Girraween High School

Year 12 Mathematics Task 1

November 2016

General Instructions

- Working Time - 1 hour & 30 minutes
- Calculators and ruler may be used
- All necessary working out must be shown

Total Marks - 70

- Attempt all questions
- Marks may be deducted for careless or badly arranged work

Question 1 (1 mark)

Two six sided dice are thrown. What is the probability that the numbers appearing are the same?

A. $\frac{1}{6}$ B. $\frac{1}{36}$ C. $\frac{1}{2}$ D. $\frac{1}{4}$

Question 2 (1 mark)

A geometric sequence with common ratio r will have a limiting sum if:

A. $r < 1$ B. $r > 1$ C. $|r| < 1$ D. $|r| > 1$

Question 3 (1 mark)

The fourth term of an arithmetic series is 27 and the seventh term is 12. What is the common difference?

A. 5 B. -5 C. 13 D. -13

Question 4 (1 mark)

The parabola with equation $(x-3)^2 = 12y$ has:

A. vertex at $(0, -3)$ and focus at $(3, -3)$

B. vertex at $(-3, 0)$ and focus at $(-3, 3)$

C. vertex at $(3, 0)$ and focus at $(3, 3)$

D. vertex at $(0, 3)$ and focus at $(3, 3)$

Question 5 (1 mark)

A bag contains 4 red marbles and 6 blue marbles. Three marbles are selected at random without replacement. What is the probability that at least one of the marbles selected is red?

A. $\frac{1}{6}$ B. $\frac{1}{2}$ C. $\frac{5}{6}$ D. $\frac{29}{30}$

The exam continues on the next page

Question 6 (11 marks)

(a) For the arithmetic sequence: 58, 53, 48, ...

i. Find the values of a and d. [2]

ii. Find the expression for the n^{th} term. [2]

iii. Find the 30^{th} term. [1]

iv. Find the first negative term in the sequence. [2]

(b) Consider the sequence 6, 11, 16, ..., 211.

i. Determine the number of terms in this sequence. [2]

ii. Find the value of $6 + 11 + 16 + \cdots + 211$ [2]

Question 7 (10 marks)

(a) The 3^{rd} term of a geometric sequence is 8 and the 7^{th} term is 128.

i. Find the 1^{st} term and the common ratio. [2]

ii. Find the sum of the first 20 terms. [1]

(b) Using a limiting sum, express $0.\dot{2}\dot{5}$ as a fraction. [2]

(c) For what values of x does the series $1 - 2x + 4x^2 - 8x^3 + \cdots$ have a limiting sum? [2]

(d) A ball is dropped from a height of $8m$. At subsequent rebounds, it rises to a height of 80% of the preceding attained height. Find the total distance travelled (up and down) by the ball. [3]

Question 8 (12 marks)

For the parabola: $(x + 2)^2 = 8(y + 4)$

i. Find the focal length [1]

ii. Find the coordinate of the vertex [1]

iii. Find the equation of the axis of symmetry [1]

iv. Find the equation of the directrix [1]

v. Find the x and y intercepts of the parabola [2]

vi. Find the equation of the focal chord which passes through the point $(2, 5)$ [2]

vii. Sketch the parabola showing the vertex, directrix, axis of symmetry, focus and the intercepts [4]

The exam continues on the next page

Question 9 (8 marks)

(a) Given that a parabola has its focus at $(1, -1)$ and its directrix is given by $y = 3$, find:

i. the focal length [1]

ii. the equation of the parabola [2]

(b) A parabola has equation $x = \frac{y^2}{12} + \frac{y}{6} - \frac{23}{12}$. Find:

i. the coordinate of its vertex [3]

ii. the coordinate of its focus [1]

iii. the equation of the directrix [1]

Question 10 (7 marks)

(a) Find the locus of a point $P(x, y)$ such that P is equidistant from the x-axis and the point $F(0, -3)$. [3]

(b) The locus of a point $P(x, y)$ which moves such that PA is always perpendicular to PB where $A(3, 4)$ and $B(-1, 2)$ is a circle. Find the centre and radius of this circle. [4]

Question 11 (9 marks)

(a) Two six sided dice are thrown, find that probability that:

i. the sum of the numbers on the two dice is 6 [1]

ii. the sum of the numbers on the two dice is less than 6 [1]

(b) In a raffle, 20 tickets are sold and there are 2 prizes, first and second prize.

i. What is the probability that someone buying 5 tickets wins the first prize? [1]

ii. What is the probability that someone buying 5 tickets wins at least one prize? [2]

(c) Out of 27 students, 18 play volleyball and 12 play basketball.

i. By drawing a venn diagram, state how many students play both sports. [3]

ii. If a student is chosen at random, what is the probability that the student plays either sport, but not both? [1]

The exam continues on the next page

Question 12 (8 marks)

(a) A card is drawn from a standard deck of cards, and a six sided die is thrown.

i. By drawing a probability tree, find the probability that the card is a heart and the number on the die is greater than 3. [2]

ii. Find the probability that the number on the card matches with the number on the die. (assuming ace corresponds to 1). [2]

(b) Pam and Quin are playing a turn based game. The probability of Pam winning on any given turn is p and probability of Quin winning on any given turn is q. The probability of any given turn ending in a draw is r. The game continues until a winner is determined and Pam takes the first turn.

i. Find the probability that Pam wins on her first or second turn. [1]

ii. Show that Pam's probabilty of winning this game is given by $\dfrac{p}{(p+q)(2-p-q)}$. [3]

End of exam

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MC: A, C, B, C, C.

Q1

$P = 6 \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{6}$ ∴ (A)

Q2

$|r| < 1$ ∴ (C)

Q3

$a + 3d = 27$

$a + 6d = 12$

∴ $3d = -15$ ∴ $d = -5$ ∴ (B)

Q4

∴ (C)

Q5

$= 1 - P(\text{all blue})$

$= 1 - \left(\frac{6}{10} \times \frac{5}{9} \times \frac{4}{8}\right) = \frac{5}{6}$ ∴ (C)

Q6

(a)

(i) $a = 58 \quad d = -5$

(ii) $T_n = 58 - 5(n-1)$

$= 58 - 5n + 5$

$T_n = 63 - 5n$

(iii)

$T_{30} = 63 - 5 \times 30$

$= -87$

(iv) $63 - 5n < 0$

$5n > 63$

$n > \frac{63}{5}$

$n > 12.6$

∴ $n = 13$

$T_{13} = 63 - 5 \times 13$

$= -2$

(b)

(i) $T_n = a + (n-1)d$.

$211 = 6 + 5(n-1)$

$211 = 6 + 5n - 5$

∴ $5n = 210$

∴ $n = 42$

∴ 42 terms in sequence.

(ii) $S_{42} = \frac{42}{2}(6 + 211)$

$= 4557$

Q7

(a) $8 = ar^2$

$128 = ar^6$

(i) $\therefore r^4 = \frac{128}{8} = 16$

$\therefore r = \pm 2$

$\therefore a = 2$

(ii) $S_{20} = \frac{2(2^{20}-1)}{2-1}$ or $S_{20} = \frac{2(1-2^{20})}{1+2}$

$= 2097150$ $= -699050$

(b) $0.\dot{2}\dot{5} = 0.2525\overline{25}\ldots$

$= 0.25 + 0.0025 + 0.000025 + \ldots$

$S_\infty = \frac{0.25}{1-0.01} = \frac{25}{99}$

(c)

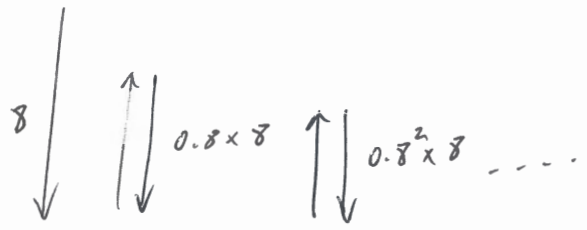
$r = -2n$

$\therefore -1 < -2n < 1$

$\therefore \frac{1}{2} > n > -\frac{1}{2}$

$\therefore -\frac{1}{2} < n < \frac{1}{2}$.

(d)



$D = 8 + 2\times0.8\times8 + 2\times0.8^2\times8 + 2\times0.8^3\times8 + \ldots$

$= 8 + 16(0.8 + 0.8^2 + 0.8^3 + \ldots)$

$= 8 + 16\left(\frac{0.8}{1-0.8}\right)$

$= 72m$

Q8

$(x+2)^2 = 8(y+4)$.

i. $4a = 8 \quad \therefore a = 2$

ii. $V = (-2, -4)$

iii. $x = -2$

iv. $y = -6$.

v. $y = 0$ when

$(x+2)^2 = 32$.

$\therefore x+2 = \pm 4\sqrt{2}$.

$\therefore x = -2 \pm 4\sqrt{2}$

$\therefore$ x-intercepts are $(-2+4\sqrt{2}, 0)$ & $(-2-4\sqrt{2}, 0)$

$x = 0$ when

$4 = 8(y+4)$

$\therefore y+4 = \frac{1}{2} \quad \therefore y = -3\frac{1}{2}$.

$\therefore$ y-intercept is $(0, -3\frac{1}{2})$.

vi. $F = (-2, -2)$

$m = \frac{5+2}{2+2} = \frac{7}{4}$.

$y+2 = \frac{7}{4}(x+2)$

$4y + 8 = 7x + 14$

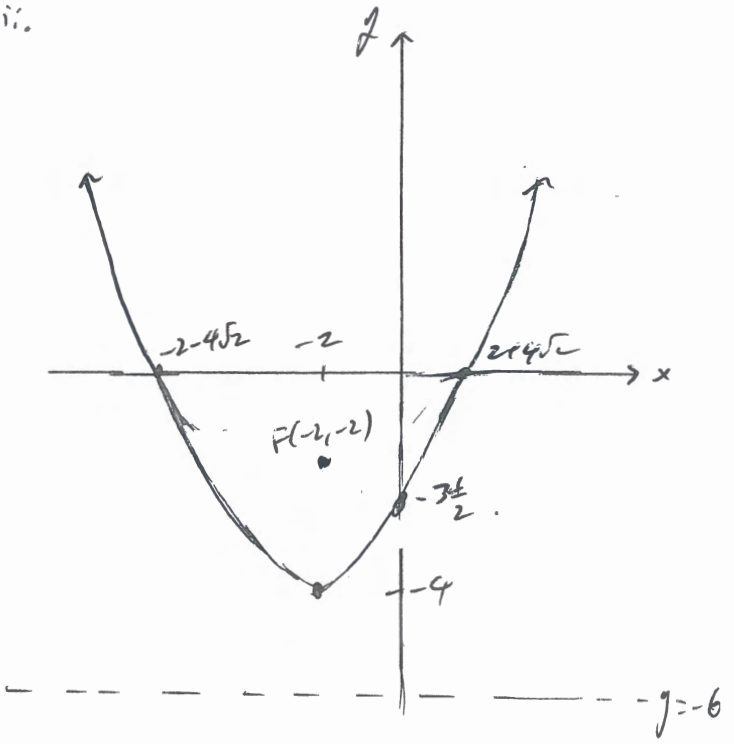
$7x - 4y + 6 = 0$.

or

$4y = 7x + 6$

$y = \frac{7}{4}x + \frac{3}{2}$.

vii.



Q9

(a) $y=3$

(i) $2a=4 \quad \therefore a=2.$

(ii) $(x-1)^2=-8(y-1)$

(b)

(i) $x=\frac{y^2}{12}+\frac{y}{6}-\frac{23}{12}$

$12x=y^2+2y-23$

$12x+23=y^2+2y$

$12x+23+1=y^2+2y+1$

$12x+24=(y+1)^2$

$12(x+2)=(y+1)^2$

$\therefore V=(-2,-1)$

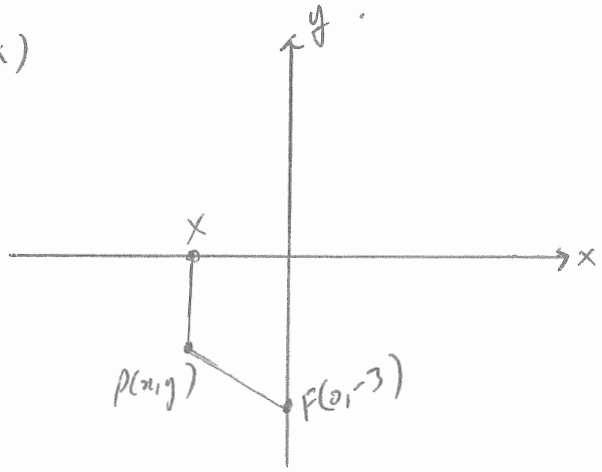
(ii) $a=3$

$\therefore F=(1,-1)$

(iii) $x=-5$

Q10

(a)



$PX=PF$

$\therefore -y=\sqrt{x^2+(y+3)^2}$

$y^2=x^2+(y+3)^2$

$x^2=y^2-(y+3)^2$

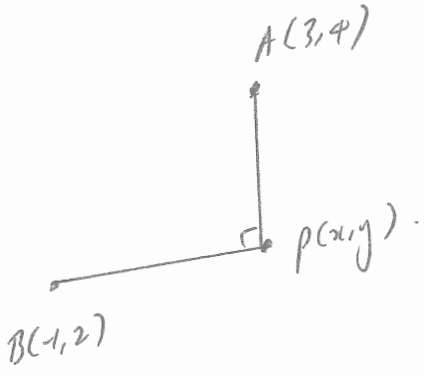
$x^2=(y+y+3)(y-y-3)$

$x^2=-3(2y+3)$

$x^2=-6\left(y+\frac{3}{2}\right).$

Q10

(b)



$$m_{PA} = \frac{y-4}{x-3} \qquad m_{PB} = \frac{y-2}{x+1}$$

$$m_{PA} \times m_{PB} = -1$$

$$\therefore \frac{y-4}{x-3} \times \frac{y-2}{x+1} = -1.$$

$$\therefore (y-4)(y-2) = (3-x)(x+1)$$

$$y^2 - 2y - 4y + 8 = 3x + 3 - x^2 - x$$

$$x^2 - 2x + y^2 - 6y = -5$$

$$x^2 - 2x + 1 + y^2 - 6y + 9 = -5 + 1 + 9$$

$$(x-1)^2 + (y-3)^2 = 5$$

$$\therefore C = (1,3) \quad r = \sqrt{5}.$$

Q11

(a)

(i)

	1	2	3	4	5	6
1						X
2					X	
3				X		
4			X			
5		X				
6						

$$\therefore p = \frac{5}{36}$$

(ii) $$p = \frac{10}{36} = \frac{5}{18}$$

(b)

(i) $$\frac{5}{20} = \frac{1}{4}.$$

(ii) $$p = 1 - p(\text{no prize})$$

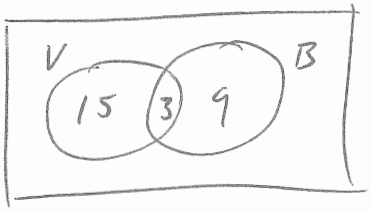
$$= 1 - \frac{15}{20} \times \frac{14}{19}$$

$$= \frac{17}{38}$$

(c)

(i) $$18 - x + x + 12 - x = 27$$

$$\therefore x = 3 \qquad \therefore 3 \text{ plays both.}$$

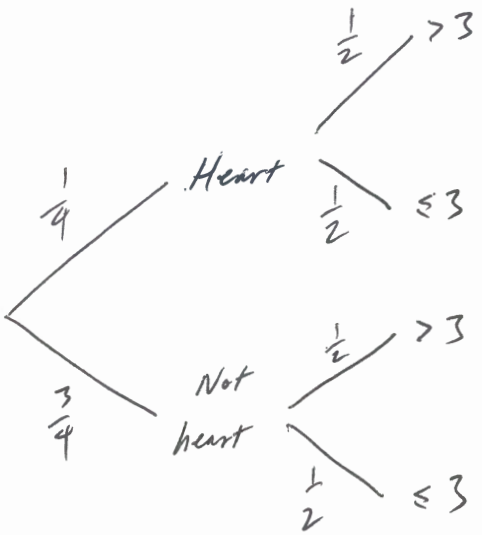


(ii) $$p = \frac{15+9}{27} = \frac{24}{27.} = \frac{8}{9}$$

Q12

(a)

(i)



$P(\text{heart} \ \& > 3) = \frac{1}{8}$

(ii) $P(\text{ace} \ \& \ 1) = \frac{4}{52} \times \frac{1}{6}$

$\therefore P(\text{same number}) = 6 \times \frac{1}{6} \times \frac{4}{52}$

$= \frac{1}{13}$

(b)

(i)

$p = p + r^2 p$

(ii)

$P_{win} = p + r^2 p + r^4 p + r^6 p + \dots$

$= p(1 + r^2 + r^4 + r^6 + \dots)$

$= p \times \frac{1}{1 - r^2}$

$= \frac{p}{1 - r^2}$

But $p + q + r = 1$

$\therefore r = 1 - p - q$

$\therefore 1 - r^2 = (1 - r)(1 + r)$

$= (p + q)(2 - p - q)$.

$\therefore P_{win} = \frac{p}{(p+q)(2-p-q)}$.