YEAR 11, HSC
MATHEMATICS - ASSESSMENT 1

Marks

Question 1 (13 marks)

(a) Determine a quadratic equation in general form with roots equal to 2 and -1. **2**

(b) Differentiate each of the following

i) $\sqrt{6x-5}$ 2

ii) $3x(2x-1)^2$ 3

(c) If α and β are the roots of the equation $2x^2+3x+5=0$, find the value of

i) $\alpha+\beta$ 1

ii) $\alpha\beta$ 1

iii) $\frac{1}{\alpha}+\frac{1}{\beta}$ 2

iv) $\alpha^2+\beta^2$ 2

Marks

Question 2 (14 marks) ***(Start a new page)***

(a) Consider the parabola with equation $x^2 = -8(y + 3)$ 4

i) Find the coordinates of the vertex of the parabola.

ii) Find the coordinates of the focus of the parabola.

(b) Solve the quadratic inequality $2x^2 - x - 10 \leq 0$ 3

(c) Find two numerical values of p so that $x^2 + 2px + (7p + 8) = 0$ has equal roots. 4

(d) Solve the equation $(x^2 - x) - 18(x^2 - x) + 72 = 0$ 3

Question 3 **(15 marks)** ***(Start a new page)***

(a) Find the value of m if one root of the equation $x^2 + 6x + m = 0$ is double the other. 3

(b) Find the values of A, B, and C if $x^2 - x \equiv A(x-4)^2 + B(x-4) + C$. 4

(c) Let A and **B** be the fixed points (-1, 0) and (2, 0) and let P be the variable point (x, y). 8

(i) Write down expressions for the distances PA^2 and PB^2

(ii) Suppose that P moves so that $PA = 2PB$. Show that the equation of this locus of points is the circle $x^2 - 6x + y^2 + 5 = 0$

(iii) Hence find the centre and radius of the circle.

End of paper

Q1 a) $(x-2)(x+1) = x^2 - x - 2 = 0$ ✓ ✓

b) i) $\frac{d}{dx}(6x-5)^{\frac{1}{2}} = \frac{1}{2}(6x-5)^{-\frac{1}{2}} \times 6$ ✓ ✓

ii) $\frac{d}{dx}\, 3x(2x-1)^7 = 3(2x-1)^7 + 3x \times 7(2x-1)^6$ ✓ ✓

$= 3(2x-1)^7 + 21x(2x-1)^6$

iii) $\frac{d}{dx}\,\frac{x}{2x-3} = \frac{2x-(2x-3)\times 1}{(2x-3)^2}$ ✓ ✓

$= \frac{3}{(2x-3)^2}$

c) i) $\alpha+\beta = \frac{-b}{a} = \frac{-3}{2}$ ✓

ii) $\alpha\beta = \frac{c}{a} = \frac{5}{2}$ ✓

iii) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta+\alpha}{\alpha\beta}$ ✓

$= \frac{-3/2}{5/2}$

$= -\frac{3}{2} \times \frac{2}{5}$

$= -3/5$ ✓

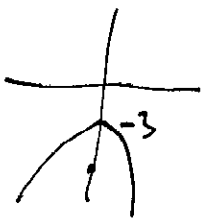
iv) $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$ ✓

$= (-3/2)^2 - 2 \times \frac{5}{2}$

$= \frac{9}{4} - 5$

$= -\frac{11}{4}$ ✓

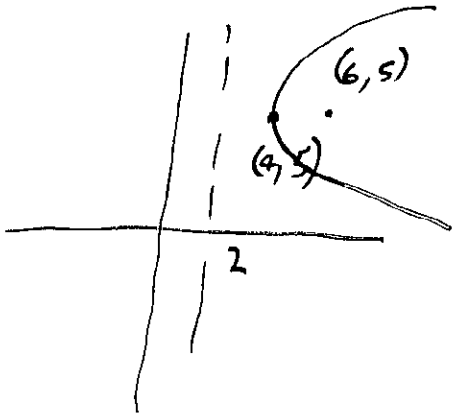
Q2

a) 

i) vertex $(0,-3)$ ✓

ii) $a = 2$ ✓

focus $(0,-5)$ ✓

b) 

$a = 2$

$(y-5)^2 = 8(x-4)$

$a = 2$ ✓

vertex $(4,5)$ ✓

$(y-k)^2 = 4a(x-h)^2$ ✓

c) $2x^2 - x - 10 \leq 0$

$P \ -20$

$S \ -1$

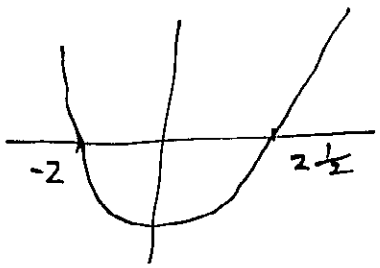
$F \ -5,4$

$2x^2 - 5x + 4x - 10 \leq 0$

$x(2x-5) + 2(2x-5) \leq 0$

$(2x-5)(x+2) \leq 0$

$-2 \leq x \leq 2\frac{1}{2}$ ✓



2 marks for factorising,
1 mark for answer.

---

d) Let $u = 5^x$

$u^2 - 6u + 5 \leq 0$

$(u-1)(u-5) \leq 0$ ✓

$u \leq 1, \ u \leq 5$ ✓

$5^x \leq 1 \quad 5^x \leq 5$

$x \leq 0 \quad x \leq 1$ ✓

Question 3

a) let $u = 5^x$

$u^2 - 6u + 5 = 0$

$(u-1)(u-5) = 0$ ✓

$u = 1, \quad u = 5$ ✓

$5^x = 1 \qquad 5^x = 5$

$x = 0 \qquad x = 1$ ✓

b) let the root be α & 2α

$\alpha + 2\alpha = \frac{-6}{1}$ ✓ $\qquad \alpha \times 2\alpha = \frac{m}{1}$

$3\alpha = -6 \qquad -2 \times -4 = m$

$\alpha = -2$ ✓ $\qquad m = 8$ ✓

c) $\Delta = b^2 - 4ac = 0$

$(2p)^2 - 4(7p+8) = 0$ ✓

$4p^2 - 28p - 32 = 0$

$p^2 - 7p - 8 = 0$ ✓

$(p-8)(p+1) = 0$

$p = 8$ or -1 ✓

Q4 a) $x^2 - x \equiv A(x-4)^2 + B(x-4) + C$

$x^2 = Ax^2 \qquad A = 1$

$x^2 - x \equiv (x-4)^2 + B(x-4) + C$

Let $x = 4$

$16 - 4 = 0 + 0 + C$

$C = 12$

$x^2 - x \equiv (x-4)^2 + B(x-4) + 12$

Let $x = 0$

$0 \equiv 16 - 4B + 12$

$4B = 28$

$B = 7$

$A = 1, \quad B = 7, \quad C = 12$

$x^2 - x = (x-4)^2 + B(x-4) + C$

$= x^2 - 8x + 16 + Bx - 4B + C$

$= x^2 - 8x + Bx + 16 - 4B + C$

$= x^2 + x(B-8) + 16 - 4B + C$

$\therefore B - 8 = -1$

$B = 7 \qquad A = 1$

$16 - 4B + C = 0$

$16 - 4 \times 7 + C = 0$

$16 - 28 + C = 0$

$C = 12$

$\therefore A = 1, B = 7, C = 12$

Q4 cont

b) i) $PA = 2PB$

$$\sqrt{(x+1)^2+(y-0)^2} = 2\sqrt{(x-2)^2+(y-0)^2} \quad \checkmark\checkmark$$

$$(x+1)^2+y^2 = 4((x-2)^2+y^2)$$

$$0 = 4(x-2)^2-(x+1)^2+3y^2$$

$$0 = 4(x^2-4x+4)-(x^2+2x+1)+3y^2 \quad \checkmark$$

$$0 = 3x^2-18x+15+3y^2$$

$$0 = x^2-6x+5+y^2 \quad \checkmark$$

$$0 = x^2-6x+y^2+5$$

ii)

$$0 = x^2-6x+9+y^2+5-9 \quad \checkmark$$

$$0 = (x-3)^2+y^2-4$$

$$(x-3)^2+y^2 = 4$$

centre $(3,0)$ ✓

radius 2 ✓