GOSFORD HIGH SCHOOL

2008 HIGHER SCHOOL CERTIFICATE

MATHEMATICS ASSESSMENT TASK #1

DECEMBER 2007

Time Allowed – 70 minutes

All necessary working should be shown.

Full marks may not be awarded for unnecessarily untidy work or work that is poorly organized.

Students must begin each new question on a new page.

Students need to place their name and/or HSC candidate number at the top of each new page.

Questions will be collected separately at the conclusion of the assessment task.

All questions are to be attempted.

Question 1 (12 marks)

(a) Find $\frac{d}{dx}\left[x^3 - 4x^2 + 12\right]$ (1)

(b) Find $\lim_{x \to 0}\left[\frac{x}{x^2 - 2x}\right]$ (2)

(c) Write a quadratic equation whose roots are $1 \pm \sqrt{3}$. (2)

(d) A population (P) is increasing but at a decreasing rate.
Describe the signs of $\frac{dP}{dt}$ and $\frac{d^2P}{dt^2}$ where t is time. (2)

(e) Find the equation of the locus of a point P (x, y) which moves so that it is equidistant from the point $(0,6)$ and the line $y = -6$. (1)

(f) Find the primitive function of $4x^3 - 6x^2 + x$. (2)

(g) It is given that a stationary point occurs at $x = 0$ on a continuous curve with $f''(x) = x^2(x-2)(x-4)$. Determine the nature of the stationary point at $x = 0$. (2)

Question 2 (12 marks)

(a) Find $f'(x)$ if $f(x) = x\sqrt{x} - \frac{3}{x^2}$ (3)

(b) Find $\frac{dy}{dx}$ if $y = \frac{2x-1}{x^2+1}$ (3)

(c) If α and β are the roots of the quadratic equation $2x^2 - 6x + 7$

(i) find the value of $\alpha + \beta$ (1)

(ii) find the value of $\alpha\beta$ (1)

(iii) find the value of $(\alpha - \beta)^2$ (2)

(iv) Are the roots of the equation Real or Unreal? Explain your answer. (2)

Question 3 (12 marks)

a) Find the coordinates of the focus of the parabola with equation $(x-4)^2 = 16(y+6)$ (2)

b) Find the centre and radius of a circle with equation $x^2 + y^2 - 8x = 0$. (2)

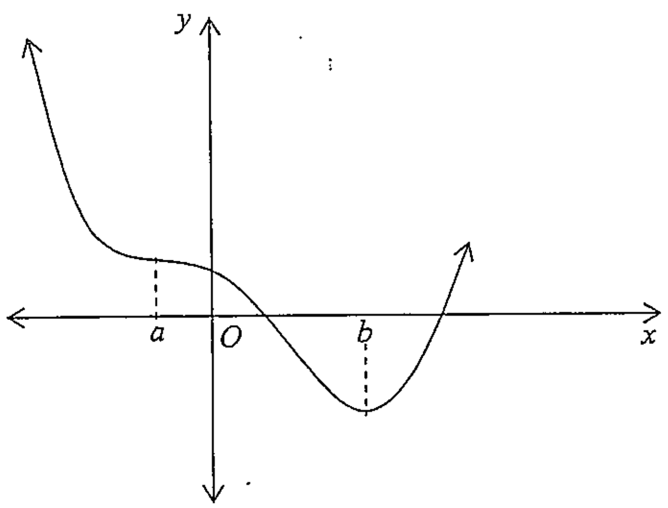
c) Write down the equation of the parabola with axis the x axis and vertex the origin and passing through the point $(-2, 6)$. (2)

d) Find the constants k and g such that $x^2 + 10x + 10 \equiv k(x+2)^2 + g(x+1)$. (3)

e) For what values of m are the roots of the equation $x^2 + 2mx + 2(m+12) = 0$ real? (3)

Question 4 (12 marks)

a) The graph below represents the function $y = f(x)$.
Using the provided enlarged copy of this diagram, on the same set of axes graph the gradient function $y = f'(x)$ (2)



(b) Find the equation of the curve with gradient function $(1+2x)^3$ if the curve passes through the point $(\frac{1}{2}, 0)$. (3)

(c) Find the equation of the tangent to the parabola $y = x^2 - 3x - 4$ at the point on the parabola where the tangent is parallel to the line $y = 2 - x$. (4)

(d) A and B are the points $(-3, 0)$ and $(3, 0)$ respectively.
Find the equation of the locus of the point P(x,y) which moves such $PA = 2PB$ (3)

Question 5 (12 marks)

(a) (i) Find the stationary points on the curve $y = 2x^3 - 3x^2 - 12x$ and determine their nature. (4)

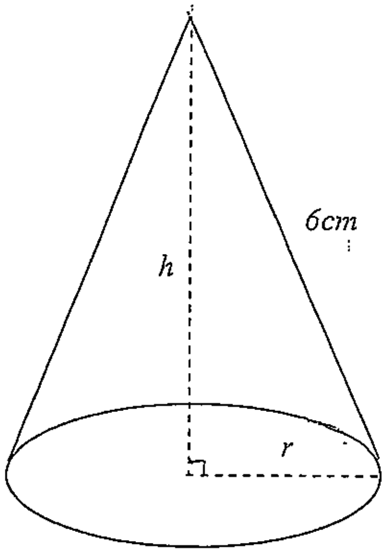
(ii) Sketch the curve in the domain $-2 \le x \le 3$ indicating on your sketch the x and y intercepts and all critical points. (do not attempt to find any points of inflexion) (3)

(b) The slant edge of a right circular cone is 6cm in length.

The volume(V) of a cone is given by the formula $V = \frac{1}{3}\pi r^2 h$

(i) Show that for the given cone below $V = 12\pi h - \frac{\pi h^3}{3}$. (2)

(i) Hence, or otherwise, find the height of the cone when the volume is a maximum. (3)



Q1

a) $\frac{d}{dx}\left(x^3-4x^2+12\right) = 3x^2-8x$

b) $\lim_{x\to 0}\left[\frac{x}{x^2-2x}\right] = \lim_{x\to 0}\left[\frac{1}{x-2}\right]$

$= -\frac{1}{2}$

c) $\alpha = 1+\sqrt{3} \quad \beta = 1-\sqrt{3}$

$\alpha+\beta = 2 \quad \alpha\beta = -2$

$x^2-2x-2=0$

d) $\frac{dP}{dt} > 0 \quad \frac{d^2P}{dt^2} < 0$

e) $a = 6$

$x^2 = 24y.$

f) $\int(4x^3-6x^2+x)\,dx = x^4-2x^3+\frac{x^2}{2}+c$

g) $f''(x) = x^2(x-2)(x-4)$

Test

x	$f''(x)$
0^-	>0
0^+	>0

$\therefore$ No change in concavity

$\therefore$ Min st pt at $x=0$.

Q2 a) $f(x) = x\sqrt{x} - \frac{3}{x^2}$

$= x^{3/2} - 3x^{-2}$

$f'(x) = \frac{3}{2}x^{1/2} + 6x^{-3}$

b) $y = \frac{2x-1}{x^2+1}$

$\frac{dy}{dx} = \frac{(x^2+1)2-(2x-1)2x}{(x^2+1)^2}$

$= \frac{-2x^2+2x+2}{(x^2+1)^2}$

c) $2x^2-6x+7$

(i) $\alpha+\beta = 3$

(ii) $\alpha\beta = 3\frac{1}{2}$

iii) $(\alpha-\beta)^2 = \alpha^2-2\alpha\beta+\beta^2$

$= (\alpha+\beta)^2-4\alpha\beta$

$= 9-14$

$= -5$

iv) Roots unreal since $(\alpha-\beta)^2<0$

Q3

a) $(y-4)^2 = 16(x+6)$

$V(4,-6) \quad a=4$

$F(4,-2)$

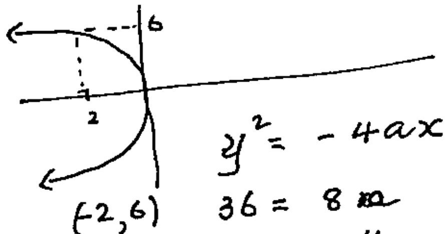
b) $x^2+y^2-8x \overset{+16}{} = 0 \overset{+16}{}$

$(x-4)^2+y^2 = 16$

centre $(4,0)$

radius $= 4$.

c)



$y^2 = -4ax$

$36 = 8a$

$a = 4\frac{1}{2}$

$\therefore y^2 = -18x$

Q5

a) (i) $y = 2x^3 - 3x^2 - 12x$

$y' = 6x^2 - 6x$

$= 6x(x-1)$

$= 0$ at $(0,0)$ $(1,-14)$

$y'' = 12x - 6$

$= -6$ at $(0,0)$

$= 6$ at $(1,-14)$

$\therefore$ Min st pt $(1,-14)$

Max st pt $(0,0)$

when $x = -2$

$y = -16 - 12 + 24$

$= -4$

when $x = 3$ $y = 54 - 27 - 36$

$= -9.$

Inflexion pts occur when $y'' = 0$

ie $x = \frac{1}{2}$ $y = -6\frac{1}{2}$

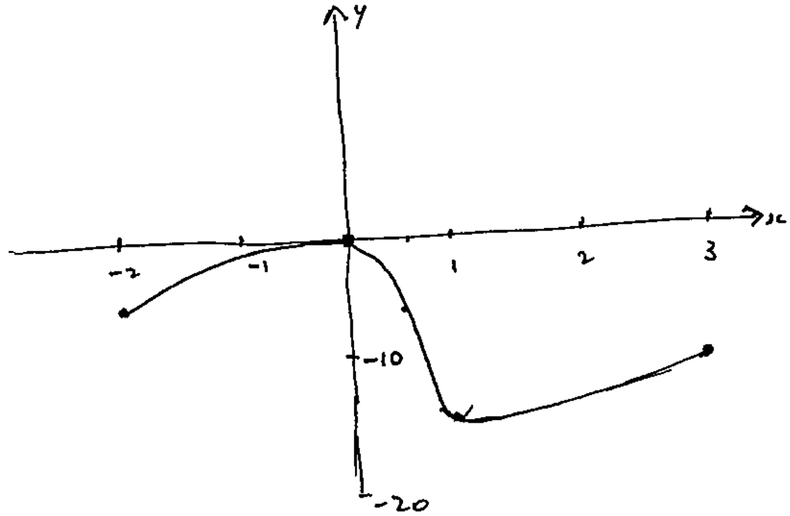
Test:

x	y''
$\frac{1}{2}^-$	<0
$\frac{1}{2}^+$	>0

$\therefore$ change in concavity at $(\frac{1}{2}, -6\frac{1}{2})$

$\therefore$ Inflexion pt.

ii)



b) $h^2 + r^2 = 36$

$r = \sqrt{36 - h^2}$

(i) $V = \frac{1}{3} \pi r^2 h$

$= \frac{1}{3} \times \pi \times (36 - h^2) h$

$= 12\pi h - \frac{\pi h^3}{3}$

$\frac{dV}{h} = 12\pi - \pi h^2$

$= 0$ when $h = \sqrt{12}$

$\frac{d^2V}{dh^2} = -2\pi h$

< 0 for $h = \sqrt{12}$

$\therefore$ Max V when $h = \sqrt{12}$ cm