St Ignatius' College Riverview

Mathematics
Assessment Task I
Year 12

(Time allowed – 60 minutes)

INSTRUCTIONS: * Answer each question on a separate answer sheet.
* Write your name and your teacher's name on each answer sheet.
* Answer ALL questions.
* Approved calculators may be used.

QUESTION 1 : SEQUENCES & SERIES (20 marks)

a) The formula for the ***n*th term** of an arithmetic sequence is : $T_n = 2n + 4$

i) What are the **4th** and **5th** terms? **(2 marks)**

ii) What is the **common difference**? **(1 mark)**

b) Find the values of x such that $\{3, x+4, x+10.....\}$ forms a geometric sequence. **(2 marks)**

c) The 3rd term of an arithmetic progression is 16, and the 12th term is 79.

i) Find the **first term** and **common difference**. **(2 marks)**

ii) Find the **sum** of the first 25 terms. **(2 marks)**

d) A super-ball drops from a height of 9 metres and bounces continually, each successive height being $\frac{2}{3}$ of the previous height.

i) Show that the first distance travelled down and up is 15 metres. **(1 mark)**

ii) When the ball finally comes to rest, through what distance will it have travelled in total? **(2 marks)**

e) How many terms of the series $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + ...$ give a sum of $\frac{1023}{1024}$? **(3 marks)**

f) The price of windows in a house is \$500 for the first window, then \$300 for each additional window.

i) Find a formula for the cost of *n* windows. **(1 marks)**

ii) How much will fifteen windows cost? **(2 marks)**

iii) What is the maximum number of windows whose total cost is less than \$10,000? **(2 marks)**

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QUESTION 2 : THE QUADRATIC FUNCTION (20 marks)

a) By observing the roots in fig. 1, write the equation of the parabola. **(2 marks)**

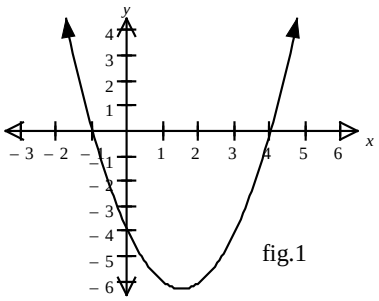


fig.1

b) Show that equation $2x^2 + x + 4 = 0$ has no real roots. **(3 marks)**

c) Find all values of k for which the expression $kx^2 + 3kx + 6$ is positive definite. **(3 marks)**

d) If α and β are the roots of the quadratic equation $x^2 - 3x - 6 = 0$, find the value of:

i) $\alpha + \beta$ **(1 mark)**

ii) $\alpha\beta$ **(1 mark)**

iii) $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$ **(2 marks)**

iv) $\alpha^2 + \beta^2$ **(2 marks)**

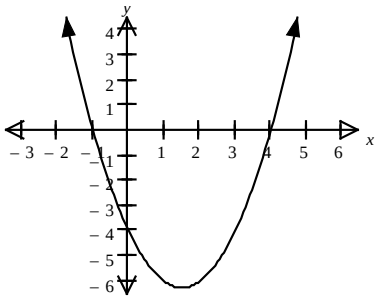
e) Solve: $x^2 + \dfrac{4}{x^2} = 5$ **(3 marks)**

f) Find values of *a*, *b* and *c* if $x^2 - x \equiv a(x+3)^2 + bx + c - 1$ **(3 marks)**

Solutions

			Markers Comments
a)	The formula for the ***n*th term** of an arithmetic sequence is : $T_n = 2n + 4$		
i)	What are the **4th** and **5th** terms?		
	$T_4 = 2(4) + 4$		
	$T_4 = 12$	1	
	$T_5 = 2(5) + 4$		
	$T_5 = 14$	1	
ii)	What is the **common difference**?		
	$d = T_5 - T_4$		
	$d = 14 - 12$	1	
	$d = 2$		
b)	Find the value of x such that $\{3, x+4, x+10.....\}$ forms a geometric sequence.		
	$\frac{x+10}{x+4} = \frac{x+4}{3}$	1	
	$3(x+10) = (x+4)(x+4)$		
	$3x + 30 = x^2 + 4x + 4x + 16$		
	$3x + 30 = x^2 + 8x + 16$		
	$x^2 + 5x - 14 = 0$		
	$(x+7)(x-2) = 0$		
	$x = -7 \text{ or } 2$	1	
c)	The 3rd term of an arithmetic progression is 16, and the 12th term is 79.		
	i) Find the **first term** and **common difference**.		
	$a + 2d = 16$ .....(1)		
	$a + 11d = 79$ .....(2)		
	$Eqn(2) - Eqn(1)$	1	
	$9d = 63$		
	$d = 7$		
	Sub $d = 7$ into $Eqn(1)$		
	$a + 2(7) = 16$	1	
	$a = 2$		
	ii) Find the **sum** of the first 25 terms.		
	$S_n = \frac{n}{2}(2a + (n-1)d)$		
	$S_{25} = \frac{25}{2}(2(2) + (25-1)7)$	1	
	$S_{25} = 2150$	1	
d)	A super-ball drops from a height of 9 metres and bounces continually, each successive height being $\frac{2}{3}$ of the previous height.		

	i) Show that the first distance travelled down and up is 15 metres. Distance down once = 9 metres		
d)	**i)Continued.** Distance up once = $9 \times \frac{2}{3}$ = 6 $\therefore$ Total distance = 9+6 = 15 metres	1	
ii)	When the ball finally comes to rest, through what distance will it have travelled in total? $S_\infty = \frac{a}{1-r}$ Total distance $= 2\left(\frac{6}{1-\frac{2}{3}}\right) + 9$ Total distance $= 45$ metres	1 1	
e)	How many terms of the series $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + ...$ give a sum of $\frac{1023}{1024}$? $a = \frac{1}{2}, \quad r = \frac{1}{2}$ $\frac{a(1-r^n)}{1-r} = \frac{1023}{1024}$ $\frac{\frac{1}{2}\left(1-\frac{1}{2}^n\right)}{1-\frac{1}{2}} = \frac{1023}{1024}$ $1-\frac{1}{2}^n = \frac{1023}{1024}$ $\frac{1}{2}^n = \frac{1}{1024}$ $\left(2^{-1}\right)^n = 2^{-10}$ $2^{-n} = 2^{-10}$ $n = 10$	1 1 1	
f)	The price of windows in a house is \$500 for the first window, then \$300 for each additional window. **i)** Find a formula for the cost of *n* windows. $T_n = 200 + 300n$ **ii)** How much will fifteen windows cost? $T_n = 200 + 300n$ where $n = 15$ $T_{15} = 200 + 300(15)$ $T_{15} = 4700$ $\therefore$ 15 widows will cost \$4700 **iii)** What is the maximum number of windows	1 1 1	

	whose total cost is less than \$10,000? $10000 = 200 + 300n$ $9800 = 300n$ $n = 32.6$ & $\therefore$ 32 windows	 1 1	
2a)	By observing the roots in fig. 1, write the equation of the parabola. $\alpha = -1$ and $\beta = 4$ $(x-\alpha)(x-\beta) = 0$ $\therefore (x+1)(x-4) = 0$ $\therefore x^2 - 4x + x - 4 = 0$ $\therefore x^2 - 3x - 4 = 0$  	 1 1	
b)	Show that equation $2x^2 + x + 4 = 0$ has no real roots. No real roots $\therefore \Delta < 0$ $b^2 - 4ac < 0$ $(1)^2 - 4(2)(4) < 0$ $1 - 32 < 0$ $-31 < 0$ $\therefore \Delta < 0$ $\therefore$ No real roots	 1 2	
c)	Find all values of k for which the expression $kx^2 + 3kx + 6$ is positive definite. Positive Definite $\therefore a > 0$ *and* $\Delta < 0$ $b^2 - 4ac < 0$ $(3k)^2 - 4(k)(6) < 0$ $9k^2 - 24k < 0$ $3k(3k-8) < 0$ $\therefore 0 < k < \frac{8}{3}$	 1 1 1	
d)	If α and β are the roots of the quadratic equation $x^2 - 3x - 6 = 0$, find the value of: **i)** $\alpha + \beta$ $\alpha + \beta = \frac{-b}{a}$ $\alpha + \beta = \frac{-(-3)}{1}$ $\alpha + \beta = 3$ **ii)** $\alpha\beta$	 1	

	$\alpha\beta = \frac{c}{a}$ $\alpha\beta = \frac{-6}{1}$ $\alpha\beta = -6$	1	
	iii) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta}{\alpha\beta} + \frac{\alpha}{\alpha\beta}$ $= \frac{\alpha+\beta}{\alpha\beta}$ $= \frac{3}{-6}$ $= -\frac{1}{2}$	1 1	
	iv) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$ $= (3)^2 - 2(-6)$ $= 9 + 12$ $= 21$	1 1	
e)	Solve: $x^2 + \frac{4}{x^2} = 5$ $x^4 + 4 = 5x^2$ $x^4 - 5x^2 + 4 = 0$ Let $m = x^2$ $\therefore m^2 - 5m + 4 = 0$ $\therefore (m-4)(m+1) = 0$ $\therefore m = 4 \text{ or } -1$ But $m = x^2$ $\therefore x^2 = 4$ or $x^2 = -1$ $\therefore x = \pm 2$ or $x = \pm\sqrt{-1}$ (invalid) $\therefore x = 2 \text{ or } -2$	1 1 1	
f)	Find values of *a*, *b* and *c* if $x^2 - x \equiv a(x+3)^2 + bx + c - 1$ $a(x+3)^2 + bx + c - 1 = a(x^2 + 6x + 9) + bx + c - 1$ $= ax^2 + 6ax + 9a + bx + c - 1$ $= ax^2 + (6a+b)x + 9a + c - 1$ For $x^2 - x \equiv a(x+3)^2 + bx + c - 1$ $a = 1$ .....(1) $6a + b = -1$ .....(2) $9a + c - 1 = 0$ .....(3) Substitute (1) into (2)	1 1	

$6(1)+b=-1$ $b=-7$ Substitute (1) into (3) $9(1)+c-1=0$ $c=-8$ $\therefore a=1,\ b=-7,\ c=-8$	1	