QUESTION 1 (14 marks)

(a) A parabola has equation $8y = x^2 - 6x - 3$

i) Write its equation in the form $(x-h)^2 = 4a(y-k)$ **(2)**

ii) Find the coordinates of the vertex **(1)**

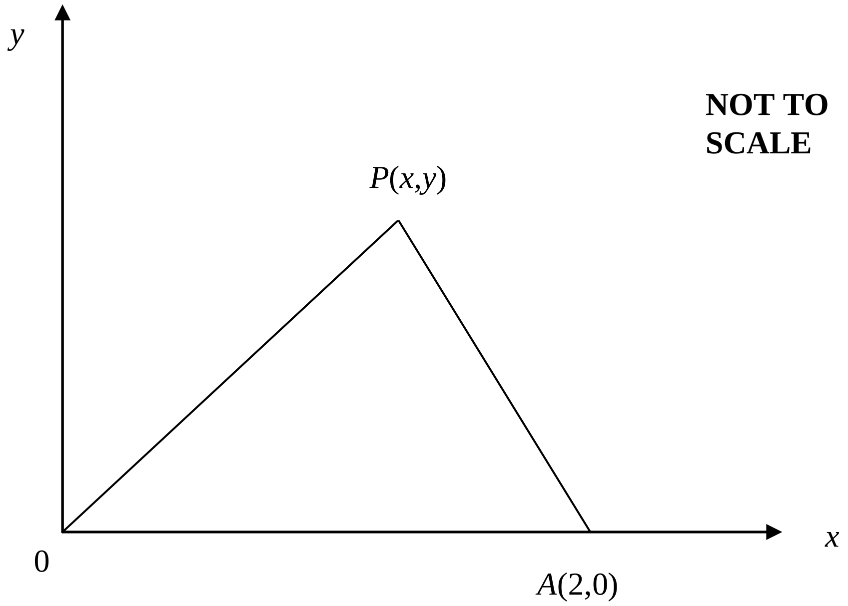
iii) Find the coordinates of the focus **(1)**

iv) What is the equation of the directrix? **(1)**

v) Sketch the parabola labelling the above features. **(2)**

(b) Find the equation of the parabola with vertex (-5,0) and focus (0,0). **(2)**

(c)



i) Write down the gradient of *AP* in terms of *x* and *y* **(1)**

ii) Show that the equation of the locus of all points *P* such that *OP* is perpendicular to *AP* is $x^2 - 2x + y^2 = 0$

(2)

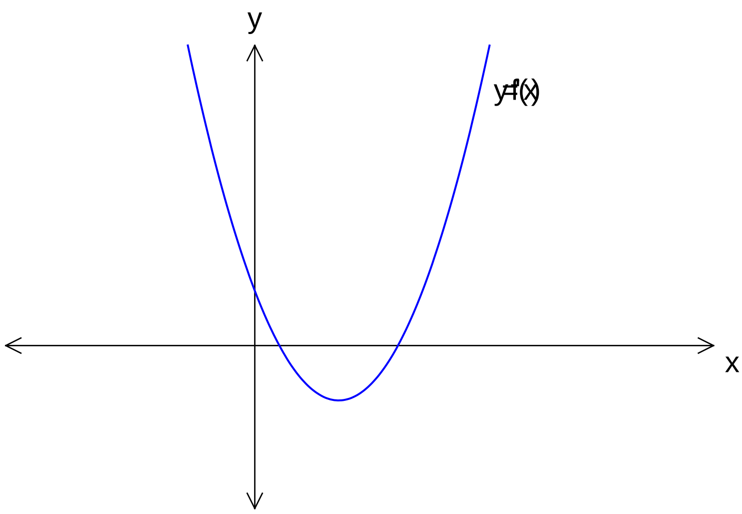
iii) Hence show that the locus of all points *P* such that *OP* is perpendicular to *AP* is a circle. Write down the centre and radius of this circle.

(2)

QUESTION 2 **(12 marks)**

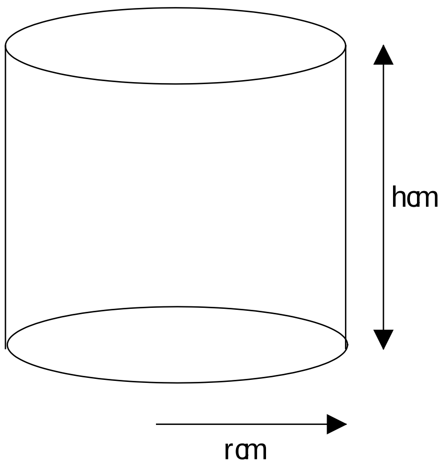
(a) Find the domain for which $f(x) = 6 - 2x - x^2$ is a decreasing function. **(2)**

(b) Copy carefully or trace, the sketch of $y = f'(x)$ given below. On the same set of axes, draw a possible graph of $y = f(x)$. **(1)**



(c) Sketch the curve which passes through the origin with the equation $y = f(x)$ given that $\frac{dy}{dx} = 0$ at the points $(0,0)$ $(2,-4)$ and $(4,0)$ and that $\frac{dy}{dx} = -1$ when $x = -1$. **(2)**

(d) A cylinder is closed at both ends. Its surface area is given by $SA = 2\pi r(r+h)$ and is known to be 600π cm^2.



i) Show that the height is given by $h = \frac{300}{r} - r$ **(2)**

ii) The volume of the cylinder is given by $V = \pi r^2 h$. Show that the volume of the cylinder may also be written as $V = 300\pi r - \pi r^3$. **(1)**

iii) Find the radius which gives the maximum volume. **(3)**

iv) Show that the maximum volume is 2000π cm^3. **(1)**