Student Number

NEWINGTON COLLEGE

2014
HSC Assessment 1

Year 12 Mathematics

General Instructions:

- Date of task – Tuesday 18th November (Period 6)
- Working time – 45 mins
- Weighting - 15%
- Board-approved calculators may be used.
- Start each question in a new booklet.
- Show all relevant mathematical reasoning and/or calculations.

Total marks – 30

Outcomes to be assessed:

P6	Relates the derivative of a function to the slope of its graph.
P7	Determines the derivative of a function through routine application of the rule of differentiation.
P8	Understands and uses the language and notation of calculus.
H6	Uses the derivative to determine the features of the graph of a function.
H7	Uses the features of a graph to deduce information about the derivative.

Section A (4 marks)

Multiple choice, answer on the multiple choice answer sheet provided.

1) What is the equation of the directrix of the parabola $y^2 = -12x$?

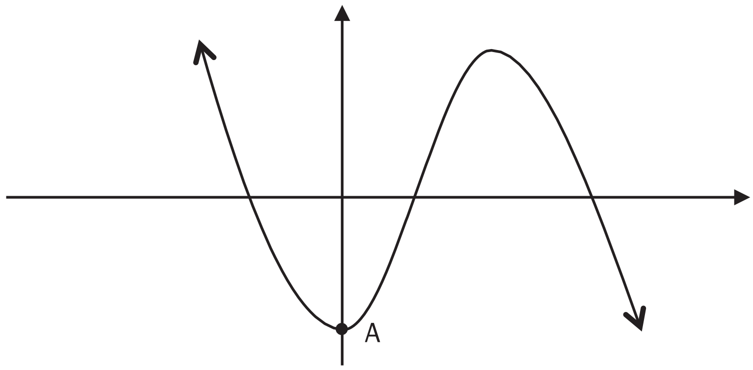
(A) $x = -3$

(B) $x = 3$

(C) $y = -3$

(D) $y = 3$

2) At the point A on the function shown below which of the following is true?



(A) $f'(x) < 0,\ f''(x) < 0$

(B) $f'(x) = 0,\ f''(x) > 0$

(C) $f'(x) = 0,\ f''(x) = 0$

(D) $f'(x) = 0,\ f''(x) < 0$

3) Which of the following is the second derivative of $\dfrac{1}{\sqrt{x}}$?

(A) $-\dfrac{1}{2\sqrt{x^3}}$

(B) $-\dfrac{3}{4\sqrt{x^5}}$

(C) $\dfrac{3}{4}\sqrt{x^5}$

(D) $\dfrac{3}{4\sqrt{x^5}}$

4) For what values of x is the curve $f(x) = 2x^3 + x^2$ concave down?

(A) $x < -\dfrac{1}{6}$

(B) $x > -\dfrac{1}{6}$

(C) $x < -6$

(D) $x > -6$

Section B (13 marks)

Answer in a new booklet.

1) If the quadratic equation $2x^2 - 3x + 4 = 0$ has the roots α and β, find

(i) $\alpha\beta$ (1 mark)

(ii) $\alpha + \beta$ (1 mark)

(iii) $4\alpha^2\beta + 4\alpha\beta^2$ (2 marks)

2) Find A and B given that $x^2 + x \equiv A\left(x^2 - 3x\right) + Bx$. (2 marks)

3) Consider the quadratic equation $(k+3)x^2 + 4x + k = 0$.

(i) Show that the discriminant is $16 - 12k - 4k^2$ (1 mark)

(ii) Find the value(s) of k for which the equation $(k+3)x^2 + 4x + k = 0$ has real roots. (2 marks)

4) A and B are the points $(1,\ 4)$ and $(-3,\ 2)$ respectively. The point $P(x, y)$ moves such that PA is perpendicular to PB.

(i) Show that the equation of the locus of P is the circle $x^2 + 2x + y^2 - 6y + 5 = 0$ (2 marks)

(ii) Find the radius and centre of this circle. (2 marks)

(End of section B)

Section C (13 marks)

Answer in a new booklet.

1) Find the first derivative of $f(x) = x(5x-1)^4$, giving your answer in its factorised form. (2 marks)

2) For the parabola $(x+2)^2 = -6y+24$, find the

   (i) coordinates of the vertex (1 mark)

   (ii) coordinates of the focus (1 mark)

3) Consider the curve given by $y = 3x^2 - x^3 + 9x - 2$.

   (i) Find the co-ordinates of the stationary points and determine their nature. (3 marks)

   (ii) Determine the co-ordinates of any point(s) of inflexion. (2 marks)

   (iii) Sketch the graph of $y = 3x^2 - x^3 + 9x - 2$, clearly indicating its stationary points, inflexion point(s) and *y*-intercept. (2 marks)

4) The line $5x+2y+14=0$ forms a focal chord of the parabola $(y-3)^2 = -4a(x+2)$. By using the focus, find a, the focal length of the parabola. (2 marks)

(End of section C)

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Multiple Choice Answer Sheet - Mathematics

Completely fill the response oval representing the most correct answer.

1	A ○	B ○	C ○	D ○
2	A ○	B ○	C ○	D ○
3	A ○	B ○	C ○	D ○
4	A ○	B ○	C ○	D ○

Year 12 Mathematics Assessment Task 1 Solutions and Marking Criteria

Section A

Q.	*Solution*	*Marking criteria*
1	$x=3 \therefore$ B	1 mark for correct answer
2	$f'(x)=0, f''(x)>0 \therefore$ B	1 mark for correct answer
3	$y=x^{-\frac{1}{2}}$ $y'=-\frac{1}{2}x^{-\frac{3}{2}}$ $y''=\frac{3}{4}x^{-\frac{5}{2}}$ $=\frac{3}{4\sqrt{x^5}} \therefore$ D	1 mark for correct answer
4	$f'(x)=6x^2+2x$ $f''(x)=12x+2$ $12x+2>0$ $12x>-2$ $x>-\frac{1}{6} \therefore$ A	1 mark for correct answer

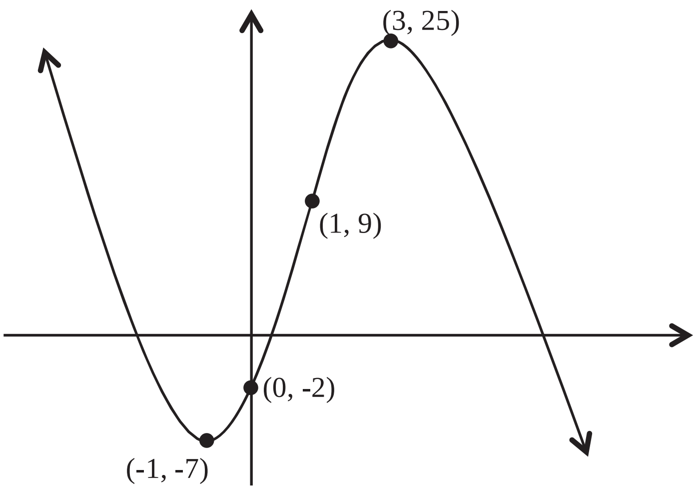
Section B

Q.	*Solution*	*Marking criteria*
1 (i)	$\alpha\beta = \frac{c}{a} = \frac{4}{2}$ $= 2$	1 mark for correct answer
(ii)	$\alpha + \beta = -\frac{b}{a} = -\frac{-3}{2}$ $= \frac{3}{2}$	1 mark for correct answer
(iii)	$4\alpha^2\beta+4\alpha\beta^2 = 4\alpha\beta(\alpha+\beta)$ $=(4\times2)\times\frac{3}{2}$ $=12$	1 mark for correct factorisation 1 mark for correct answer
2	$A(x^2-3x)+Bx$ $= Ax^2-3Ax+Bx$ $= Ax^2+(B-3A)x$ $\therefore A=1$ $\therefore B-3A=1$ $B-3=1$ $B=4$	1 mark for $A=1$ 1 mark for $B=4$

3 (i)	$\Delta = b^2 - 4ac$ $= 4^2 - 4\times(k+3)\times k$ $= 16 - 4k(k+3)$ $= 16 - 4k^2 - 12k$ $= 16 - 12k - 4k^2$	1 mark for correctly showing the discriminant
(ii)	$\Delta \geq 0$ $16 - 12k - 4k^2 \geq 0$ $4 - 3k - k^2 \geq 0$ $k^2 + 3k - 4 \leq 0$ $(k+4)(k-1) \leq 0$ $\therefore\ -4 \leq k \leq 1$	1 mark for $16-12k-4k^2 \geq 0$ or equivalent 1 mark of solution follows
4 (i)	$m_{PA} = \frac{y-4}{x-1}, \quad m_{PB} = \frac{y-2}{x+3}$ $m_{PA} \times m_{PB} = -1$ $\frac{y-4}{x-1} \times \frac{y-2}{x+3} = -1$ $(y-4)(y-2) = -(x-1)(x+3)$ $y^2 - 6y + 8 = -(x^2 + 2x - 3)$ $y^2 - 6y + 8 = -x^2 - 2x + 3$ $\therefore\ x^2 - 2x + y^2 - 6y + 5 = 0$	1 mark for $\frac{y-4}{x-1} \times \frac{y-2}{x+3} = -1$ or equivalent 1 mark for showing the correct equation of the circle
(ii)	$(x+1)^2 + (y-3)^2 = -5 + 1 + 9$ $= 5$ Centre $(-1,\ 3)$ Radius $\sqrt{5}$ units	1 mark for centre 1 mark for radius

Section C

Q.	*Solution*	*Marking criteria*
1	$f'(x) = x \times \left[4(5x-1)^3 \times 5\right] + 1 \times (5x-1)^4$ $= 20x(5x-1)^3 + (5x-1)^4$ $= (5x-1)^3(20x + 5x - 1)$ $= (5x-1)^3(25x-1)$	1 mark for correct application of the product rule and chain rule 1 mark for correct factorisation
2 (i)	$(x+2)^2 = -6y + 24$ $(x+2)^2 = -6(y-4)$ vertex is at $(-2, 4)$	1 mark for correct answer
(ii)	$a = \frac{6}{4} = \frac{3}{2}$ focus is at $\left(-2, \frac{5}{2}\right)$	1 mark if focus follows using $a = \frac{3}{2}$

3 (i)	$y' = 6x - 3x^2 + 9$ $y'' = -6x + 6$ Stationary points when $y' = 0$ $-3x^2 + 6x + 9 = 0$ $3x^2 - 6x - 9 = 0$ $x^2 - 2x - 3 = 0$ $(x-3)(x+1) = 0$ $\therefore\ x = 3$ or $x = -1$ When $x = 3,\ y'' = -6 \times 3 + 6 = -12$ $\therefore$ curve is concave down at $x = 3$ When $x = -1,\ y'' = -6 \times -1 + 6 = 12$ $\therefore$ curve is concave up at $x = -1$ When $x = 3,\ y = 25$ When $x = -1,\ y = -7$ $\therefore$ local minimum turning point at $(-1, -7)$ local maximum turning point at $(3, 25)$	1 mark for correct *x* values of stationary points 1 mark for correctly determining the nature of the stationary points 1 mark for correctly stating the co-ordinates of the stationary points
(ii)	Let $y'' = 0$ $-6x + 6 = 0$ $-6x = -6$ $x = 1$ Possible point of inflexion at $x = 1$ Testing, let $\varepsilon = 0.1$ At $x + \varepsilon,\ y'' < 0$ $x - \varepsilon,\ y'' > 0$ $\therefore$ point of inflexion exists At $x = 1,\ y = 9$ $\therefore$ point of inflexion at $(1, 9)$	1 mark for correct co-ordinates of point of inflexion 1 mark for checking for concavity change and hence concluding point of inflexion occurs
(iii)		1 mark for correct shape of curve indicating the stationary points 1 mark for labelling inflexion point and *y*-intercept

4	Vertex $(-2,\ 3)$ Focus $(x,\ 3)$ When $y=3$, $5x+6+14=0$ $5x=-40$ $x=-4$ $\therefore$ focus is at $(-4,3)$ $\therefore$ focal length, $a=2$ units	1 mark for identifying the *y* value of the focus 1 mark for finding the focal length