = 0$.

Question 5

a i) $PA = PB$ $A(-5,0)$ $B(-3,y)$

$PA^2 = PB^2$

$(x+5)^2 + (y-0)^2 = (x+3)^2 + (y-y)^2$

$x^2 + 10x + 25 + y^2 = x^2 + 6x + 9$

$y^2 + 4x + 16 = 0$ (2)

ii) $PA \perp PB$ $A(4,2)$ $B(-1,2)$

$m_{PA} = \frac{y-2}{x-4}$ $m_{PB} = \frac{y-2}{x+1}$

$\frac{y-2}{x-4} \times \frac{y-2}{x+1} = -1$

$(y-2)^2 = -(x-4)(x+1)$ (3)

$y^2 - 4y + 4 = -x^2 + 3x + 4$

$\therefore x^2 - 3x + y^2 - 4y = 0$

ii) $PB = 2$ $B(3,4)$

$PB^2 = 4$

$(x-3)^2 + (y-4)^2 = 4$ (2)

b i) $PA^2 = 4 \times PB^2$ $A(5,0)$ $B(2,0)$

$(x-5)^2 + (y-0)^2 = 4[(x-2)^2 + (y-0)^2]$

$2 - 10x + 25 + y^2 = 4[x^2 - 4x + 4 + y^2]$

$c^2 - 10x + 25 + y^2 = 4x^2 - 16x + 16 + 4y^2$

$3x^2 - 6x + 3y^2 - 9 = 0$

$x^2 - 2x + y^2 - 3 = 0$

$x^2 + y^2 - 2x - 3 = 0$ (2)

ii) $x^2 + y^2 - 2x - 3 = 0$

$x^2 - 2x + 1 + y^2 = 3 + 1$

$(x-1)^2 + y^2 = 4$

$\therefore$ Centre $(1,0)$

Question 6

a) $a = 100$ $r = 0.7$

$S_n = \frac{a(1-r^n)}{1-r}$

$S_8 = \frac{100 \cdot (1 - (0.7)^8)}{1 - 0.7} + 420$ (2)

$= 314.11\ldots\ldots + 420$

$\therefore$ The plant grows to, 734mm.

b) $r = \frac{2}{3}$ $S_\infty = \frac{3}{2}$.

$\frac{3}{2} = \frac{a}{1 - \frac{2}{3}}$

$\frac{3}{2} = a \times 3$.

$\frac{3}{2} = 3a$. (2)

$3 = 6a$.

$\therefore a = \frac{1}{2}$.

c) common ratio $= (r+2)$.

limiting sum $|r| < 1$

$|r+2| < 1$.

$-1 < r+2 < 1$. (2)

$\therefore -3 < r < -1$.

d) $r = (1-x)^{\frac{1}{2}}$.

$|r| < 1$.

$|(1-x)^{\frac{1}{2}}| < 1$ (2)

$0 < (1-x)^{\frac{1}{2}} < 1$

$0 < (1-x) < 1$

$0 > x - 1 > 1$

$1 > x > 0$.

$\therefore 0 < x < 1$

Question 6 cont.

c ii) $a = \sqrt{1-x}$

$r = \sqrt{1-x}$.

$$\frac{\sqrt{1-x}}{1-\sqrt{1-x}} = 2\sqrt{3}+3$$

$$\sqrt{1-x} = (2\sqrt{3}+3)(1-\sqrt{1-x})$$

$$\sqrt{1-x} = 2\sqrt{3} - 2\sqrt{3}(\sqrt{1-x}) + 3 - 3\sqrt{1-x}$$

$$2\sqrt{3}(\sqrt{1-x}) + 3\sqrt{1-x} + \sqrt{1-x} = 2\sqrt{3}+3.$$

$$(4+2\sqrt{3})(\sqrt{1-x}) = 2\sqrt{3}+3.$$

$$\sqrt{1-x} = \frac{2\sqrt{3}+3.}{4+2\sqrt{3}.}$$

$$\sqrt{1-x} = \frac{2\sqrt{3}+3}{4+2\sqrt{3}} \times \frac{4-2\sqrt{3}.}{4-2\sqrt{3}.}$$

$$\sqrt{1-x} = \frac{(2\sqrt{3}+3)(4-2\sqrt{3})}{16-12.}$$

$$\sqrt{1-x} = \frac{8\sqrt{3} - 12 + 12 - 6\sqrt{3}}{4}$$

$$\sqrt{1-x} = \frac{2\sqrt{3}}{4}$$

(4)

$$\sqrt{1-x} = \frac{\sqrt{3}}{2.}$$

$$1-x = \frac{3}{4}$$

$$\therefore x = \frac{1}{4}.$$