SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2013
November
Year 11 Assessment
Task 1

Mathematics (2 Unit)

General Instructions

- Reading Time – 5 Minutes
- Working time – 1 ½ Hours
- Write using black or blue pen. Pencil may be used for diagrams.
- Board approved calculators maybe used.
- Marks may NOT be awarded for messy or badly arranged work.
- EACH QUESTION IS TO BE RETURNED IN A SEPARATE BUNDLE. Q1, Q2, Q3, Q4 and Q5
- All necessary working should be shown in every question if full marks are to be awarded
- Leave all answers in simplified exact form unless indicated otherwise.

Total Marks – 80

- Attempt questions 1 – 5

Examiner: *F Nesbitt*

This is an assessment task only and does not reflect the content or format of the Higher School Certificate

Question 1 (16 marks)

(a) Show that $x^4 - 3x^2 - 2$ is an even function. 2

(b) Find $\lim_{x \to 3} \frac{3x}{x+2}$ 1

(c) Evaluate $\sum_{n=1}^{6} (2n-3)$ 2

(d) Differentiate:

(I) $y = x^2 + 2x$

(ii) $y = 3x - \frac{2}{x^3}$

(iii) $y = \frac{2x}{1+x^2}$ 5

(e) Simplify fully:

(i) $a^5 \div a^{-3}$

(ii) $\left(\frac{2n^2}{-m}\right)^3 \left(\frac{-m^3}{n}\right)^2$ 3

(iii) $\log_3 81 + \log_3 9 + \log_3 243$ 1

(f) $f(x) = 3\mathrm{x}^3 - 3x^2 + bx + 5$

Find the value of b if $f(x)$ has only one stationary point. 2

Question 2: (16 marks)

(a) The quadratic equation $3x^2 + 9x + 1 = 0$ has roots α *and* β. Write the value of:

(i) $\alpha + \beta$

(ii) $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$

(iii) $(\alpha + 2)(\beta + 2)$ 5

(b) Find the equation of the normal to $y = \dfrac{x^2}{2}$ at the point (4,8). 3

(c) Find the largest possible domain and range for each of the following functions

(i) $y = 3x^2 - 5$

(ii) $y = 3^x + 1$

(iii) $y = \dfrac{1}{x+5}$

(iv) $y = \dfrac{1}{\sqrt{2 - x^2}}$ 8

Question 3 (16 marks)

(a) A die is biased so that to throw a six is twice as likely as throwing any other number. What is the probability of throwing:

(i) a two?

(ii) a six? 2

(b) The first four terms of an arithmetic series are -5 -1+ 3+ 7 +……

(i) What is the nth term

(ii) Find the sum of the first 20 terms 3

(c) Find the equation of the locus of a point which is always $\sqrt{3}$ units From the point (-3 ,5) 1

(d) Find the equation of the locus of a point that moves so that It is always equidistant from the line $6x+8y-5=0$ and The line $5x+12y-1=0$. 3

(e) Lily bought 2 tickets in a small raffle. 20 tickets were sold. There were two prizes. What is the probability that Lily won:

(i) both prizes?

(ii) at least one prize? 3

(f) Find what value(s) of k does the equation $x^2+kx+8-k=0$ have

(i) equal roots (ii) real and distinct roots 4

Question 4 (16 marks)

(a) Express the following series in sigma notation:

$1 \times 4 \ + \ 2 \times 5 \ + \ 3 \times 6 + \ \dots.. \ 10 \times 13$ 2

(b) What is the value of $f(3a-1)$ if $f(x) = (2x-3)$? 2

(c) Simplify fully:

(i) $8^{\frac{1}{2}} \div 2^{\frac{1}{2}}$

(ii) $\log_a\left(a^2+a\right) - \log_a\left(a+1\right)$ 3

(d) For the curve $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x - 1$:

(i) Find the coordinates of any stationary points and determine their nature. 3

(ii) Find any point(s) of inflexion. 2

(iii) Sketch the curve in the domain $-4 \le x \le 3$ 3.

(iv) What is the maximum value of $f(x)$ in the given domain? 1

Question 5 (16 marks)

(a) Write the coordinates of the focus and the equation of the directrix of the parabola $3y^2 = 4x$ 2

(a) Solve the inequation: $2x^2 - 9x + 7 > 0$ 3

(b) Find, from first principles, the derivative of $y = x^2 - 6x + 5$ 3

(c) Find the domain and range of $y = \log(x-2)(x-5)$ 2

(d) George has inherited a sum of \$80 000. He deposited it in the bank in an account earning five percent per annum interest compounded annually. He enjoys an annual European holiday and plans to withdraw \$10 000 at the end of each year to finance his holiday.

Let A_n be the amount in the account after n years.

(i) Show that

$A_3 = 80000(1.05)^3 - 10000(1.05)^2 - 10000(1.05) - 10000$ 2

(ii) Find an expression for A_n.

(iii) Calculate the number of annual holidays George will be able to fully finance from his account. 3

2013 YEAR 11 MATHS (2U) TASK 1

a) Let $f(x) = x^4 - 3x^2 - 2$.

$$f(-x) = (-x)^4 - 3(-x)^2 - 2$$

(2)

$$= x^4 - 3x^2 - 2$$

$$= f(x)$$

$\therefore f(x) = x^4 - 3x^2 - 2$ is even.

b) $\lim_{x \to 3} \frac{3x}{x+2} = \frac{9}{5}$. (1)

c) $\sum_{n=1}^{6} (2n-3)$ $\Rightarrow$ first term $a = -1$.
last term $2(6) - 3 = 9$.

$$S_6 = \frac{n}{2}(a + \ell).$$

(2)

$$= \frac{6}{2}(-1 + 9)$$

$$= 24.$$

d) i) $y = x^2 + 2x$. $\frac{dy}{dx} = 2x + 2$. (1)

(2)

ii) $y = 3x - 2x^{-3}$. $\frac{dy}{dx} = 3 + 6x^{-4} = 3 + \frac{6}{x^4}$

(iii) $y = \frac{2x}{1+x^2} = \frac{u}{v}$. $\frac{dy}{dx} = \frac{vu' - uv'}{v^2}$. (2)

$$= \frac{(1+x^2)(2) - (2x)(2x)}{(1+x^2)^2}.$$

$$= \frac{2 + 2x^2 - 4x^2}{(1+x^2)^2}$$

$$= \frac{2-2x^2}{(1+x^2)^2}$$

$$= \frac{2(1-x^2)}{(1+x^2)^2} \qquad \frac{-2(x^2-1)}{(x^2+1)^2}$$

e) i) $\frac{a^5}{a^{-3}} = a^8$ 1

ii) $\left(\frac{2n^2}{-m}\right)^3 \left(\frac{-m^3}{n}\right)^2$ 2

$$= \left(\frac{8n^6}{-m^3}\right)\left(\frac{m^6}{n^2}\right)$$

$$= \frac{8n^6 m^6}{-m^3 n^2}$$

$$= -8n^4 m^3$$

iii) $\log_3 81 + \log_3 9 + \log_3 243$ 1

$4 + 2 + 5 = 11$

f) $f(x) = 3x^3 - 3x^2 + bx + 5$

$f'(x) = 9x^2 - 6x + b$ 2

for only 1 stn $\Delta = 0$

$$\Delta = b^2 - 4ac$$

$$= (-6)^2 - 4(9)(b) = 0$$

$$36 - 36b = 0$$

$$b = 1$$

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2 a) $3x^2 + 9x + 1 = 0$

$\alpha+\beta = -\frac{b}{a}$ $\quad a = 3,\ b = 9,\ c = 1$

$\alpha\beta = \frac{c}{a}$

(i) $\alpha+\beta = -\frac{9}{3} = -3$ 1

(ii) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{-3}{\frac{1}{3}} = -9$ 2

(iii) $(\alpha+2)(\beta+2)$

$= \alpha\beta + 2\alpha + 2\beta + 4$

$= \alpha\beta + 2(\alpha+\beta) + 4$

$= \frac{1}{3} + 2\times -3 + 4 = -1\frac{2}{3}$ 2 $\left(-\frac{5}{3}\right)$

(b) $y = \frac{x^2}{2}$

$y' = \frac{2x}{2} = x$

At $x = 4$ gradient tangent is $m_T = 4$ $\quad (m_1 m_2 = -1)$

$\therefore$ gradient of normal is $m_N = -\frac{1}{4}$ 1

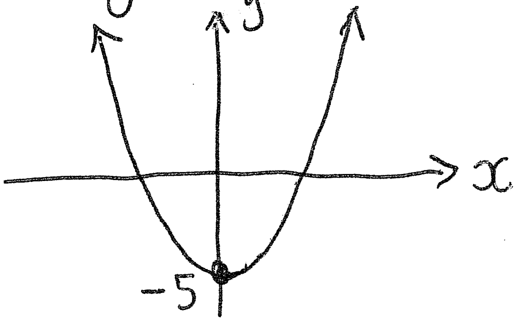
Using $(y - y_1) = m(x - x_1)$

$y - 8 = -\frac{1}{4}(x - 4)$

$y - 8 = -\frac{x}{4} + 1$

$y = -\frac{x}{4} + 9$ or $x + 4y - 36 = 0$ 2

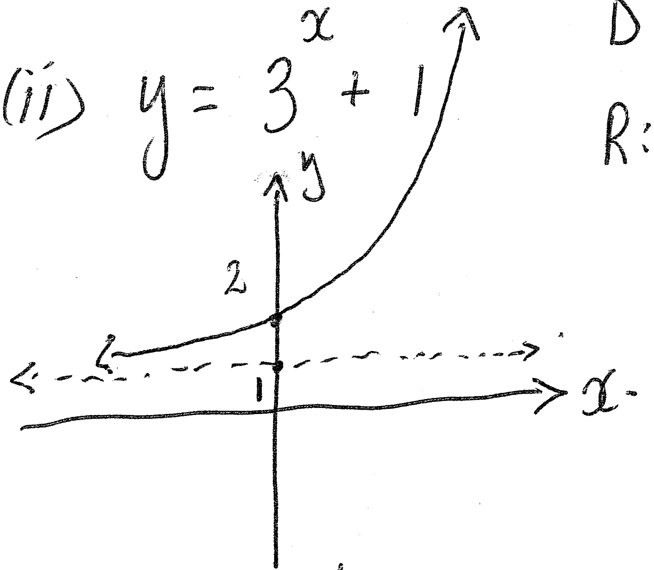
(c) (i) $y = 3x^2 - 5$



D: $\mathbb{R}$ ①

R: $y \geq -5$ ①

(ii) $y = 3^x + 1$



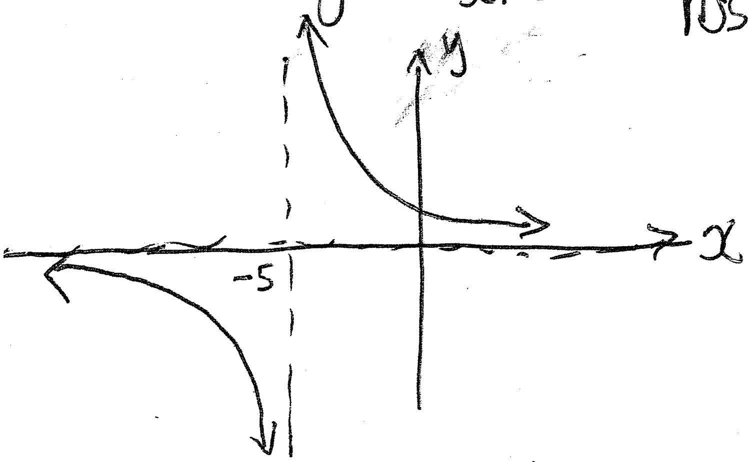
D: $\mathbb{R}$ ①

R: $y > 1$ ①

(iii) $y = \frac{1}{x+5}$

$x \neq -5$

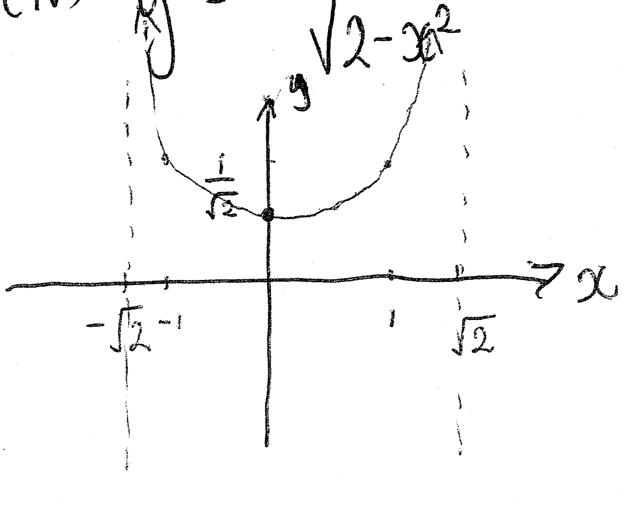
Positive hyperbola.



D: all $\mathbb{R}$, $x \neq -5$ ①

R: all $\mathbb{R}$, $y \neq 0$ ①

(iv) $y = \frac{1}{\sqrt{2-x^2}}$



$(2 - x^2 > 0)$

D: $-\sqrt{2} < x < \sqrt{2}$ ①

R: $y \geq \frac{1}{\sqrt{2}}$ ①

Question 3:

a) $\{1, 2, 3, 4, 5, 6, 6\}$

i) $P(2) = \frac{1}{7}$

ii) $P(6) = \frac{2}{7}$

b) $T_1 = -5,\ T_2 = -1,\ T_3 = 3,\ T_4 = 7$

$d = 4.$

i) $T_n = a + (n-1)d$

$= -5 + (n-1)4$

$= -5 + 4n - 4$

$= 4n - 9$

ii) $S_n = \frac{n}{2}(2a + (n-1)d)$

$S_{20} = \frac{20}{2}\left(2(-5) + (20-1)4\right)$

$= 10(-10 + 76)$

$= 660$

c) $(x - x_1)^2 + (y - y_1)^2 = r^2$

$\therefore (x-(-3))^2 + (y-5)^2 = (\sqrt{3})^2$

$(x+3)^2 + (y-5)^2 = 3$

d) $d_1 = \frac{|6x + 8y - 5|}{\sqrt{6^2 + 8^2}}$

$= \frac{|6x + 8y - 5|}{10}$

$d_2 = \frac{|5x + 12y - 1|}{\sqrt{5^2 + 12^2}}$

$= \frac{|5x + 12y - 1|}{13}$

When $d_1 = d_2$

$\frac{|6x + 8y - 5|}{10} = \frac{|5x + 12y - 1|}{13}$

<u>case 1:</u> (Positive)

$13(6x + 8y - 5) = 10(5x + 12y - 1)$

$78x + 104y - 65 = 50x + 120y - 10$

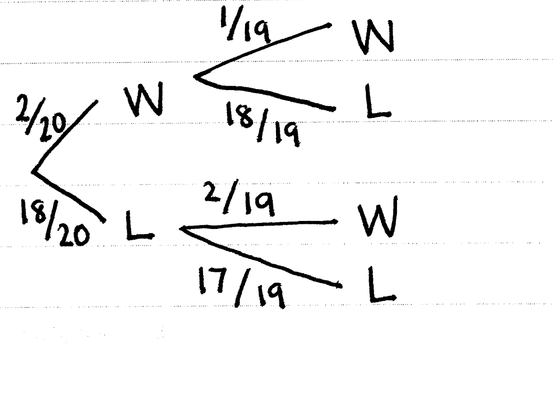
$\underline{28x - 16y - 55 = 0}$

<u>case 2:</u> (one negative)

$78x + 104y - 65 = -50x - 120y + 1$

$\underline{128x + 224y - 75 = 0}$

e) 1st Prize 2nd Prize



i) $P(WW) = \frac{2}{20} \times \frac{1}{19}$

$= \frac{1}{190}$

ii) At least one

$\therefore 1 - P(LL)$

$= 1 - \left(\frac{18}{20} \times \frac{17}{19}\right)$

$= \frac{37}{190}$

f) $x^2 + kx + 8 - k = 0$

i) equal roots $\Delta = 0$

$\Delta \Rightarrow k^2 - 4(1)(8-k) = 0$

$k^2 - 32 + 4k = 0$

$k^2 + 4k - 32 = 0$

$(k-4)(k+8) = 0$

$\therefore k = 4, -8$

ii) real and distinct

$\Delta > 0$

$\therefore k > 4, k < -8$

QUESTION 4

(a) $\displaystyle\sum_{r=1}^{10} r(r+3)$

(b) $f(3a-1) = 2(3a-1) - 3$
$= 6a - 5$

(c) (i) $8^{\frac{1}{2}} \div 2^{\frac{1}{2}} = \left(\frac{8}{2}\right)^{\frac{1}{2}}$
$= 2.$

(ii) $\log_a \frac{a^2+a}{a+1} = \log_a a$
$= 1.$

(d)(i) $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x - 1$

$f'(x) = x^2 + x - 2$

$f''(x) = 2x + 1.$

For st. points let $f'(x) = 0$

$(x+2)(x-1) = 0$

$x = 1, -2$

$\therefore (1, -\frac{13}{6})$ and $(-2, 2\frac{1}{3})$ are stationary pts.

Test nature $f''(1) = 3 \quad \therefore (1, -\frac{13}{6})$ is a rel. min. ~~max~~ turning pt.

$f''(-2) = -3 \quad \therefore (-2, 2\frac{1}{3})$ is a rel. max. turning pt.

Q4 (CONT'D)

(ii) For possible inflexions

let $f''(x) = 0$

i.e. $2x + 1 = 0$

$x = -\frac{1}{2}$

$\therefore \left(-\frac{1}{2}, \frac{1}{12}\right)$ is a possibility

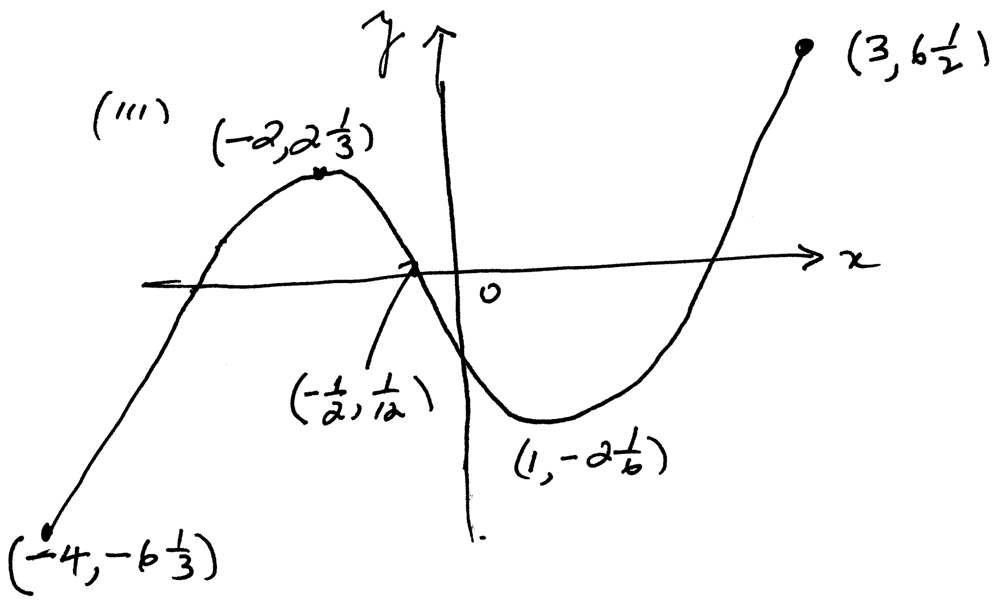
Test

x	-1	$-\frac{1}{2}$	0
y''	-1	0	1

$\therefore$ change in concavity

$\therefore \left(-\frac{1}{2}, \frac{1}{12}\right)$ is a point of inflexion.

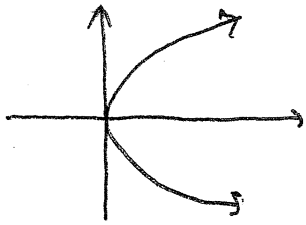
(iii)



(iv) $6\frac{1}{2}$

5 (a) $3y^2 = 4x$

$y^2 = \frac{4}{3}x$

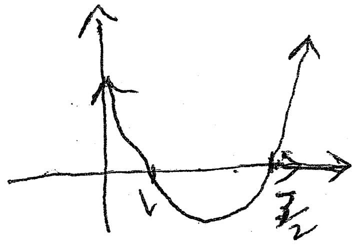


$\therefore\ a = \frac{1}{3}\quad V(0,0)$ [2]

$\therefore$ Focus $S(\frac{1}{3}, 0)$ Directrix $x = -\frac{1}{3}$

(b) $2x^2 - 9x + 7 > 0$

$(2x-7)(x-1) > 0 \qquad x = \frac{7}{2}, 1$



$x < 1,\ x > \frac{7}{2}$ [3]

(c) $f(x) = x^2 - 6x + 5$

$$f'(x) = \lim_{h\to 0} \frac{(x+h)^2 - 6(x+h) + 5 - (x^2 - 6x + 5)}{h}$$

$$= \lim_{h\to 0} \frac{x^2 + 2xh + h^2 - 6x - 6h + 5 - x^2 + 6x - 5}{h}$$

$$= \lim_{h\to 0} \frac{h(2x + h - 6)}{h} = 2x - 6$$ [3]

(d) (i) $A_0 = 80K$

$A_1 = 80K \times 1.05 - 10K$

$A_2 = A_1 \times 1.05 - 10K$ [2]

$= 80K(1.05)^2 - 10K(1.05) - 10K$

$A_3 = 80K(1.05)^3 - 10K(1.05)^2 - 10K(1.05) - 10K$

(ii) $A_n = 80K(1.05)^n - \dfrac{10K(1.05^n - 1)}{1.05 - 1}$ [3]

(iii) We seek $\lfloor n \rfloor$ for $A_n < 0$

ie $0 < 80k(1.05)^n(0.05) - 10k(1.05^n - 1)$

$0 < 80(1.05)^n(0.05) - 10(1.05^n) + 10$ [3]

$6(1.05^n) < 10 \qquad \therefore\ n < 10.469$

$n < \dfrac{\ln 10/6}{\ln 1.05} \qquad \therefore$ 10 holidays are possible.