Name : ________________________

HURLSTONE AGRICULTURAL HIGH SCHOOL
YEAR 12 2009 MATHEMATICS
ASSESSMENT TASK 1

Examiners P. Biczo, S. Hackett

General Instructions

- Reading time : 3 minutes
- **Working time : 40 minutes**
- Attempt **all** questions
- **Start a new sheet of paper for each question**
- All necessary working should be shown
- This paper contains 4 questions worth 8 marks each. Total Marks: **32 marks**
- Marks may not be awarded for careless or badly arranged work
- Board approved calculators may be used
- This examination paper must **not** be removed from the examination room

---

Question 1 (Start a new sheet of paper) **Marks**

(i) Evaluate $\lim\limits_{x\to\infty} \dfrac{3-4x}{x^2-5x}$ **1**

(ii) Use the definition of the derivative, $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$, to find $f'(x)$ when $f(x) = x^2 - 3$ **2**

(iii) Explain why $y = \dfrac{1}{x+2}$ is not a continuous function **1**

(iv) Differentiate the following with respect to x:

(a) $\dfrac{3x^3+2x^2+x}{x}$ **1**

(b) $\dfrac{1-x}{2x+3}$ **2**

(c) $(3+2x)^3$ **1**

Question 2 (Start a new sheet of paper) **Marks**

(i) Show that the equation of the tangent to the curve $y = x^3 - 5x + 2$ at the point $(-1, 6)$ is given by $2x + y - 4 = 0$ **2**

(ii) Consider the curve $f(x) = x^3 - x^2 - x + 1$. Find the values of x for which $f'(x) = 0$ **2**

(iii) Consider the curve $xy = 4$. Find $\frac{dy}{dx}$ when $x = 2$ **2**

(iv) Show that the derivative of $y = x\sqrt{2x+1}$ is given by $\frac{dy}{dx} = \frac{3x+1}{\sqrt{2x+1}}$ **2**

Question 3 (Start a new sheet of paper)

(i) For the parabola $y = 2x - x^2$, find

(a) the equation of the axis of symmetry **1**

(b) the coordinates of the vertex **1**

(ii) Solve $(3x-1)^2 = 7$ leaving your answer in surd form. **2**

(iii) Solve $x^4 - 7x^2 + 12 = 0$ **2**

(iv) What values of m will make the expression $x^2 + 6x + m$ positive definite? **2**

Question 4 (Start a new sheet of paper)

(i) If α and β are the roots of $2x^2 + 3x + 4 = 0$ find the value of $\alpha^2 + \beta^2$. **3**

(ii) (a) Sketch the graph of $y = x^2 - 6$ and label all intercepts with the axes. **1**

(b) On the same set of axes, carefully sketch the graph of $y = |x|$ **1**

(c) Find the coordinates of the two points where the graphs intersect. **2**

(d) Hence solve the inequality $x^2 - 6 \le |x|$. **1**

Outcomes Addressed in this Question

P8 understands and uses the language and notation of calculus
P7 determines the derivative of a function through routine application of the rules of differentiation

Outcome	Sample Solution	Marking Guidelines
(i) P8	$\lim_{x\to\infty}\frac{3-4x}{x^2-5x}=\lim_{x\to\infty}\frac{\frac{3}{x^2}-\frac{4}{x}}{1-\frac{5}{x}}=\frac{0-0}{1-0}=0$	**1 mark ~** Correct answer
(ii) P8	$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to 0}\frac{(x+h)^2-3-\left(x^2-3\right)}{h}$ $=\lim_{h\to 0}\frac{x^2+2xh+h^2-x^2+3}{h}$ $=\lim_{h\to 0}\frac{2xh+h^2}{h}$ $=\lim_{h\to 0}(2x+h)$ $=2x$	**2 mark ~** Correct solution **1 mark ~** Attempt to apply definition
(iii) P8	$y=\frac{1}{x+2}$ is not a continuous function because at $x=-2$ the function does not exist.	**1 mark ~** Correct explanation.
(iv) (a) P7	$\frac{d}{dx}\left(\frac{3x^3+2x^2+x}{x}\right)=\frac{d}{dx}\left(3x^2+2x+1\right)=6x+2$	**1 mark ~** Correct answer.
(b) P7	$\frac{d}{dx}\left(\frac{1-x}{2x+3}\right)=\frac{(2x+3).-1-(1-x).2}{(2x+3)^2}=\frac{-5}{(2x+3)^2}$	**2 marks ~** Correct solution **1 mark ~** Attempt to use the quotient rule or equivalent.
(c) P7	$\frac{d}{dx}\left\{(3+2x)^3\right\}=3.(3+2x)^2.2=6(3+2x)^2$	**1 mark ~** Correct answer.

Year 12 | Mathematics | Task 1 ~ 2008/9

Question No. 2 | Solutions and Marking Guidelines

Outcomes Addressed in this Question

P6 Relates the derivative of a function to the slope of its graph
P7 Determines the derivative of a function through routine application of the rules of differentiation
P8 Understands and uses the language and notation of calculus
H5 Applies appropriate techniques from the study of calculus to solve problems

Outcome	Solutions	Marking Guidelines
P6, H5	(i) $y = x^3 - 5x + 2$ $\frac{dy}{dx} = 3x^2 - 5$ When $x = -1$, $\frac{dy}{dx} = 3(-1)^2 - 5 = -2$ $\therefore$ gradient of tangent is -2 Equation of tangent through $(-1, 6)$ is $y - 6 = -2(x+1)$ $y - 6 = -2x - 2$ $\therefore$ tangent is $2x + y - 4 = 0$	**2 marks:** finding derivative plus gradient of tangent and showing equation given **1 mark:** one of above
P8, H5	(ii) $f(x) = x^3 - x^2 - x + 1$ $f'(x) = 3x^2 - 2x - 1$ $f'(x) = 0$ when $3x^2 - 2x - 1 = 0$ $\qquad \begin{array}{r\|l} -3 & 1 \\ & -3 \\ \hline & -2 \end{array}$ $\therefore 3x^2 - 3x + x - 1 = 0$ $\therefore 3x(x-1) + 1(x-1) = 0$ $\therefore (x-1)(3x+1) = 0$ $\therefore x = 1$ or $x = \frac{-1}{3}$	**2 marks:** putting derivative equal to 0 and correctly solving **1 mark:** one of above
P8	(iii) $xy = 4 \;\therefore\; y = \frac{4}{x} = 4x^{-1}$ $\frac{dy}{dx} = -4x^{-2} = -\frac{4}{x^2}$ When $x = 2$, $\frac{dy}{dx} = -\frac{4}{2^2} = -1$	**2 marks:** correctly putting in a form which can be differentiated and correctly finding answer **1 mark:** one of above
P7	(iv) $y = x\sqrt{2x+1} = x(2x+1)^{\frac{1}{2}}$ Using product rule, $\frac{dy}{dx} = x.\frac{d}{dx}(2x+1)^{\frac{1}{2}} + (2x+1)^{\frac{1}{2}}.\frac{d}{dx}(x)$ $= x.\frac{1}{2}(2x+1)^{-\frac{1}{2}}.2 + (2x+1)^{\frac{1}{2}}.1($ $= x.(2x+1)^{-\frac{1}{2}} + (2x+1)^{\frac{1}{2}}$ $= (2x+1)^{-\frac{1}{2}}\left(x + (2x+1)^1\right)$ $\therefore \frac{dy}{dx} = (2x+1)^{-\frac{1}{2}}(3x+1) = \frac{3x+1}{\sqrt{2x+1}}$	**2 marks:** correctly use product rule plus function of a function rule and correctly simplify to required form or equivalent **1 mark:** one of above

Year 12 Question No. 3	Mathematics Solutions and Marking Guidelines	Task 1 ~ 2008/9

Outcomes Addressed in this Question

P4 chooses and applies appropriate arithmetic, algebraic, graphical, trigonometric and geometric techniques

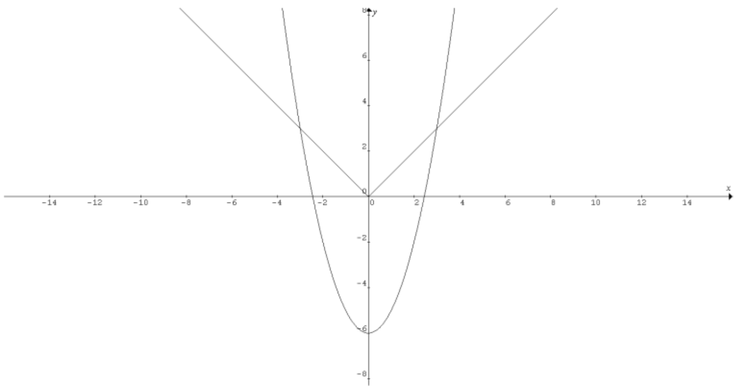
Outcome	Sample Solution	Marking Guidelines
P4	(i) (a) $x = -\dfrac{2}{2(-1)}$ $x = 1$ (b) Vertex (1,1)	**1 mark ~** Correct equation of the line **1 mark ~** Correct coordinates of vertex
P4	(ii) $(3x-1)^2 = 7$ $3x - 1 = \pm\sqrt{7}$ $3x = 1 \pm \sqrt{7}$ $x = \dfrac{1 \pm \sqrt{7}}{3}$	**2 marks ~** Correct solution **1 mark ~** Correct solution not fully simplified eg. $x = \dfrac{2 \pm 2\sqrt{7}}{6}$
P4	(iii) $x^4 - 7x^2 + 12 = 0$ $let\ u = x^2$ $u^2 - 7u + 12 = 0$ $(u-3)(u-4) = 0$ $u = 3, 4$ $\therefore x^2 = 3 \qquad x^2 = 4$ $x = \pm\sqrt{3} \qquad x = \pm 2$ $\therefore x = \pm\sqrt{3}, \pm 2$	**2 marks ~** All four correct solutions **1 mark ~** Making an appropriate substitution and finding solutions 3 and 4.
P4	(iv) For positive definite, $a > 0\, and\, \Delta < 0$ $\Delta = 36 - 4m$ $36 - 4m < 0$ $4m > 36$ $\therefore m > 9$	**2 marks ~** Correct conditions for positive definite and correct answer for m **1 mark ~** Correct conditions for positive definite.

Year 11/12	Mathematics	Task 1 2008/9
Question No. 4	Solutions and Marking Guidelines	

Outcomes Addressed in this Question

P4 chooses and applies appropriate arithmetic, algebraic, graphical, trigonometric and geometric techniques

H9 communicates using mathematical language, notation, diagrams and graphs

Outcome	Sample Solution	Marking Guidelines
(i) P4	$2x^2+3x+4=0$ $\alpha+\beta=-\frac{3}{2}$ $\alpha\beta=\frac{4}{2}=2$ $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=\left(-\frac{3}{2}\right)^2-2(2)$ $=\frac{9}{4}-4$ $=-\frac{7}{4}$	**3 marks ~** Correct answer **2 marks** ~ Correctly determines values of $\alpha+\beta$ and $\alpha\beta$ only. **1 mark** ~ Correctly determines value of $\alpha+\beta$ or $\alpha\beta$.
(ii) (a) P4,H9 (b) P4,H9		**(a)** **1 mark ~** Correct graph **(b)** **1 mark ~** Correct graph
(c) P4	$x=x^2-6$ $x^2-x-6=0$ $(x-3)(x+2)=0$ $x=3,\ -2$ However, the graph indicates that $x=3$ is the only valid solution. By symmetry, the other point of intersection is where $x=-3$. Hence, the points of intersection are $(3,\ 3)$ and $(-3,\ 3)$	**2 marks ~** Correct points stated. **1 mark ~** Indicates that $x=3$ and $x=-3$ only.
(d) P4	From the graph, $-3\le x\le 3$	**1 mark ~** Correct answer.