# GIRRAWEEN HIGH SCHOOL

## MATHEMATICS

**YEAR 12 HSC** **Task 1, 2015**

**Time Allowed: 90 minutes** **Name:** _____

Instructions: Examiner: C. McMillan

- Attempt all questions. Write using **blue** or **black** pen **only**
- Circle the best response for the questions in Part A
- Start each question in Part B on a new page
- All necessary working must be shown
- Board approved calculators can be used.
- Marks may be deducted for careless or badly arranged work

## PART A (5 marks)

**For questions 1-5 circle the best response from the following:**

**Question 1:** The probability that a person selected at random has a gene linked with a particular disease is 0.12. Of people with that gene, the probability that they will get that disease in their lifetime is 0.4. What is the probability that a person selected at random has the gene but will not get the disease?

A) 0.072 B) 0.48 C) 0.72 D) 0.6

**Question 2:** The limiting sum of the series $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+....$ is?

A) $\frac{1}{2}$ B) 1 C) $1\frac{1}{2}$ D) 2

**Question 3:** The roots of the quadratics equation $ax^2 - bx - 10 = 0$ are -1 and 5. The values of *a* and *b* are:

A) $a = -2, b = -8$ B) $a = 2, b = 8$ C) $a = 2, b = 6$ D) $a = -2, b = 6$

**Question 4:** The parabola with equation $x^2 = -8(y-2)$ has its focus at:

A) (0, -2) B) (0, -4) C) (-2, 0) D) (0, 0)

**Question 5:** If events *A* and *B* are mutually exclusive and $P(A) = \frac{2}{5}$ and $P(B) = \frac{3}{10}$ then

*P(A or B)* =

A) $\frac{9}{10}$ B) $\frac{9}{20}$ C) $\frac{7}{10}$ D) $\frac{13}{20}$

## PART B

**Question 6 (21 marks)**

**(a)** A box contains 10 unlabelled CDs, 2 of which are blank and the remainder used. Jack chooses one at random, puts it aside and then chooses another.

i) Draw a probability tree diagram to show all possible outcomes. (2)

ii) Find the probability that both CDs are blank. (2)

iii) Find the probability that at least one CD is blank. (2)

**(b)** In a group of 50 students, 30 study Mathematics, 25 study Physics and 20 study both Mathematics and Physics.

i) Draw a Venn diagram to represent this information. (2)

ii) How many students study Mathematics but not Physics? (1)

iii) If a student is selected at random what is the probability they only study Mathematics or only study Physics? (2)

iv) What would be the probability they study neither Mathematics or Physics? (2)

**(c)** A President and Vice-President are to be randomly chosen from the members of a committee. If the committee consists of 5 women and 7 men, what is the probability that:

i) the President is a woman? (1)

ii) the Vice-President is a woman? (2)

iii) both the President and Vice-President are women? (2)

**(d)** Of two coins A and B, A is a fair coin while B is loaded with a probability of 0.6 for heads. A coin is chosen at random and tossed twice. What is the probability of tossing two heads? (3)

**Question 7 (16 marks)**

**(a)** Evaluate $\sum_{n=1}^{6}(5n+3)$ : (3)

**(b)** For the series 2+19+36+53+.....+444:

i) Find the common difference (1)

ii) Find the $8^{th}$ term (2)

iii) Find the number of terms in the series (2)

iv) Find the sum of the series (2)

**(c)** The first term of a geometric series is 8000 and the second term is 7200. Find:

(i) the common ratio. (2)

(ii) the $4^{th}$ term. (2)

(iii) the sum of the first 10 terms. (2)

**Question 8 (16 marks)**

**(a)** $\alpha$ and $\beta$ are the roots of the equation $x^2+4x+1=0$ .

(i) $\alpha+\beta$ (1)

(ii) $\alpha\beta$ (1)

(iii) $\frac{1}{\alpha}+\frac{1}{\beta}$ (2)

(iv) $\alpha^2+\beta^2$ (2)

(v) $(\alpha+2)(\beta+2)$ (3)

**(b)** Find the value of $k$ for the equation $x^2-(k+2)x+4k=0$ if one root is 2 more than the other root. (4)

**(c)** Find $a$, $b$ and $c$ if: $x^2 \equiv a(x+1)^2+bx+c$ . (3)

**Question 9 (12 marks)**

**(a)** Find the locus of a point $P(x, y)$ that moves such that $PA=PB$ where $A(2, 4)$ and $B(2, -6)$. (3)

**(b)** Given $A(4, 2)$ and $B(-2, -8)$, $P(x, y)$ moves so that the $\angle APB$ is a right angle. Show that the locus of $P$ is a circle, giving its centre and radius. (3)

**(c)** For the parabola $(x-1)^2 = 4(y+2)$, find:

(i) the vertex. (1)
(ii) the focus. (1)
(iii) the equation of the directrix. (1)
(iv) the axis of symmetry. (1)

**(d)** Find the equation of the parabola with vertex $(2, -1)$ and focus $(1, -1)$. (2)

**Question 10 (17 marks)**

**(a)** On the first day of harvest an orchid produces 560kg of fruit. On the next day, the orchard produces 543kg and the amount produced continues to decrease by the same amount each day.

(i) How much fruit is produced on the 14th day of the harvest? (2)
(ii) What is the total amount of fruit that is produced in the first 14 days of harvest? (2)
(iii) On what day does the daily production first fall below 60kg? (3)

**(b)** Using the limiting sum express $0\dot{3}\dot{2}$ as a fraction. (2)

**(c)** A girl tosses a die until she gets a 6.

(i) What is the probability that she first gets a 6 on the 4th toss? (2)
(ii) What is the probability that she first gets a 6 on the 1st, 2nd, 3rd or 4th toss? (3)

**(d)** For what values of m does the series $1 - \frac{m^2}{4} + \frac{m^4}{16} - \frac{m^6}{64} + \ldots$ have a limiting sum? (3)

**Question 11 (19 marks)**

**(a)**

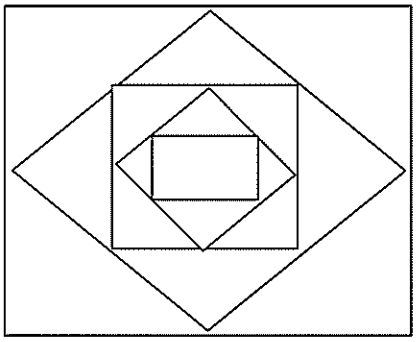
(i) Find the locus of a point which moves so that it is equidistant from the point $(0, 1)$ and the line $y = -1$. (2)

(ii) Find the co-ordinates of the points $A$ and $B$ where the line $y=1$ meets the locus. (2)

(iii) Show that the tangents at A and B are perpendicular. (3)

**(b)** The roots of $x^2 - (k + 5)x + 4k = 0$ are such that their product is 2 more than twice their sum. Find the value of $k$. (3)

**(c)** A courtyard is paved, and a pattern is created, with bricks. The pattern consists of 8 rectangles and 7 rhombuses positioned as shown in the diagram.



The first rectangle consists of 7 bricks and each subsequent rectangle uses twice as many as the previous one. The first rhombus uses 10 bricks and each subsequent rhombus uses twice as many as the previous one.

(i) How many bricks are in the $8^{th}$ rectangle? (2)

(ii) How many bricks are used in all the rhombuses? (3)

(iii) How many bricks are used altogether in the pattern? (4)

**END OF PAPER.**

# SOLUTIONS TASK 1 HSC 2015 2U

## PART A

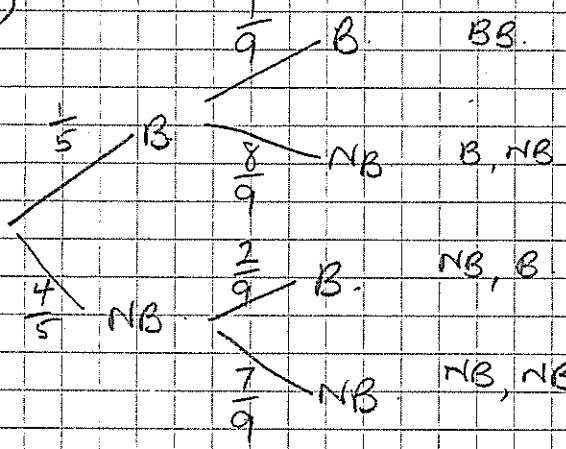
Q1 A Q2 D Q3 B Q4 D.

Q5 C

## PART B

### Question 6

a i)

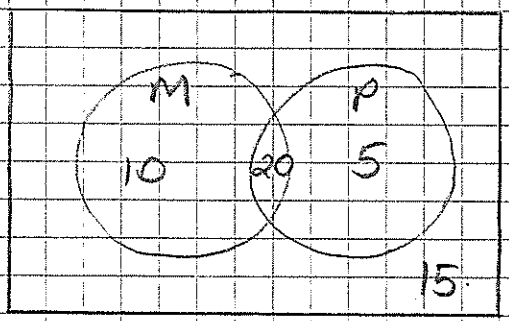



ii) $P(B,B) = \frac{1}{5} \times \frac{1}{9} = \frac{1}{45}$

iii) $P(\text{at least one is blank}) = 1 - P(NB, NB)$

$= 1 - \frac{28}{45} = \frac{17}{45}$.

b i)




ii) 10.

iii) $P(\text{Only M or P}) = \frac{10}{50} + \frac{5}{50}$

$= \frac{15}{50} = \frac{3}{10}$.

iv) $P(\text{study neither}) = \frac{15}{50} = \frac{3}{10}$.

c i) $P(\text{Prac. is W}) = \frac{5}{12}$.

ii) $P(\text{V.P is W}) = \left(\frac{5}{12} \times \frac{4}{11}\right) + \left(\frac{7}{12} \times \frac{5}{11}\right)$

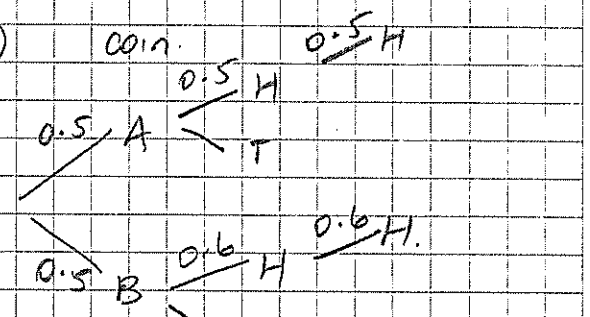
$= \frac{20}{132} + \frac{35}{132}$

$= \frac{55}{132} = \frac{5}{12}$

iii) $P(\text{Both W}) = \frac{5}{12} \times \frac{4}{11}$

$= \frac{20}{132} = \frac{5}{33}$.

d)




$\therefore P(HH) = (0.5)^3 + 0.5(0.6)^2$

$= 0.305$

### Question 7

a) $\sum_{n=1}^{6} (5n+3)$

$= (5+3) + (10+3) + (15+3)$

$+ (20+3) + (25+3) + (30+3)$

$= 8 + 13 + 18 + 23 + 28 + 33$

$= 123$.

b i) $d = 19 - 2$

$= 17$.

ii) $T_8 = 2 + 7(17)$

$= 121$

Question 7 cont.

iii) $T_n = 444$

$444 = 2 + (n-1)17$

$444 = 2 + 17n - 17$

$459 = 17n$.

$\therefore n = 27$

iv) $S_{27} = \frac{27}{2}(2 + 444)$.

$= \frac{27}{2}(446)$

$= 27(223)$

$= 6021$

c) $a = 8000 \quad T_2 = 7200$

i) Common ratio $= \frac{7200}{8000}$

$= \frac{9}{10}$.

ii) $T_4 = ar^3$

$= 8000\left(\frac{9}{10}\right)^3$

$= 5832$.

iii) $S_{10} = \frac{a(1-r^n)}{1-r}$

$= \frac{8000(1 - 0.9^{10})}{1 - 0.9}$

$= 52105.72479$

Question 8

a) $x^2 + 4x + 1 = 0$.

i) $\alpha + \beta = -4$.

ii) $\alpha\beta = 1$

iii) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta}$.

$= \frac{-4}{1} = -4$

iv) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$.

$= (-4)^2 - 2(1)$

$= 16 - 2 \quad = 14$

v) $(\alpha + 2)(\beta + 2)$

$= \alpha\beta + 2\alpha + 2\beta + 4$

$= \alpha\beta + 2(\alpha + \beta) + 4$

$= 1 + 2(-4) + 4$

$= 1 - 8 + 4$

$= -3$.

Question 8 cont.

b) Let one root be $\alpha$, and the other $\alpha+2$.

$(\alpha+2)+\alpha = \frac{-b}{a}$ $\quad \alpha(\alpha+2) = \frac{c}{a}$

$2\alpha+2 = \frac{(k+2)}{1}$ $\quad \alpha^2+2\alpha = \frac{4k}{1}$

$2\alpha+2 = k+2$ $\quad \left(\frac{k}{2}\right)^2+2\left(\frac{k}{2}\right) = 4k$

$k = 2\alpha$

$\alpha = \frac{k}{2}$

$\frac{k^2}{4}+k = 4k$

$\frac{k^2}{4} = 3k$

$k^2 = 12k$

$k^2-12k = 0$

$k(k-12) = 0$

$\therefore k = 0, 12.$

$\therefore k = 12, k = 0.$

c) $x^2 \equiv a(x+1)^2+bx+c$

$RHS = ax^2+2ax+a+bx+c$

$= ax^2+(2a+b)x+a+c.$

$\therefore a = 1.$

$2a+b = 0.$

$2+b = 0$

$b = -2.$

$a+c = 0.$

$1+c = 0.$

$c = -1.$

$\therefore a = 1, \ b = -2, \ c = -1.$

Question 9.

a) $PA = PB.$

$PA^2 = PB^2$

$(x-2)^2+(y-4)^2 = (x-2)^2+(y+6)^2.$

$y^2-8y+16 = y^2+12y+36$

$-20y+20 = 0.$

$y+1 = 0.$

$\therefore y = 1.$

b) $m_{PA} \times m_{PB} = -1$

$m_{PA} = \frac{y-2}{x-4}$ $\quad m_{PB} = \frac{y+8}{x+2}$

$\left(\frac{y-2}{x-4}\right) \times \left(\frac{y+8}{x+2}\right) = -1.$

$(y-2)(y+8) = -(x-4)(x+2)$

$y^2+6y-16 = -x^2+2x+8$

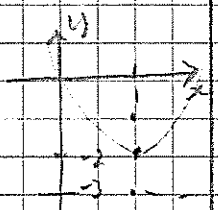
$x^2-2x+y^2+6y-24 = 0.$

$(x-1)^2+(y+3)^2 = 34$

$\therefore$ locus is a circle centre $(1,-3)$ and radius $\sqrt{34}$

Question 9 cont.

c) $(x-1)^2 = 4(y+2)$



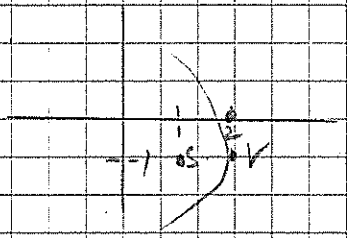

i) Vertex: $(1, -2)$

ii) Focus: $S(1, -1)$

iii) Directrix: $y = -3$.

iv) Axis of Symmetry: $x = 1$.

d) Vertex $(2, -1)$ $S(1, -1)$




$(y+1)^2 = -4(x-2)$

Question 10.

a) $a = 560$ $T_2 = 543$ $d = -17$.

i) $T_{14} = 560 + (13)(-17)$

$= 339$.

ii) $S_{14} = \frac{14}{2}(560 + 339)$

$= \frac{14}{2}(899)$

$= 6293$.

iii) $T_n < 60$

$560 + (n-1)(-17) < 60$

$560 - 17n + 17 < 60$.

$577 - 17n < 60$

$-17n < -517$

$n > 30.41\ldots$

∴ On the 31st day.

b) $0.3232\ldots$

$0.32 + 0.0032 + \ldots$

$a = 0.32$ $r = 0.01$.

$S_\infty = \frac{0.32}{1 - 0.01}$.

$= \frac{0.32}{0.99}$

$= \frac{32}{99}$

c) i) $P(\text{6on 4H 2Toss})$

$= \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}$.

$= \frac{125}{1296}$

ii) $P(1^{st}, 2nd, 3rd \text{ or } 4th)$

$= \frac{1}{6} + \left(\frac{5}{6} \times \frac{1}{6}\right) + \left(\frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}\right)$

$+ \left(\frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}\right)$

$= \frac{1}{6} + \frac{5}{36} + \frac{25}{216} + \frac{125}{1296}$

$= \frac{671}{1296}$

d) $a = 1$ $r = -\frac{m^2}{4}$.

$S_\infty \Rightarrow |r| < 1$.

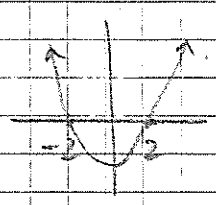
$\left|-\frac{m^2}{4}\right| < 1$.

$\frac{m^2}{4} < 1$

$m^2 < 4$

$m^2 - 4 < 0$

∴ $-2 < m < 2$.



Question 11

a) i) $(0,1)$ $y=-1 \Rightarrow (x,-1)$

$\therefore (x-0)^2+(y-1)^2=(x-x)^2+(y+1)^2$

$x^2+y^2-2y+1=y^2+2y+1$

$\therefore x^2=4y$

ii) $y=1$ $x^2=4y$.

$\therefore x^2=4(1)$.

$x^2=4$.

$x=\pm 2$

$\therefore A(2,1)$ $B(-2,1)$

iii) $x^2=4y$ $\therefore y=\frac{x^2}{4}$

$y'=\frac{x}{2}$

When $x=2$ $y'=1$

When $x=-2$ $y'=-1$

$\therefore m_{tan\alpha} \times m_{tan\beta}=1$.

$\therefore$ tan A is perpendicular tan B.

b) $x^2-(k+5)x+4k=0$

$\alpha\beta=2(\alpha+\beta)+2$ $\alpha\beta=4k$

$\therefore 4k=2k+10+2$. $\alpha+\beta=k+5$

$4k=2k+12$.

$2k=12$

$\therefore k=6$.

c) 8 rectangles, 7 rhombuses

1st rectangle - 7 bricks.

$a=7$ $r=2$.

1st rhombus - 10 bricks.

$a=10$ $r=2$.

i) $T_8=(7)2^7$

$=896$

$\therefore$ 896 bricks are used in the 8th rectangle

ii) $S_7=\frac{10(2^7-1)}{2-1}$

$=1270$

$\therefore$ 1270 bricks are used in the rhombuses.

iii) $S_8=\frac{7(2^8-1)}{2-1}$

$=1785$

$\therefore$ Total number of bricks $=1270+1785$

$=3055$.

$\therefore$ 3055 bricks are used in total.