NORTH SYDNEY BOYS HIGH SCHOOL

2010 YEAR 12 HSC
ASSESSMENT TASK 1

Mathematics

General Instructions

- Working time – 50 minutes
- Write on one side of the paper (with lines) in the booklet provided
- Write using blue or black pen
- Board approved calculators may be used
- All necessary working should be shown in every question
- Each new question is to be started on a **new page.**

Total Marks (50)

- Attempt all questions

Class Teacher:
(Please tick or highlight)

- O Mr Berry
- O Mr Fletcher
- O Mr Rezcallah
- O Mr Lowe
- O Mr Ireland
- O Mr Barrett
- O Mr Trenwith
- O Mr Weiss

Student Number: ....

Question	1	2	3	4	5	Total	Total
Mark	$\frac{}{9}$	$\frac{}{13}$	$\frac{}{8}$	$\frac{}{11}$	$\frac{}{9}$	$\frac{}{50}$	$\frac{}{100}$

2010 - Past paper.

Question 1 (9 marks) *Start a new page* **Marks**

(a) Differentiate with respect to x:

(i) $\frac{x^2+2x}{x}$ **2**

(i) $(4-x^2)^3$ **2**

(iii) $\frac{3x+1}{2x-1}$ **2**

(b) Find the equation of the tangent to the curve $y = x^3 - 2x + 2$ at the point (2, 6) **3**

Question 2 (13 marks) *Start a new page*

(a) Evaluate $\sum_{n=1}^{5} n^2$ **1**

(b) The third term of an arithmetic progression is 21 and the 9th term is 57.

(i) Find the value of the first term and the common difference. **2**

(ii) Find the sum of the first 80 terms. **2**

(c) Find the 10th term of the geometric series $-5,\ 10, -20, ...$ **3**

(d) Frank keeps a big stack of paint cans in his hardware store. The stack has 3 cans on the top row and each succeeding row has 2 more cans than the previous row. (A diagram may be helpful here.)

(i) Find the number of cans in the 12th row. **2**

(ii) If there are 224 cans in the stack, how many rows are there? **3**

Question 3 (8 marks) *Start a new page*

(a) Consider the graph of $y = x^3 + 3x^2 - 9x - 11$

(i) Show that this function has stationary points at $(1, -16)$ and $(-3, 16)$ and determine their nature. **3**

(ii) Find any points of inflexion. **2**

(iii) Sketch the curve, showing all important features (not *x*-intercepts) **2**

(iv) For what values of *x* is the curve increasing? **1**

Question 4 (11 marks) *Start a new page*

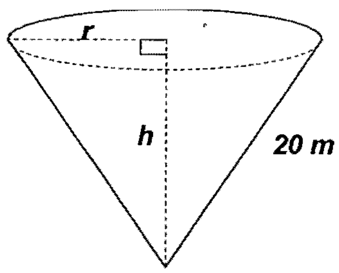
(a) A particle is moving in a straight line so that its distance, x metres, from a fixed point, O, after t seconds is given by $x = \frac{1}{3}t^3 - \frac{7}{2}t^2 + 6t$.

(i) Find the equation for its velocity. **1**

(ii) Find its initial position and velocity. **2**

(ii) When is the particle at rest? **2**

(b) A cone shaped storage tank is constructed. It has a slant height of 20 metres. The top has radius r metres and the whole tank is h metres high, as shown in the diagram:



(i) Express r in terms of h. **1**

(i) Show that the volume, V m^3 of the tank can be expressed by

$$V = \frac{\pi}{3}(400h - h^3)$$ **2**

(ii) Hence find the value of h which will give the cone its maximum volume. **3**

Question 5 (9 marks) *Start a new page*

(a) "The inflation level I is still increasing, but at a reduced rate of increase."

(i) Sketch a graph to illustrate the above statement. **1**

(ii) What does the above statement mean with reference to

$\frac{dI}{dt}$ and $\frac{d^2I}{dt^2}$? **2**

(b) A container holds 50 litres of water. A pump extracts 10 litres on the first cycle and $7 \cdot 5$ litres on the second. In each future pumping cycle the pump extracts $\frac{3}{4}$ of the amount that the previous cycle extracted.

Show that the container will never be emptied, and find how much water will finally remain in the container. **3**

(c) If the sum of the first n terms of a sequence is given by $S_n = n\,(2n - 1)$, find a formula for the n^{th} term of the sequence, T_n. **3**

Q1 (a) (i) $\frac{d}{dx}\left(\frac{x^2+2x}{x}\right) = \frac{d}{dx}(x+2)$ ✓

$= 1$ ✓

(ii) $\frac{d}{dx}\left(\left(4-x^2\right)^3\right) = 3\cdot\left(4-x^2\right)^2\cdot -2x$ ✓

$= -6x\left(4-x^2\right)^2$ ✓

(iii) $\frac{d}{dx}\left(\frac{3x+1}{2x-1}\right) = \frac{(2x-1)(3)-(3x+1)(2)}{(2x-1)^2}$ ✓

$= \frac{6x-3-6x-2}{(2x-1)^2}$

$= \frac{-5}{(2x-1)^2}$ ✓

(b) $y = x^3 - 2x + 2$

$y' = 3x^2 - 2$ ✓

$\therefore$ at $(2,6)$, $y' = 3\cdot 2^2 - 2$

$= 10$ ✓

$\therefore y - 6 = 10(x-2)$ ✓

$y = 10x - 14.$

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Q2 (a) $\sum_{n=1}^{5} n^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2$

$= 55$ ✓

(b) (i) $T_3 = a + 2d = 21$

$T_9 = a + 8d = 57$

$\therefore 6d = 36$

$d = 6$ ✓

$a = 9$ ✓

(ii) $S_{80} = \frac{80}{2}\left(2 \times 9 + 79 \times 6\right)$ ✓

$= 19\,680$ ✓

(c) $r = \frac{10}{-5} = -2$ ✓

$\therefore T_{10} = -5 \cdot (-2)^9$ ✓

$= 2560$ ✓

(d) (i) $a = 3, \quad d = 2$

$T_{12} = 3 + 11 \times 2$ ✓

$= 25$ ✓

(ii) $224 = \frac{n}{2}\left(6 + 2(n-1)\right)$ ✓

$= n(3 + n - 1)$

$= n(n+2$

$\therefore n^2 + 2n - 224 = 0$ ✓

$(n+16)(n-14) = 0$ $\quad \therefore n = 14$ (as $n > 0$)

ie. 14 rows ✓

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Q3 (a) $y = x^3 + 3x^2 - 9x - 11$

(i) $y' = 3x^2 + 6x - 9$

$= 3(x+3)(x-1)$ ✓

$y'' = 6x + 6$

$= 6(x+1)$

For stat. point, $y' = 0$ $\therefore$ $x = -3$, $y = 16$ or $x = 1$, $y = -16$ ✓

At $(-3, 16)$, $y'' = -12 < 0$ $\therefore$ max. turn. pt. at $(-3, 16)$

At $(1, -16)$, $y'' = 6(2) > 0$ $\therefore$ min. turn. pt. at $(1, -16)$ ✓

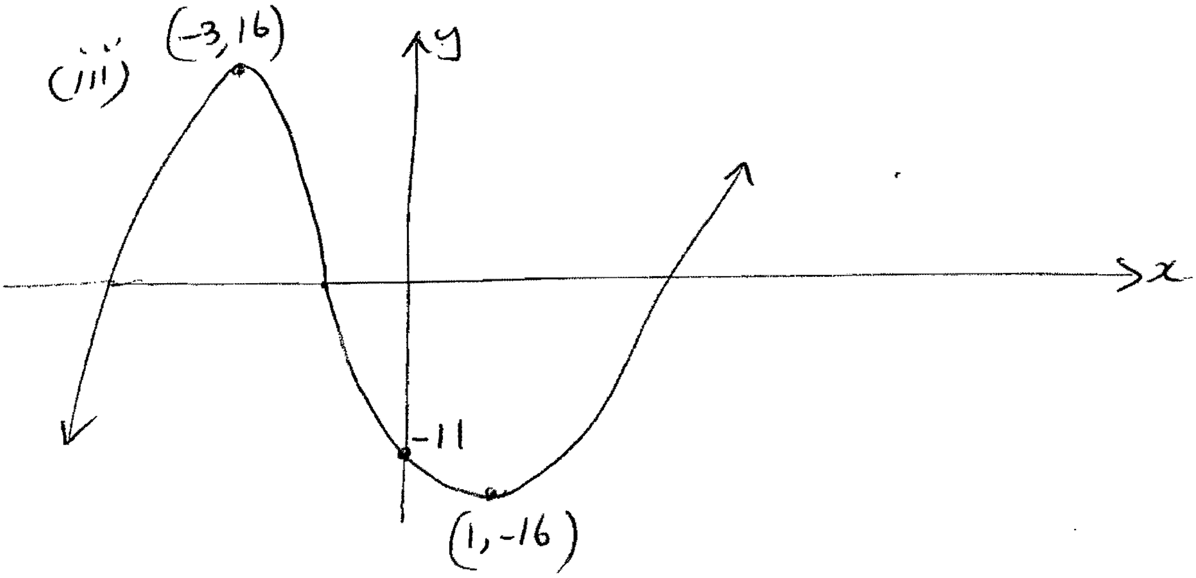
(ii) For inflexions, $y'' = 0$ $\therefore$ $x = -1$, $y = 0$ ✓

Test :-

x	-2	-1	0
y''	-	0	+

$\therefore$ change of concavity $\therefore$ inflexion at $(-1, 0)$. ✓

(iii)



✓✓

(iv) From the graph, y is increasing for $x < -3$ or $x > 1$ ✓

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Q4 (a) $x = \frac{1}{3}t^3 - \frac{7}{2}t^2 + 6t$

(i) $v = \frac{dx}{dt} = t^2 - 7t + 6$ ✓

(ii) at $t=0$, $x = 0$ $\therefore$ at origin ✓

$v = 6$ $\therefore$ 6m/s to right. ✓

(iii) "at rest" $\Rightarrow v = 0$

$\therefore (t-1)(t-6) = 0$ ✓

$\therefore t = 1, 6$

So $\therefore$ at rest at 1 second & at 6 seconds. ✓

(b) (i) $r^2 = 20^2 - h^2$

$\therefore r = \sqrt{400 - h^2}$ ✓

(ii) $V = \frac{1}{3} \cdot \pi \cdot r^2 \cdot h$

$= \frac{1}{3}\pi(400 - h^2)h$

$V = \frac{\pi}{3}(400h - h^3)$ ✓✓

(iii) For max., $\frac{dV}{dh} = 0$

$\therefore \frac{\pi}{3}(400 - 3h^2) = 0$ ✓

$3h^2 = 400$

$h = \frac{20}{\sqrt{3}}$ (as $h > 0$) ✓

Test: $\frac{d^2V}{dh^2} = \frac{\pi}{3}(-6h) < 0$

$\therefore$ max. at $h = \frac{20}{\sqrt{3}}$ m. ✓

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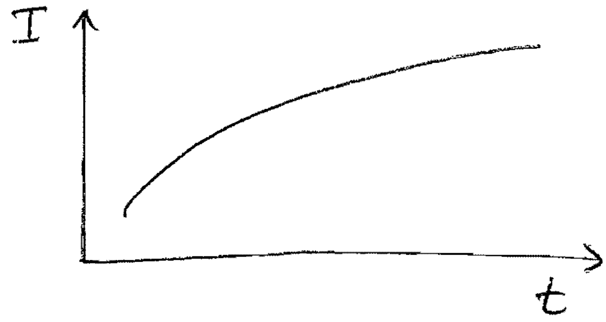
Q5 (a) (i)



(ii) $\frac{dI}{dt} > 0$ means inflation is increasing.

$\frac{d^2I}{dt^2} < 0$ means the rate of increase is slowing down.

(b) The amounts extracted form an infinite G.P. with $a = 10$, $r = \frac{3}{4}$,

Since water extracted $= 10 + 7.5 + \cdots$

$\therefore S_\infty = \frac{10}{1 - \frac{3}{4}}$

$= 40$ litres

$\therefore$ 10 litres will remain.

(c) $T_n = S_n - S_{n-1}$

$= n(2n-1) - (n-1)(2(n-1)-1)$

$= 2n^2 - n - (n-1)(2n-3)$

$= 2n^2 - n - (2n^2 - 5n + 3)$

$= 4n - 3$

$\therefore T_n = 4n - 3$

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