

SYDNEY TECHNICAL HIGH SCHOOL

HSC ASSESSMENT

TASK 1

MATHEMATICS

December 2003

Time allowed: 70 minutes

Weighting 10%

Name: \_\_\_\_\_ Class: \_\_\_\_\_

Instructions

- Start each question on a new page
- Marks indicated are approximate only
- Answer all questions – marks may be deducted for poorly arranged work

*leave out circled questions*

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Total
----	----	----	----	----	----	----	----	-------

Question 1 (8 marks)

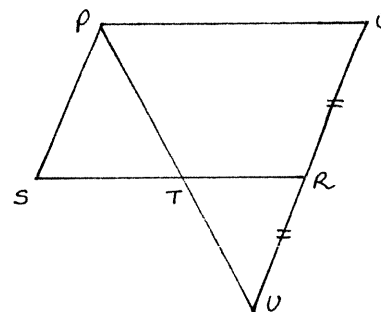
- a) For the series  $-12 + 13 + 38 + \dots$
- Find the value of the 12<sup>th</sup> term (2)
  - Find the sum of the first 12 terms (2)
  - Which term has the value 963? (2)
- b) Write down the coordinates of the focus for the parabola  $x^2 = 12 - 12y$  (2)

Question 2 (8 marks)

- a) The roots of  $3x^2 - 5x - 4 = 0$  are  $\alpha$  and  $\beta$  (6)
- Find the value of,
- $\alpha\beta$
  - $\alpha + \beta$
  - $\frac{1}{\alpha} + \frac{1}{\beta}$
  - $\alpha^3\beta + \alpha\beta^3$
- b) The first two terms of a geometric series are 18 and 12 respectively. (2)
- Find the limiting sum of the series.

Question 3 (8 marks)

- a) In the diagram, PQRS is a parallelogram. QR is produced to U so that QR = RU (4)

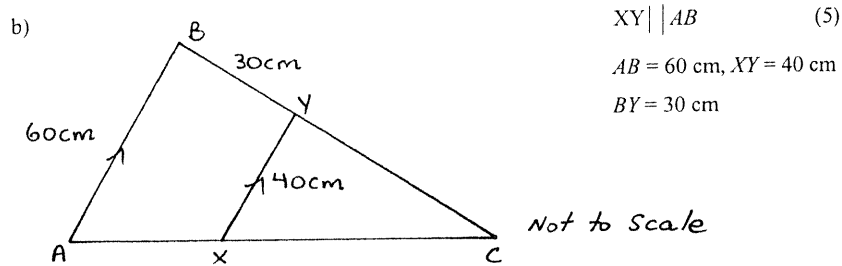


*Not to scale*

- Giving clear reasons, show that the triangles PST and URT are congruent.
  - Hence or otherwise, show that T is the midpoint of SR
- b) Write  $0.\dot{1}0\dot{2}$  as a geometric series, and state the value of  $a$  and  $r$ . Hence, write  $0.\dot{1}0\dot{2}$  as a (4)
- fraction in its simplest form.

**Question 4 (8 marks)**

a) Evaluate  $\sum_{n=4}^{20} 2n - 5$  (3)



- i Prove  $\triangle ABC \sim \triangle XYC$
- ii Calculate the length  $CY$

**Question 5 (8 marks)**

a) Solve for  $K$  if the zeros of the parabola  $y = 2x^2 - 7x + K$  are not real. (3)

b) If  $x + 2, 2x - 4, 4x + 1$  are the first three terms in an Arithmetic sequence, find  $x$ . (2)

c) Find the values of  $P, Q$  and  $R$  if  $3x^2 + 5x - 1 \equiv P(x+1)^2 + Q(x+1) + R$  (3)

**Question 6 (8 marks)**

a) A parabola has equation  $x^2 = -12y$  (6)

- i Find the coordinates of the vertex of the parabola
- ii Write down the coordinates of the focus of the parabola
- iii Find the equation of the tangent to the parabola at the point where  $x = 6$ .
- iv Find the coordinates of  $M$ , the point where the tangent cuts the  $y$  axis

b) Write down the equation of the parabola whose focus is  $(1, 3)$  and directrix is  $x = 5$ . (2)

**Question 7 (8 marks)**

a) Consider the points  $A(-2, 1)$ ,  $B(4, 1)$  and  $P(x, y)$  (6)

- i Find expressions for the gradients of the two intervals  $PA$  and  $PB$
- ii Show that the locus of  $P$  is  $x^2 - 2x + y^2 - 2y = 7$  if  $\angle APB = 90^\circ$
- iii Show that the locus represents a circle and give its centre and radius.

b) Find the sum of the first ten terms of the series  $1 - \sqrt{2} + 2 - \dots$  (2)

Express your answer in simplest exact form with a rational denominator.

**Question 8 (8 marks)**

a) In an arithmetic series the eight term is 32 and the sum of the first ten terms is 400. (3)

Find,

- i the value of the common difference
- ii the value of the first term

b) At the beginning of each year Xena the Warrior princess invests \$1200 in a superannuation fund, on which she is paid 8.5% pa interest, compounded annually. (5)

Find,

- i the amount of interest earned in the first year
- ii the total of her investments at the end of 25 years.

Teacher's Name:

Student's Name/N<sup>o</sup>:

2 unit HSC Ass one - 2003 - Answers

Question 1

a)  $a = -12$   $d = 25$

i.  $T_{12} = -12 + 11 \times 25$  - ① or formula  
 $= 263$  - ① Ans

ii.  $S_n = \frac{n}{2}(a + t)$  ① formula  
 $= \frac{12}{2}(-12 + 263)$  ① Ans  
 $= 1506$

iii.  $963 = a + (n-1)d$

$963 = -12 + (n-1) \times 25$  - ①

$975 = 25n - 25$

$1000 = 25n$

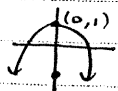
$n = 40$  - ①

$\therefore$  40<sup>th</sup> term has value 963

b)  $x^2 = -12y + 12$

$x^2 = -12(y-1)$  - ①

$V = (0, 1)$   $a = 3$



$S = (0, -2)$  - ①

Question 2

a)  $a = 3$   $b = -5$   $c = -4$

i.  $\alpha\beta = c/a = -4/3$  ①

ii.  $\alpha + \beta = -b/a = 5/3$  ①

iii.  $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta}$  - ①

$= 5/3 \div -4/3$

$= -5/4$  - ①

iv.  $\alpha^3\beta + \alpha\beta^3 = \alpha\beta(\alpha^2 + \beta^2)$

$= \alpha\beta(\alpha + \beta)^2 - 2\alpha\beta$  - ①

$= -4 \left[ \left(\frac{5}{3}\right)^2 - 2 \times \frac{-4}{3} \right]$  - ①

$= -7 \frac{1}{27}$

Q2 (b)  $18 + 12 + \dots$

GP  $a = 18$   $r = 12/18$

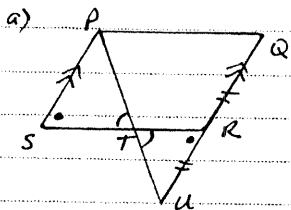
$= 2/3$  - ①

$S_{\infty} = \frac{a}{1-r}$

$= \frac{18}{1-2/3}$

$= 54$  - ①

Question 3



i.  $PS \parallel QR$  opp sides of parallelogram equal

$PS = QR$  opp sides of parallelogram equal.

In  $\Delta PST$  and  $\Delta QRT$

$PS = QR$  (= QR)

$\angle PST = \angle QRT$  (alt.  $\angle$ 's  $PS \parallel QR$ ) ①

$\angle PTS = \angle QTR$  (vert opp) - ①

$\therefore \Delta PST \cong \Delta QRT$  (AAS)

ii.  $ST = TR$  corresp sides of  $\cong \Delta$ 's  $\therefore T$  is the midpoint of  $SR$  - ①

b)  $0.102 = \frac{102}{1000} + \frac{102}{1000000} + \dots$

$\therefore a = \frac{102}{1000}$   $r = \frac{1}{1000}$  ①

$S_{\infty} = \frac{a}{1-r}$  ①

$= \frac{102}{1000} \div \frac{999}{1000}$

$= \frac{102}{999}$

$= 34/225$  - ①

Teacher's Name:

Student's Name/N<sup>o</sup>:

Question 4

a)  $\sum_{n=4}^{20} 2n - 5$

$n=4$   $n=5$   $n=6 \dots n=20$

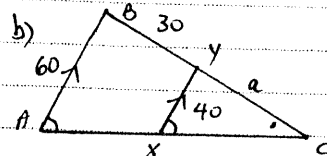
$3 + 5 + 7 + \dots + 35$  ①

AP  $a=3$   $d=2$   $n=20-4+1$   
 $= 17$  ①

$S_n = \frac{n}{2}(a + t)$

$= \frac{17}{2}(3 + 35)$

$= 323$  ①



i. In  $\Delta ABC \cong \Delta XYC$

$\angle C$  is common ①

$\angle BAC = \angle YXC$  (Corresp  $\angle$ 's  $AB \parallel XY$ ) ①

$\therefore \Delta ABC \parallel \Delta XYC$  (all  $\angle$ 's =) ①

ii.  $\frac{60}{40} = \frac{30+a}{a}$  ratio of ①  
 sides in  $\parallel \Delta$ 's

$6a = 120 + 4a$

$2a = 120$

$a = 60$

$\therefore CY = 60$  ①

Question 5

a) Not real  $\Delta < 0$  ①  $a = 2$

$b^2 - 4ac < 0$   $b = -7$

$49 - 4 \times 2 \times K < 0$   $c = K$

$49 - 8K < 0$

$-8K < -49$

$8K > 49$

$K > 49/8$  ①

b)  $x+2, 2x-4, 4x+1, \dots$

AP  $\rightarrow T_2 - T_1 = T_3 - T_2$  ①

$2x-4 - (x+2) = (4x+1) - (2x-4)$

$2x-4-x-2 = 4x+1-2x+4$

$x-6 = 2x+5$

$x = -11$  ①

c)  $3x^2 + 5x - 1 \equiv P(x+1)^2 + Q(x+1) + R$

RHS =  $P(x^2 + 2x + 1) + Qx + Q + R$

equating coefficients.

\*  $3 = P$  ①

\*  $5 = 2P + Q$

$5 = 6 + Q$   $Q = -1$  ①

\*  $-1 = P + Q + R$

$-1 = 3 - 1 + R$

$-1 = 2 + R$

$R = -3$  ①

$\therefore P = 3, Q = -1, R = -3$

## Question 6

a)  $x^2 = -12y$

I.  $V = (0, 0)$  ①

II.  $S = (0, -3)$  ①

III.  $y = \frac{x^2}{-12}$

$y' = \frac{2x}{-12}$  at  $x=6$   $y=-3$  ①

$m_T = -1$  ①

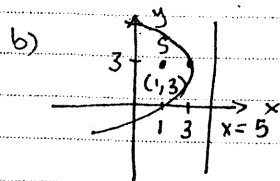
∴ equation

$y + 3 = -1(x - 6)$  — ①

$y + 3 = -x + 6$

$x + y - 3 = 0$

IV.  $x = 0$   $y = 3$  ①

Vertex  $(3, 3)$ 

$a = 2$

$(y - y_1)^2 = 4a(x - x_1)$

$(y - 3)^2 = -8(x - 3)$

① ↑ ① Neg.

## Question 7

a)  $M_{PA} = \frac{y-1}{x+2}$   $M_{PB} = \frac{y-1}{x-4}$  ① ①

II.  $\frac{y-1}{x+2} \times \frac{y-1}{x-4} = -1$  ①

$\frac{y^2 - 2y + 1}{x^2 - 2x - 8} = -1$

$y^2 - 2y + 1 = -x^2 + 2x + 8$

$x^2 - 2x + y^2 - 2y = 7$  ①

III.  $x^2 - 2x + 1 + y^2 - 2y + 1 = 7 + 2$

$(x-1)^2 + (y-1)^2 = 9$

$C = (1, 1)$   $r = 3$

① ①

b)  $1 - \sqrt{2} + 2 \dots$

G.P.  $a = 1$   $r = \sqrt{2}$

$n = 10$

$S_n = \frac{a(r^n - 1)}{r - 1}$

$= \frac{1((- \sqrt{2})^{10} - 1)}{-\sqrt{2} - 1}$

$= \frac{1(32 - 1)}{-\sqrt{2} - 1}$

$= \frac{31}{-\sqrt{2} - 1}$

$= \frac{-31}{\sqrt{2} + 1} \times \frac{\sqrt{2} - 1}{\sqrt{2} - 1}$

$= \frac{-31\sqrt{2} + 31}{1}$

$= -31\sqrt{2} + 31$

## Question 8

a)

$T_8 = a + 7d = 32$

$S_{10} = \frac{10}{2}(2a + 9d) = 400$

$2a + 9d = 80$  ①

$2a + 14d = 64$

$\therefore 5d = 16$

$d = \frac{-16}{5}$  ①

$a + 7 \times \frac{-16}{5} = 32$

$a = 54\frac{2}{5}$  ①

b)

I.  $A = 1200(1.085)^1$

$= 1302$  ①

$\therefore \text{Interest} = \$102$  ①

II.  $A_1 = 1200(1.085)^{25}$

$A_2 = 1200(1.085)^{24}$

$A_3 = 1200(1.085)^{23}$

⋮

$A_{25} = 1200(1.085)^1$  ①

Total =  $1200(1.085 + 1.085^2 + \dots + 1.085^{25})$

$= 1200 \times \frac{a(r^n - 1)}{r - 1}$

$a = 1.085$

$r = 1.085$

$n = 25$

① - setting up G.P.

$= 1200 \times \frac{1.085(1.085^{25} - 1)}{0.085}$

$= \$10\,2425.47$  ①