Name:..

HSC Mathematics

Assessment Task 1 - 2013

Time Allowed - 60 minutes + 5minutes reading

Instructions: Calculators may be used in any parts of the task. For 1 Mark Questions, the correct answer is sufficient to receive full marks. For Questions worth more than 1 Mark, necessary working MUST be shown to receive full marks.

Multiple Choice Questions 1 - 5	/5
Question 6	/12
Question 7	/11
Question 8	/12
Question 9	/11
Total	/51

Circle the correct answer - Questions 1 - 5 are worth 1 mark each

1. A parabola has a focus of $(0, -a)$ and a directrix at $y = a$. Its equation is:

A $y^2 = 4ax$

B $y^2 = -4ax$

C $x^2 = 4ay$

D $x^2 = -4ay$

2. The derivative of $\frac{1}{2x^2}$ is

A $-4x^{-3}$

B $\frac{1}{4x}$

C $\frac{-1}{x^3}$

D $\frac{-1}{2x^3}$

3. A parabola is the locus of points equidistant from:

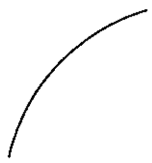
A two points

B two intersecting lines

C a point and a line

D two parallel lines

4. For the curve,



A $f'(x) > 0$ and $f''(x) < 0$

B $f'(x) > 0$ and $f''(x) > 0$

C $f'(x) < 0$ and $f''(x) < 0$

D $f'(x) < 0$ and $f''(x) > 0$

5. The gradient of the normal to $y = 2x^3 - 3x - 5$ at $(-1, -4)$ is

A -2

B 3

C $\frac{1}{2}$

D $-\frac{1}{3}$

Question 6 12 Marks (Begin a new sheet of paper) **Marks**

a) Differentiate i) $y = 4x^3 - 7x - 2$ 1

ii) $y = x^2\sqrt{x}$ 2

b) Find the derivative in simplest factored form of

i) $y = (7 - 5x)^4$ 2

ii) $y = \dfrac{(3x-4)^4}{(2x+3)^2}$ 4

c) Differentiate from first principles $f(x) = 5 - 2x - 3x^2$ 3

Question 7 12 Marks (Begin a new sheet of paper)

a) For the curve $y = 4x^3 - x^4$ find

i) The x intercepts **1**

ii) The coordinates and nature of the stationary points **4**

iii) Any points of inflexion **2**

iv) Sketch the curve **2**

b) Find the equation of the tangent to the curve $y = 2\sqrt{2x + 5}$ at the point where $x = 2$ **2**

Question 8 12 Marks (Begin a new sheet of paper)

a) Find the equation of the locus of a point which is always 5 units from the point $(-1, 2)$ **1**

b) i) Sketch the straight lines $3x - 4y = 12$ and $5x + 12y = 12$ on the same number plane. Clearly label each line and your number plane should be at least $\frac{1}{3}$ page in size. **2**

ii) Find the equation of the locus of a point which is equidistant from these two lines. **4**

iii) Sketch and clearly label the locus on the same number plane as in i).. **1**

c) For the parabola $(x-1)^2 = 4(y+2)$ find

i) The coordinates of the vertex **1**

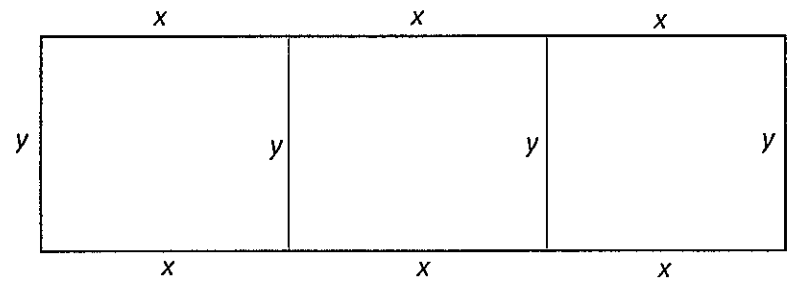
ii) The coordinates of the focus **1**

iii) The equation of the dirextrix **1**

iv) Sketch the parabola **1**

Question 9 (11 Marks) (Begin a new sheet of paper)

a) A 20 cm piece of wire is cut to form the shape below



i) Show that $y = 5 - \frac{3}{2}x$. **1**

ii) Show that the total enclosed area is given by $A = 15x - \frac{9}{2}x^2$ **1**

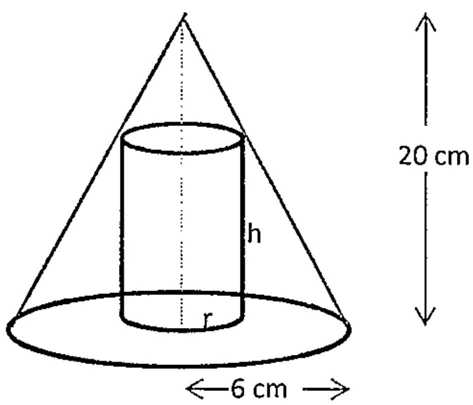
iii) Find the value of x such that the enclosed area is a maximum. **3**

b) A cylinder of radius r cm and height h cm is inscribed in a cone with base radius 6 cm and height 20 cm.

i) Use similar triangles to show that $h = \dfrac{60-10r}{3}$ **2**

ii) Show that the volume of the cylinder is given by $V = \dfrac{10\pi r^2(6-r)}{3}$ **1**

iii) Hence find the values of r and h for the cylinder which has maximum volume. **3**

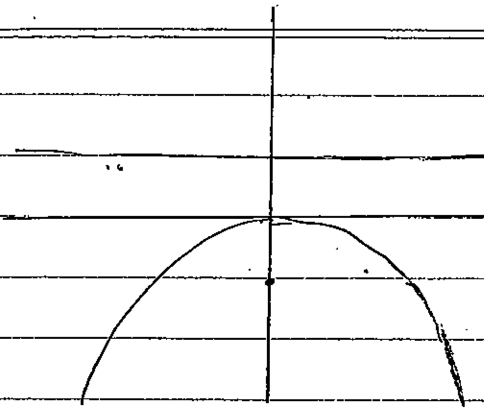


END OF TEST

Solutions to 2013 HSC Mathematics Task 1

1. $x^2 = -4ay$

(D)



2. $y = \frac{1}{2}x^{-2}$

$y' = \frac{1}{2} \cdot -2x^{-3} = -\frac{1}{x^3}$ (C)

3. (C)

4. Increasing $\therefore f'(x) > 0$

Concave down $\therefore f''(x) < 0$

(A)

5. $y = 2x^3 - 3x - 5$ $\quad -2+3-5 = -4$

$y' = 6x^2 - 3$

m tangent $= 6 - 3$

$= 3$

m normal $= -\frac{1}{3}$

(D)

Question 6.

a) i) $y' = 12x^2 - 7$

ii) $y = x^{\frac{5}{2}}$

$y' = \frac{5}{2} x^{\frac{3}{2}} = \frac{5}{2} x\sqrt{x}$

b) i) $y' = 4(7-5x)^3 . -5$

$y' = -20(7-5x)^3$

ii) $y' = \dfrac{(2x+3)^2 . 4(3x-4)^3 . 3 - (3x-4)^4 . 2(2x+3)^1 . 2}{(2x+3)^4}$

$y' = \dfrac{4(2x+3)(3x-4)^3 \{3(2x+3) - (3x-4)\}}{(2x+3)^4}$

$y' = \dfrac{4(3x-4)^3(3x+13)}{(2x+3)^4}$

c) $f(x) = 5 - 2x - 3x^2$

$f(x+h) = 5 - 2(x+h) - 3(x+h)^2$

$= 5 - 2x - 2h - 3x^2 - 6xh - 3h^2$

$f(x+h) - f(x) = -2h - 6xh - 3h^2$

$\dfrac{f(x+h) - f(x)}{h} = -2 - 6x - 3h$

$f'(x) = \lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h} = -2 - 6x$

7b) $y = 2(2x+5)^{\frac{1}{2}}$

$y' = 2 . \frac{1}{2}(2x+5)^{-\frac{1}{2}} . 2 = \dfrac{2}{\sqrt{2x+5}}$

$x = 2, \quad y = 6, \quad m = \frac{2}{3}$

$\therefore y - 6 = \frac{2}{3}(x-2)$

$3y - 18 = 2x - 4$

$2x - 3y + 14 = 0$

Q7

a) $y = 4x^3 - x^4$

$y = x^3(4 - x)$

i) $y = 0 \Rightarrow x = 0$ or 4

ii) $y' = 12x^2 - 4x^3$

$= 4x^2(3 - x)$

$y' = 0 \Rightarrow \left.\begin{matrix} x = 0 \\ y = 0 \end{matrix}\right\}$ or $\left.\begin{matrix} 3 \\ 27 \end{matrix}\right\}$

Nature of $(0, 0)$

x	0^-	0	0^+
y'	$+$	0	$+$

horizontal point of inflexion

Nature of $(3, 27)$

x	3^-	3	3^+
y'	$+$	0	$-$

Maximum turning point

iii) For pts of inflexion $y'' = 0$ & changes sign

$y'' = 24x - 12x^2$

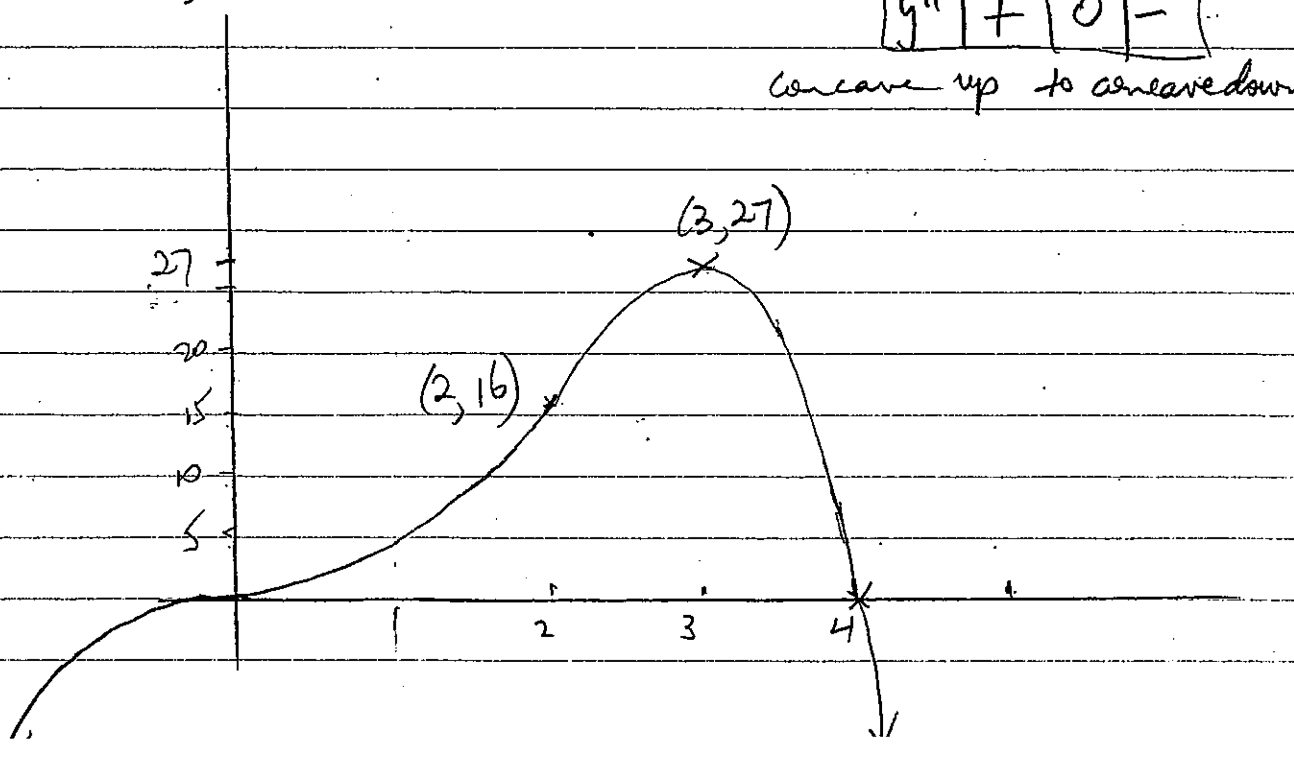
$= 12x(2 - x)$

$\left.\begin{matrix} x = 0 \\ y = 0 \end{matrix}\right\}$ or $\left.\begin{matrix} x = 2 \\ y = 16 \end{matrix}\right\}$ are possible pts of inflexion

already examined

x	2^-	2	2^+
y''	$+$	0	$-$

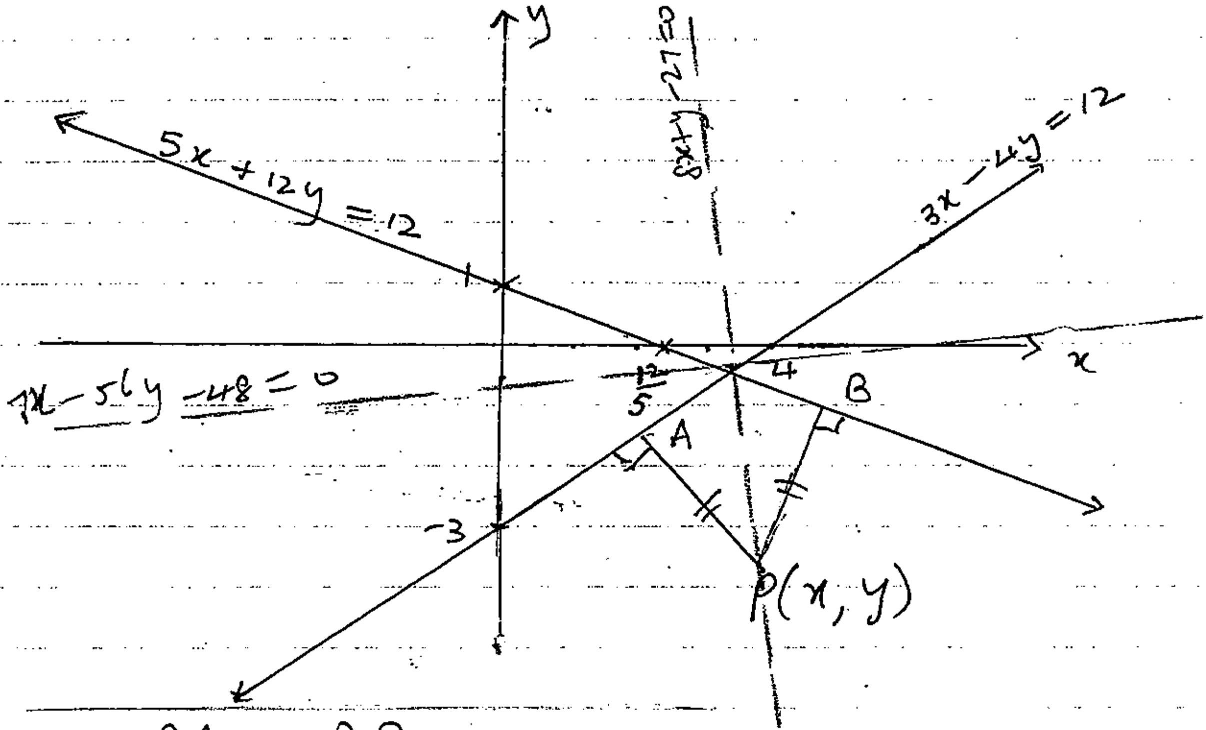
concave up to concave down.



Question 8

a) $(x+1)^2 + (y-2)^2 = 25$

b)



$$PA = PB$$

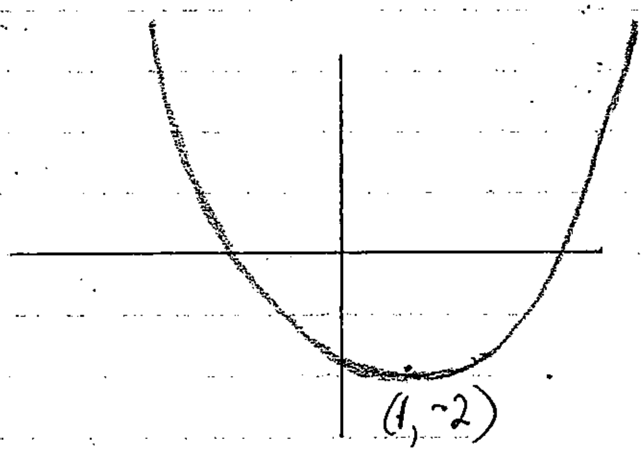
$$\left|\frac{3x-4y-12}{5}\right| = \left|\frac{5x+12y-12}{13}\right|$$

$39x-52y-156 = 25x+60y-60$ or $39x-52y-156 = -25x-60y+60$

$14x-112y-96=0$ or $64x+8y-216=0$

$7x-56y-48=0$ or $8x+y-27=0$

c)



i) vertex $(1,-2)$

ii) focus $(1,-1)$

iii) directrix
$y=-3$

$a=1$

9 a) i) $6x + 4y = 20$

$3x + 2y = 10$

$2y = 10 - 3x$

$y = 5 - \frac{3x}{2}$

ii) $A = 3x \times y$

$A = 3x\left(5 - \frac{3x}{2}\right)$

$A = 15x - \frac{9x^2}{2}$

iii) $\frac{dA}{dx} = 15 - \frac{18x}{2}$

For maximum area, $\frac{dA}{dx} = 0$

$15 - 9x = 0$

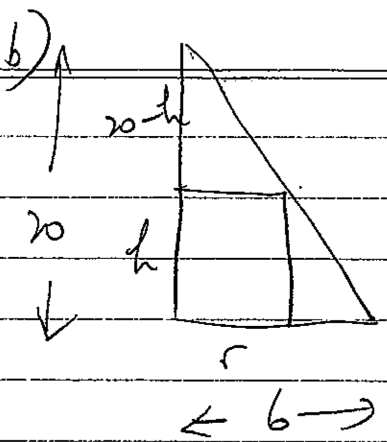
$x = \frac{15}{9} = \frac{5}{3}$

Show that A is in fact maximum

$\frac{d^2A}{dx^2} = -9$

which is negative $\therefore$ A is maximum

9 b)



i) $\dfrac{20-h}{20} = \dfrac{r}{6}$

$20r = 6(20-h)$

$10r = 3(20-h)$

$10r = 60 - 3h$

$3h = 60 - 10r$

$h = \dfrac{60-10r}{3}$

$h = \dfrac{10}{3}(6-r)$

ii) $V = \pi r^2 h$

$V = \pi r^2 \cdot \dfrac{10}{3}(6-r)$

$= \dfrac{10\pi r^2}{3}(6-r)$

iii) $V = \dfrac{10\pi}{3}(6r^2 - r^3)$

$\dfrac{dV}{dr} = 0$ for maximum volume

$\dfrac{10\pi}{3}(12r - 3r^2) = 0$

$3r(4-r) = 0$

$r = 0$ or 4

Show that V is in fact a maximum when $r = 4$

$\dfrac{d^2V}{dr^2} = \dfrac{10\pi}{3}(12 - 6r)$

$= \dfrac{10\pi}{3}(12 - 24)$ which is $-$ve

$\therefore$ concave down $\therefore$ V is maximum

When $r = 4$, $h = \dfrac{10}{3}(6-4)$

$r = 4$, $h = \dfrac{20}{3}$