Student Number:

2015

YEAR 11

HSC Assessment Point 1

Mathematics

General Instructions

- Date of task – Tuesday 17th November
- Weighting – 15%
- Reading time - 3 minutes
- Working time – 45 minutes
- Write using black or blue pen
- BOSTES approved calculators may be used
- A reference sheet is provided at the back of this paper
- Show all necessary working

Total marks - 34

Question 1-3

34 marks

- Start each question in a new booklet

Outcomes

- Tangent to a Curve and Derivative of a Function
- The Quadratic Polynomial
- Locus and the Parabola

Question 1 (10 Marks)

a) Find the equation(s) of a point $P(x,y)$ that moves so that it is always 7 units from the line y = 2. 2

b) Differentiate $5x^3 - 2x + 4$ with respect to x. 1

c) Find the equation of the tangent to the curve $y = x^3 - 3x^2$ at the point (3, 0). 3

d) Find the equation of the directrix of the parabola $x^2 = -18y$. 2

e) Find the values of k for which the quadratic equation $2x^2 + 8x + k = 0$ has 2 distinct, real roots. 2

Question 2 (14 Marks) **Start this question in a new booklet**

a) Differentiate the following with respect to *x*, leaving your answer in fully factored form:

(i) $\dfrac{x-2}{x+3}$ 2

(ii) $x^3(2x-9)^5$ 3

b) Let α and β be the solutions of $x^2 + 12x + 3 = 0$. Find:

(i) $\alpha + \beta$ 1

(ii) $\alpha\beta$ 1

(iii) $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$ 2

(iv) $\alpha + \dfrac{3}{\alpha}$ 2

c) Find the values of *a*, *b* and *c*, given that $3x^2 - 14x - 2 \equiv a(x-2)^2 + b(x+4) + 1 + c$ 3

Question 3 (10 Marks) **Start this question in a new booklet**

a) Given that $f(x)=\sqrt{4x+b}$ and that $f'(11)=\frac{2}{5}$,

(i) Show that $f'(x)=\frac{2}{\sqrt{4x+b}}$ 2

(ii) Hence, find the value of b. 2

b) Consider the parabola $y^2-4y-6x-8=0$.

(i) Find the coordinates of the vertex 2

(ii) Find the coordinates of the focus 2

(iii) Show that the line $3x+4y-3=0$ intersects the parabola exactly twice. 2

End of Examination

Question 1		
Q	*Solution*	*Suggested marking criteria*
A	$y=9$ and $y=-5$	1 mark for $y=9$ 1 mark for $y=-5$
B	$\frac{d}{dx}(5x^3-2x+4)=15x^2-2$	1 mark for correct answer
C	$y=x^3-3x^2$ $y'=3x^2-6x$ When $x=3$, $y'=9$ $y-0=9(x-3)$ $y=9x-27$	1 mark for $y'=3x^2-6x$ 1 mark for $y'=9$ at $x=3$ 1 mark for $y=9x-27$ or equivalent
D	$4a=18$ $a=\frac{18}{4}$ $=\frac{9}{2}$ Directrix equation is $y=\frac{9}{2}$	1 mark for the focal length, *a* 1 mark for the equation of the directrix
E	Two distinct real roots $\therefore\ \Delta>0$ $\Delta=8^2-4\times2\times k$ $=64-8k$ $\therefore 64-8k>0$ $-8k>-64$ $k<8$	1 mark for $64-8k>0$ 1 mark if answer follows

SECTION 2		
Q	*Solution*	*Suggested marking criteria*
A i	$\frac{d}{dx}\left(\frac{x-2}{x+3}\right)=\frac{1(x+3)-1(x-2)}{(x+3)^2}$ $=\frac{5}{(x+3)^2}$	1 mark for correct substitution in quotient or product rule 1 mark if answer follows
A ii	$u=x^3 \qquad v=(2x-9)^5$ $u'=3x^2 \qquad v'=10(2x-9)^4$ $\frac{d}{dx}x^3(2x-9)^5=3x^2(2x-9)^5+10x^3(2x-9)^4$ $=x^2(2x-9)^4\left[3(2x-9)+10x\right]$ $=x^2(2x-9)^4(6x-27+10x)$ $=x^2(2x-9)^4(16x-27)$	1 mark for correct differentiation of *u* and *v* 1 mark for correct substitution into product rule 1 mark for fully factored result

B i	$\alpha+\beta=-\frac{12}{1}$ $=-12$	1 mark for correct answer
B ii	$\alpha\beta=\frac{3}{1}$ $=3$	1 mark for correct answer
B iii	$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}$ $=\frac{-12}{3}$ $=-4$	1 mark for $\frac{\alpha+\beta}{\alpha\beta}$ 1 mark for correct answer
B iv	From ii $\beta=\frac{3}{\alpha}$ $\therefore \alpha+\frac{3}{\alpha}=\alpha+\beta$ $=-12$	1 mark for $\beta=\frac{3}{\alpha}$ 1 mark for correct answer
C	$a(x-2)^2+b(x+4)+1+c$ $=a(x^2-4x+4)+b(x+4)+1+c$ $=ax^2-4ax+4a+bx+4b+1+c$ $=ax^2+(b-4a)x+(4a+4b+1+c)$ $\therefore\ a=3$ $\therefore\ b-4a=-14$ $b-12=-14$ $b=-2$ $\therefore\ 4a+4b+1+c=-2$ $12-8+1+c=-2$ $5+c=-2$ $c=-7$ $\therefore\ a=3,\ b=-2,\ c=-7$	1 mark for correct expansion and factorising 1 mark for correct value of *a* and if *b* follows 1 mark if value of *c* follows

SECTION 3		
Q	*Solution*	*Suggested marking criteria*
A i	$f(x)=(4x+b)^{\frac{1}{2}}$ $f'(x)=\frac{1}{2}(4x+b)^{-\frac{1}{2}}\times 4$ $=2(4x+b)^{-\frac{1}{2}}$ $=\frac{2}{(4x+b)^{\frac{1}{2}}}$ $=\frac{2}{\sqrt{4x+b}}$	1 mark for correctly showing use of the chain rule 1 mark for fully showing the result

A ii	$\frac{2}{\sqrt{4x+b}}=\frac{2}{5}$ when $x=11$ $\frac{2}{\sqrt{44+b}}=\frac{2}{5}$ $\therefore \sqrt{44+b}=5$ $44+b=25$ $b=-19$	1 mark for $\frac{2}{\sqrt{44+b}}=\frac{2}{5}$ 1 mark if solution follows
B i	$y^2-4y=6x+8$ $(y-2)^2=6x+12$ $(y-2)^2=6(x+2)$ Vertex is at $(-2,2)$	1 mark for $y(y-4)=6\left(x+\frac{4}{3}\right)$ 1 mark if vertex follows
B ii	$4a=6$ $a=\frac{3}{2}$ Focus is at $\left(-\frac{1}{2},2\right)$	1 mark for focal length following from b i 1 mark if focus follows
B iii	$3x+4y-3=0$ $3x=-4y+3$ $6x=-8y+6$ Subs into parabola $y^2-4y-(-8y+6)-8=0$ $y^2-4y+8y-6-8=0$ $y^2+4y-14=0$ $\Delta=4^2-4\times1\times-14$ $=16+56$ $=72$ $\Delta>0$ $\therefore$ two points of intersection between the line and parabola	1 mark for $y^2-4y-(-8y+6)-8=0$ or equivalent 1 mark for correctly using the discriminant to show the result