Name:	
Class:	

YEAR 12

ASSESSMENT TEST 1
TERM 4, 2015

MATHEMATICS

Time Allowed – 90 Minutes
(Plus 5 minutes Reading Time)

- *All* questions may be attempted.
- *All* questions are of equal value.
- Department of Education approved calculators and templates are permitted.
- In every Question, show all relevant mathematical reasoning and/or calculations.
- Marks may not be awarded for careless or badly arranged work.

The answers to all questions are to be returned in separate bundles clearly labeled Question 1, Question 2, Question 3, Question 4.

Each question must show (in the top right hand corner) your Candidate Number.

Question 1 (15 Marks)

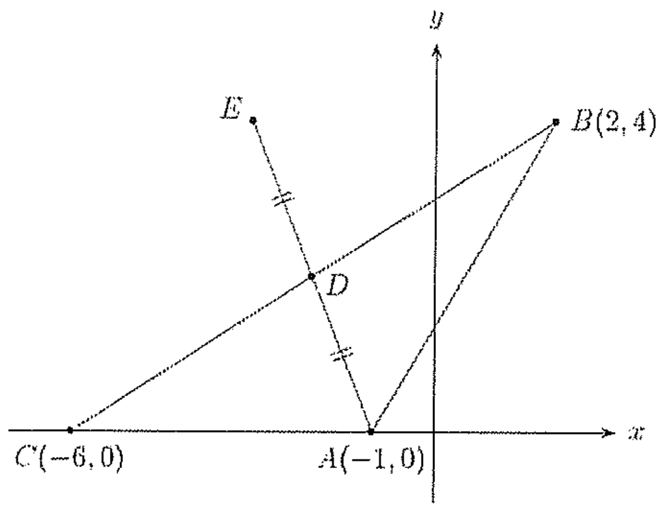
(a) Differentiate:

(i) $y = \sin^2(4x)$. **1**

(ii) $y = x^3 e^{3x}$. **1**

(iii) $y = \dfrac{e^x}{(x+3)^2}$. (Full simplification of your answer is not required.) **2**

(b)



In the diagram A,B, C and D are the points (-1,0), (2,4), (-6,0) respectively. D is the midpoint of AE

(i) Find the length of the interval AB **1**

(ii) Find the midpoint of BC **1**

(iii) Find the size of angle $\angle CAB$ **2**

(iv) Show the equation of the line BC is x-2y+6=0 **1**

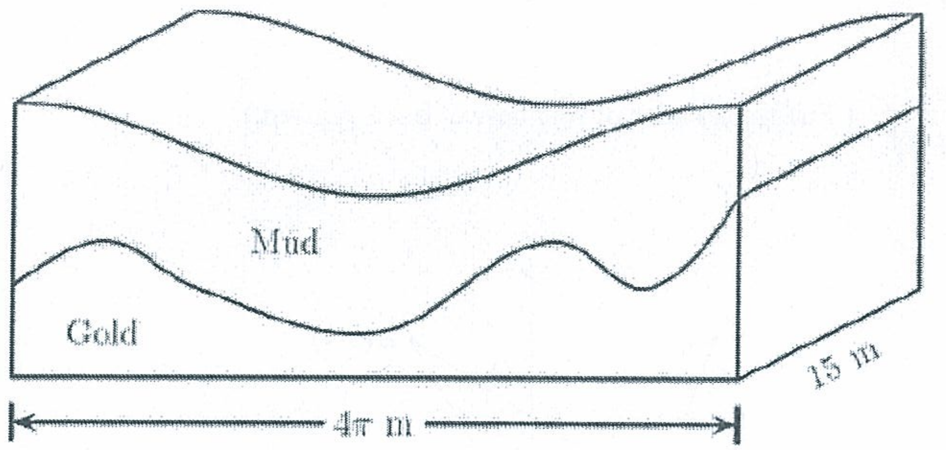
(v) Find the perpendicular distance of A from the line BC in simplest exact form. **1**

(vi) What type of quadrilateral is ABEC? Give reasons for your answer **2**

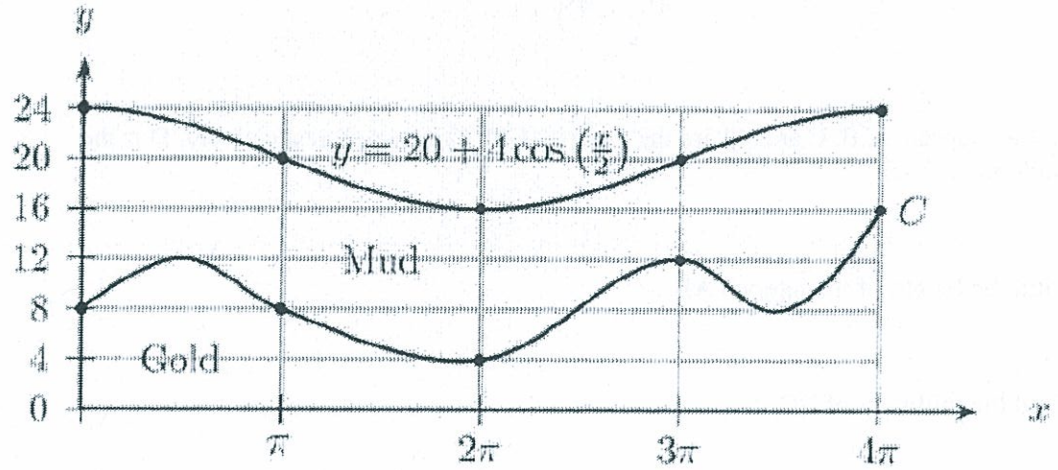
(c) Find the primitive function of $x^3 - \sqrt{x} + \dfrac{2}{x^3}$ **3**

Question 2 (15 Marks) Start a new page

(a) The diagram below shows an amount of gold, which is the shape of a prism underneath a large amoun of mud. The width of the prism is 4π metres and its length is 15 metres



The graph below shows the cross-section of the prism .The top of the mud is given by the function $y = 20 + 4\cos\frac{x}{2}$ and the top of the gold is shown by curve C



(i) Find by integration the total area of the cross-section,i.e the area of both mud and gold **2**

(ii) Using Simpson's Rule with the five function values shown on the graph. Find an estimate for the area of the cross-section of gold. **3**

(iii) Find the volume of the mud. **1**

Question 2 (continued)

(b) Find

(i) $\int \cos 4x\,dx$ **1**

(ii) $\int (4x-9)^6\,dx$ **2**

(iii) $\int \frac{7x^2-3x}{x}\,dx$ **2**

(iv) $\int_2^3 (3x^2 - e^{2x})\,dx$ (Answer in exact form) **3**

(c) The gradient function of a curve is $y' = \frac{4x}{x^2+1}$ and the curve passes through the point $(0, e)$. Find the equation of the curve. **2**

Question 3 (15 Marks) Start a new page

(a) Water started leaking out of a tank. The rate of change of V, the volume of water in the tank t days after the leak started is given by $\frac{dV}{dt} = 20t - 300$ litres per day. When the tank stopped leaking it still had 4750L of water in it.

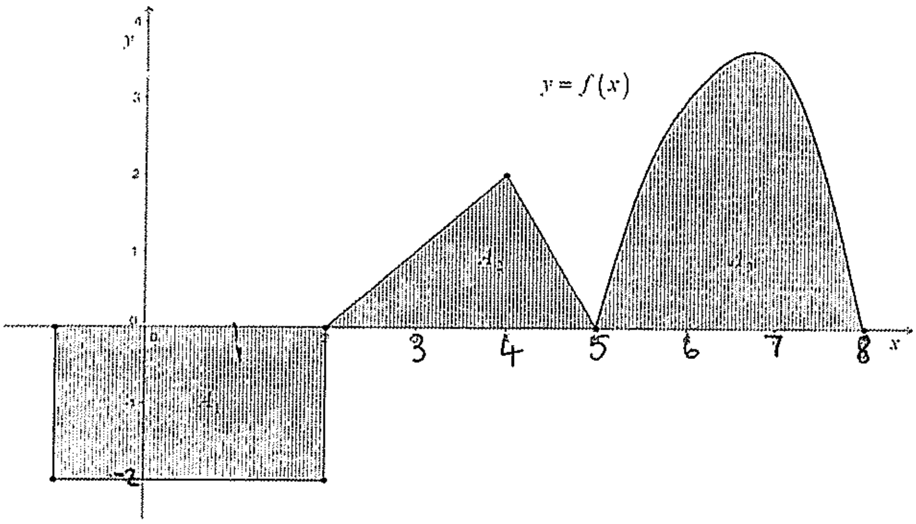
(i) For how many days was the tap leaking **1**

(ii) Find a formula for V **3**

(iii) How much water was in the tank when it started to leak **1**

(b) If $y = \ln\left(\frac{x+4}{x-5}\right)$ find $\frac{dy}{dx}$ **2**

(c)



The diagram above shows y=f(x) from x=-1 to x=8. The three distinct areas are labelled A_1, A_2 and A_3 **2**

(i) If $3A_1 = 2(A_2 + A_3)$ find A_3

(ii) Evaluate $\int_{-1}^{8} f(x)dx$ **1**

(c) Consider the function $y = \log_e(x+2)$ for $x > -2$

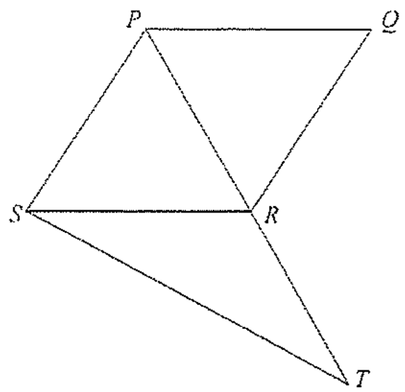
(i) Sketch the function showing its essential features **2**

(ii) Use the trapezoidal rule using 2 trapezia find an approximation for $\int_0^4 \log_e(x+2)dx$ **2**

(iii) Is this answer more or less than the actual value? Justify your answer **1**

Question 4 (15 Marks) Start a new page

(a) $PQRS$ is a rhombus. PR is produced to T such that $SR = TR$

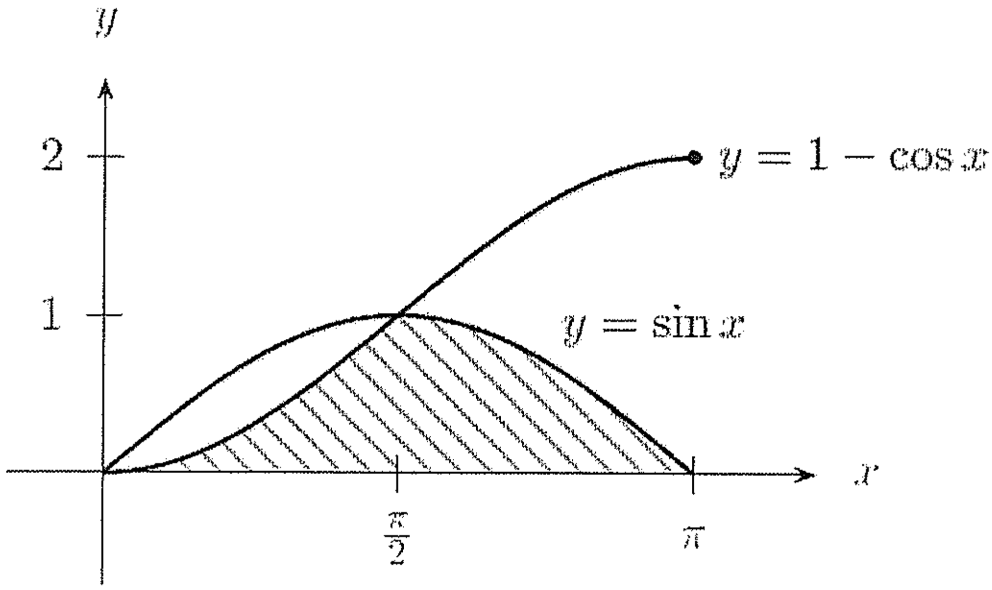


Copy the diagram onto your answer sheet

(i) Show that $\angle SPQ = 4\angle STR$ **3**

ii) Show that R is the midpoint of PT, given that $\angle PST = 90°$ **3**

(b)



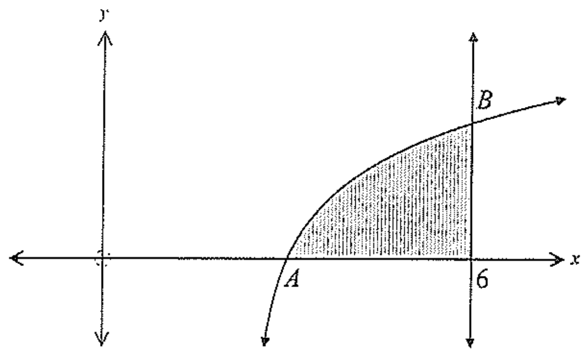
The diagram shows the graphs of the functions y=1-cosx and y=sinx between x=0 and x=π **4**

The graphs intersect at x=$\dfrac{\pi}{2}$.

Find the area of the shaded region

(c) The diagram shows a shaded region which is bounded by the curve $y = \ln(2x - 5)$, the x axis and the line $x = 6$.

The curve $y = \ln(2x - 5)$ intersect the x axis at A and the line $x = 6$ at B.



(i) Show that the coordinates of points A and B are (3, 0) and (6, ln7) respectively. **1**

(ii) Show that if $y = \ln(2x - 5)$, then $x = \dfrac{e^y + 5}{2}$. **1**

(iii) Hence find the exact area of the shaded region. **3**

END OF EXAM

Question 1 – 2U

a (i) $y' = 8\sin 4x \cos 4x$

or

$y' = 4\sin 8x$

(ii) $y' = x^3 . 3e^{3x} + e^{3x} . 3x^2$

$= e^{3x}(3x^3 + 3x^2)$ or

$= 3x^2 e^{3x}(1+x)$

(iii) $y' = \dfrac{(x+3)^2 e^x - e^x(2)(x+3)}{((x+3)^2)^2}$ Numerator Denominator

$= \dfrac{(x+3)e^x(x+1)}{(x+3)^4}$ or $\dfrac{e^x(x+1)}{(x+3)^3}$ or $\dfrac{e^x(x+3) - 2e^x}{(x+3)^2}$

b (i) $AB = \sqrt{3^2 + 4^2} = 5$ units

(ii) Midpoint $\left(\frac{-4}{2}, \frac{4}{2}\right) = (-2, 2)$

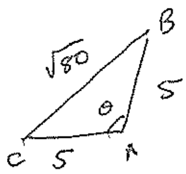
(iii) $\tan \angle BAF = \frac{4}{3}$

$\therefore \angle BAF = 53°8'$ (nearest minute)

$\therefore CAB = 180 - 53°8'$

$= 126°52'$

Alternatively – Cosine Rule



$\cos\theta = \dfrac{5^2 + 5^2 - 80}{2 \times 5 \times 5}$

$= -3/5$

$= 126° 52'$

(iv) $M_{BC} = \dfrac{4-0}{2--6} = \dfrac{4}{8} = \dfrac{1}{2}$

$B(2,4)$ and $C(-6,0)$

$\therefore y - 0 = \frac{1}{2}(x - (-6))$

$y = \frac{1}{2}(x+6)$

$y = \frac{1}{2}x + 3$

$\therefore 2y = x + 6$

$0 = x - 2y + 6 = 0$

(v) $d = \left| \dfrac{ax+by+c}{\sqrt{a^2+b^2}} \right|$

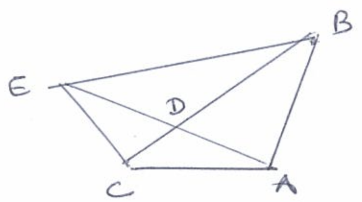
$= \dfrac{|(1)(-1)+2(0)+6|}{\sqrt{1^2+(-2)^2}}$

$= \dfrac{|-1+0+6|}{\sqrt{5}}$

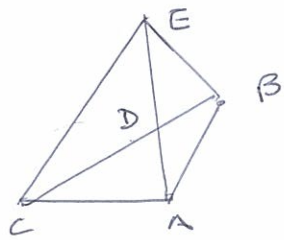
$= \dfrac{5}{\sqrt{5}}$ or $\sqrt{5}$ units ✓

(vi) One cannot say what special type of quadrilateral ABEC is as neither of the coordinates for E or D are given. We say it is irregular quadrilateral Since D is not fixed in relation to BC, D can be anywhere along the line of BC. Also point E is constructed such that AD = DE. Some possibilities include

2 marks



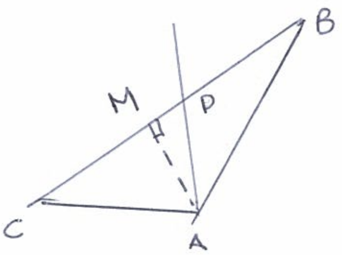
or



1 mark - Said Rhombus

Just!

Assumed D as $(-2, 2)$ which is the midpoint M of line BC. Not true.



To get one mark, one had to show that diagonals bisected each other at $90°$ and not just mention that AD was perpendicular bisector. (Note $D \neq M$)

0 marks - Parallelogram or kite

One could go further to prove it is a rhombus by looking at lengths of AB, AC BE and CE.

<u>Note</u> Some found Point E as $(-3,4)$ based on assuming D is $(-2,2)$ – Not true

(c) primitive of $x^3 - \sqrt{x} + \frac{2}{x^3}$ is

$$\frac{x^4}{4} - \frac{2x^{3/2}}{3} - \frac{1}{x^2} + C$$

✓ ✓ ✓

1 mark for each term but Lost a mark for not writing $(+C)$

MATHEMATICS: Question 2		
Suggested Solutions	**Marks**	**Marker's Comments**
(i)(a) $A = \int_0^{4\pi} 20 + 4\cos\left(\frac{x}{2}\right) dx = \left[20x + 8\sin\frac{x}{2}\right]_0^{4\pi}$ $= 80\pi + 8\sin 2\pi - [0+0]$ $= 80\pi + 0$ $= 80\pi$ Area of Mud and Gold $= 80\pi\ u^2$ $\doteqdot 251\cdot 3\ u^2$	2	1 integration 1 Answer.
(ii) <table><tr><td>x</td><td>0</td><td>π</td><td>2π</td><td>3π</td><td>4π</td></tr><tr><td>y</td><td>8</td><td>8</td><td>4</td><td>12</td><td>16</td></tr></table> $A \approx \frac{\pi}{3}\left[8 + 4\times 8 + 2\times 4 + 4\times 12 + 16\right]$ $\approx \frac{\pi}{3}\left[8 + 32 + 8 + 48 + 16\right]$ $\approx \frac{112\pi}{3}$ $\approx 117\cdot 29$ Area Gold $\approx 117\cdot 29\ u^2$	2	1 Expression 1 $\frac{\pi}{3}$ and correct calculation
(iii) Volume of Mud $\approx \left[80\pi - \frac{112\pi}{3}\right] \times 15$ $\approx 640\pi$ $\approx 2010\cdot 6\ u^3$.	1	correct answer.

MATHEMATICS: Question 2		
Suggested Solutions	**Marks**	**Marker's Comments**
(b) (i) $\int \cos 4x \, dx = \frac{1}{4}\sin 4x + C$	1	correct answer.
(ii) $\int (4x-9)^6 \, dx = \frac{1}{7\times 4}(4x-9)^7 + C$ $= \frac{1}{28}(4x-9)^7 + C$	2	1 Numerator 1 Denomenator.
(iii) $\int \frac{7x^2-3x}{x} dx = \int (7x-3)\,dx$ $= \frac{7x^2}{2} - 3x + C$	2	1 + 1 Eachterm
(iv) $\int_2^3 (3x^2 - e^{2x})\,dx = \left[x^3 - \frac{1}{2}e^{2x}\right]_2^3$ $= 27 - \frac{1}{2}e^6 - 8 + \frac{1}{2}e^4$ $= 19 - \frac{1}{2}[e^6 - e^4]$	2	1+1 each term 1 answer.
(c) $y' = \frac{4x}{x^2+1}$ $y = \int \frac{4x}{x^2+1} dx = 2\int \frac{2x}{x^2+1} dx$ $= 2\ln(x^2+1) + C$ when $x=0$ $y=e$. $e = 2\ln 1 + C \quad \therefore C = e$. $y = 2\ln(x^2+1) + e$	2	1 integration 1 answer.

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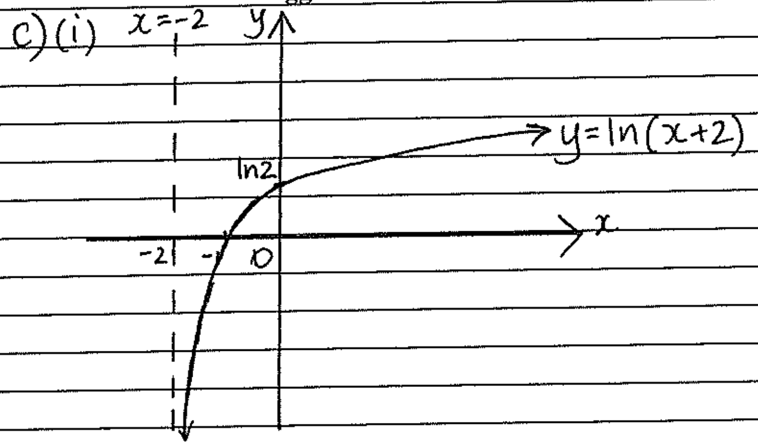
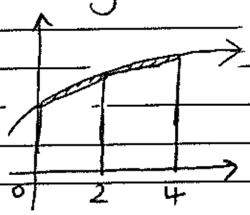
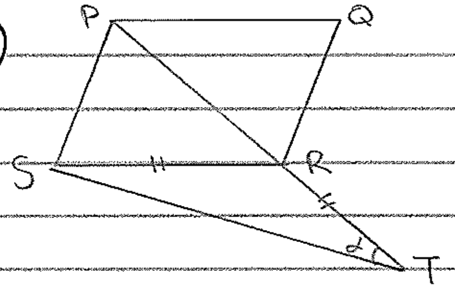
MATHEMATICS	**Question 3**	
Suggested Solutions	**Marks**	**Marker's Comments**
3a)(i) $\frac{dV}{dt} = 20t - 300$		
Tap leaking = volume changing		
$\therefore$ Tap stops leaking when $\frac{dV}{dt} = 0$		
$20t - 300 = 0$		
$20t = 300$		
$t = 15$		
$\therefore$ Tap was leaking for 15 days.	1	Answer
(ii) When $t = 15$, $V = 4750$		
$V = 10t^2 - 300t + C$	1	V
$4750 = 10 \times 15^2 - 300 \times 15 + C$	1	Substitution
$C = 7000$		
$\therefore V = 10t^2 - 300t + 7000$	1	Eqn with C
(iii) When $t = 0$		
$V = 7000L$	1	units L
b) $y = \ln\left(\frac{x+4}{x-5}\right)$		Many students did not use log laws.
$= \ln(x+4) - \ln(x-5)$		
$\frac{dy}{dx} = \frac{1}{x+4} - \frac{1}{x-5}$	2	using log laws
c) (i) $A_1 = 2 \times 3$		
$= 6u^2$		
$A_2 = \frac{1}{2} \times 3 \times 2$		
$= 3u^2$	1	for A_1 & A_2
$3A_1 = 2(A_2 + A_3)$		
$18 = 6 + 2A_3$		
$2A_3 = 12$		
$A_3 = 6u^2$	1	A_3
(ii) $\int_{-1}^{8} f(x)\,dx = -6 + 3 + 6$		Finding value of integral
$= 3$	1	not an area. Units not required.

MATHEMATICS : Question 3

Suggested Solutions	Marks	Marker's Comments
c)(i) 	1 1	shape intercept and asymptote
(ii) <table><tr><td>x</td><td>0</td><td>2</td><td>4</td></tr><tr><td>$f(x)$</td><td>$\ln 2$</td><td>$\ln 4$</td><td>$\ln 6$</td></tr><tr><td>w</td><td>1</td><td>2</td><td>1</td></tr></table>	1	values
$\int_0^4 \ln(x+2)\,dx \approx \frac{4-0}{4}(\ln 2 + 2\ln 4 + \ln 6)$		Should have integral $\approx$
≈ 5.2575 (to 4dp)	1	answer Many students left anwer in terms of log. Many students thought they were finding an area.
(ii) Less than actual area as the trapezia will lie under the graph of $y=\ln(x+2)$. OR/ curve is concave down OR/ diagram 	1	

MATHEMATICS: Question 4

Suggested Solutions	Marks	Marker's Comments
a) 		
i) let $\angle STR = \alpha$ $SR = TR$ (given) $\angle RST = \angle STR$ (equal angles are opposite equal sides in ΔSTR) $= \alpha$ $\angle PRS = \angle RST + \angle STR$ (exterior angle of ΔSTR) $= 2\alpha$	1	Always prove what you want to use first eg. $PQ // SR$ (opposite sides of a rhombus are parallel) then you can use this property.
$\angle QRP = \angle PRS$ (diagonal of rhombus bisects vertex angle) $= 2\alpha$	1	
$\angle QRS = 4\alpha$ (adjacent angle sum) $\angle SPQ = \angle QRS$ (opposite angles of a rhombus are equal) $= 4\alpha$ $\therefore \angle SPQ = 4\angle STR$	1	
ii) $\angle SPR = 2\alpha$ (diagonal of rhombus bisects vertex angle) $\angle PST + \angle STP + \angle TPS = 180°$ (angle sum of ΔSTP) $90° + \alpha + 2\alpha = 180°$ $3\alpha = 90°$ $\alpha = 30°$	1	
$\therefore \angle SPR = 2\alpha$ $= 60°$		

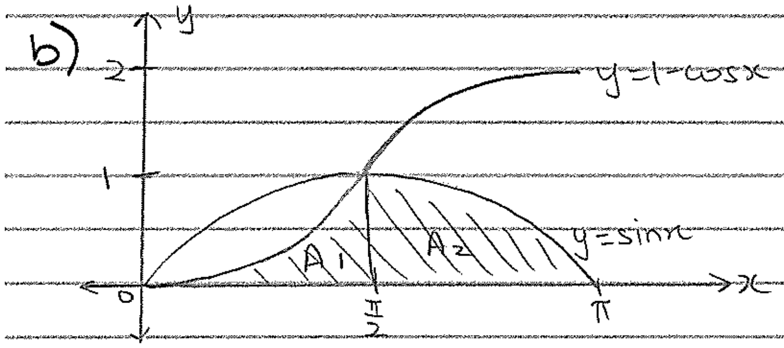
MATHEMATICS: Question 4

Suggested Solutions	Marks	Marker's Comments

$\angle PRS = 2\alpha$
$= 60^\circ$
$\angle PSR = 180 - 60 - 60$ (angle sum of ΔSPR)
$= 60^\circ$
$\therefore \Delta SPR$ is equilateral (all angles are 60°) — 1

$PR = SR$ (sides of equilateral triangle SPR are equal)
$SR = RT$ (given)
$\therefore PR = RT$
$\therefore R$ is the midpoint of PT — 1

b)



$A = A_1 + A_2$

$= \int_0^{\frac{\pi}{2}} 1 - \cos x \, dx + \int_{\frac{\pi}{2}}^{\pi} \sin x \, dx$ — 2

$= \left[x - \sin x\right]_0^{\frac{\pi}{2}} + \left[-\cos x\right]_{\frac{\pi}{2}}^{\pi}$ — 1

$= \frac{\pi}{2} - 1 - 0 + 1 - 0$

$= \frac{\pi}{2}$ units^2 — 1

c) i) when $x = 6$, $y = \ln(2 \times 6 - 5)$
$= \ln(12 - 5)$
$= \ln 7$
$\therefore B$ is $(6, \ln 7)$

Marker's comment: Need to show working NOT just state it.

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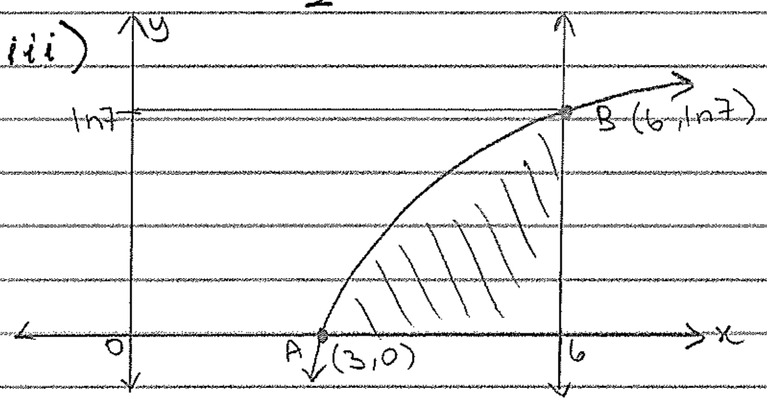
MATHEMATICS: Question 4

Suggested Solutions	Marks	Marker's Comments
when $y=0$, $0=\ln(2x-5)$		
$1 = 2x-5$		
$2x = 6$		
$x = 3$	1	
$\therefore A = (3,0)$		
ii) $y = \ln(2x-5)$		
$e^y = 2x-5$		
$2x = e^y + 5$	1	
$x = \dfrac{e^y+5}{2}$		
iii) 		
$A = 6\ln 7 - \int_0^{\ln 7} \dfrac{e^y+5}{2}\,dy$	1	1 for limits
$= 6\ln 7 - \frac{1}{2}\left[e^y + 5y\right]_0^{\ln 7}$		
$= 6\ln 7 - \frac{1}{2}\left[e^{\ln 7} + 5\ln 7 - 1\right]$		
$= 6\ln 7 - \frac{1}{2}\left[7 + 5\ln 7 - 1\right]$		
$= 6\ln 7 - \frac{5}{2}\ln 7 - 3$	1	
$= 3\frac{1}{2}\ln 7 - 3$ units^2		