Name : _____

Teacher/Class: _____

SYDNEY TECHNICAL HIGH SCHOOL

HSC ASSESSMENT TASK 1

DECEMBER 2006

MATHEMATICS

Time Allowed: **70 minutes**

Instructions:

- Write your name and class at the top of each page
- All necessary working must be shown. Marks may be deducted for careless or badly arranged work.
- Marks indicated are a guide only and may be varied if necessary.
- Start **each** question on a **new** page.
- Diagrams unless otherwise stated are not to scale.

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	TOTAL
/7	/7	/7	/7	/7	/7	/7	/7	/7	/56

QUESTION 1 (7 Marks) **(MARKS)**

a) Factorise $2x^2 + 5x - 3$ (1)

b) Solve $x^2 - 4x > 0$ (2)

c) Evaluate $\sum_{r=2}^{5} \frac{1}{r+1}$ (1)

d) Let α and β be the roots of $x^2 - 3x - 7 = 0$ find

i) $\alpha + \beta$ (1)

ii) $\alpha\beta$ (1)

e) Form a quadratic equation whose roots are -2 and 3 (1)

QUESTION 2 (7 Marks) (start a new page) **(MARKS)**

a) If $x = -2$ is a root of $4x^2 + x + k = 0$, find k (1)

b) Find the limiting sum of the geometric series

$$\frac{13}{5} + \frac{13}{25} + \frac{13}{125} + \ldots\ldots\ldots\ldots$$ (2)

c) For the sequence -8 , -1 , 6 ……..

i) Find the 29th term (1)

ii) Find the sum of the first 29 terms (1)

iii) Which term of the sequence has a valve of 167? (2)

QUESTION 3 (7 Marks) (start a new page) **(MARKS)**

a) i) Draw a neat sketch (using a ruler for the axes) of the parabola $x^2 = 8y$ (1)

ii) Find the co-ordinates of the focus (1)

iii) Find the equation of the directrix (1)

iv) Find the equation of the tangent to the parabola $x^2 = 8y$ that passes through the point (-8 , 8) (2)

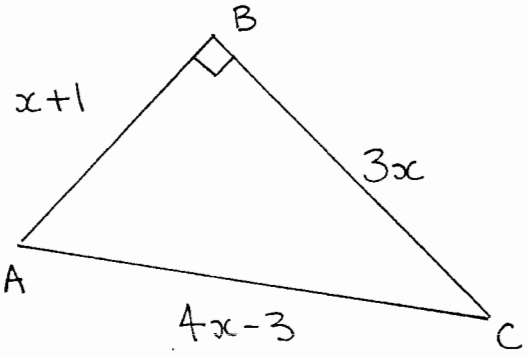
b) In solving a quadratic equation a student wrote his solution as

$$x = \frac{4 \pm \sqrt{16+96}}{6} \qquad (2)$$

What was the original equation?

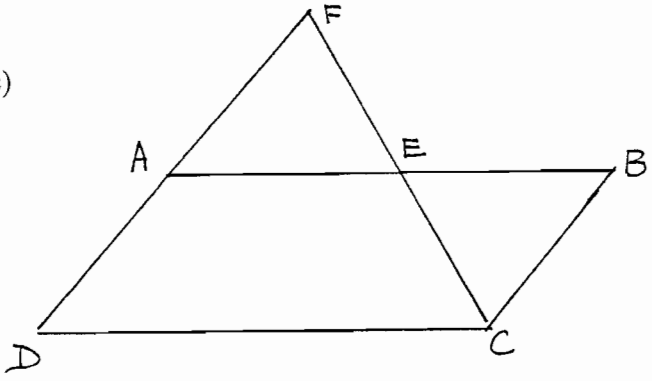
QUESTION 4 (7 Marks) (start a new page) **(MARKS)**

a) Triangle ABC below has angle ABC equal to $90°$. Find all possible value(s) for x. (2)



b) How much will \$500 grow to at 12% p.a. if compounded quarterly for 5 years (1)

c)



ABCD is a parallelogram *AE=EB*

DA produced intersects

CE produced at *F*

i) Copy the diagram onto your answers sheet

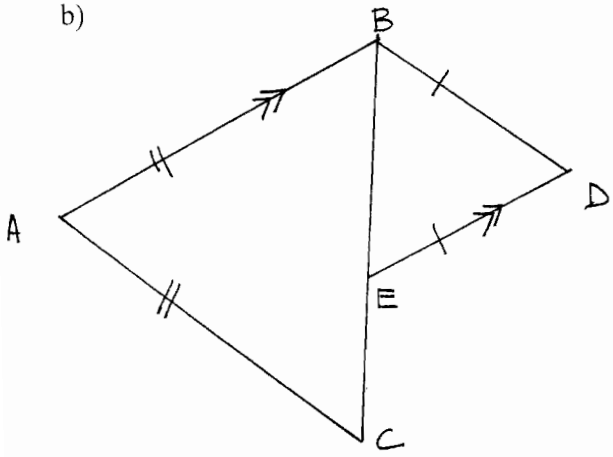
ii) Prove $\Delta AFE \equiv \Delta BCE$ (2)

iii) Hence explain why $DA = AF$ (2)

QUESTION 5 (7 Marks) (start a new page) **(MARKS)**

a) For what values of k does the equation $x^2 + kx + 3 - k = 0$ have real, different roots? (3)

b)



Triangles ABC and BDE are isosceles

$AB=AC$ and $BD=ED$ and AB ll ED

i) Copy the diagram onto your answer sheet.

ii) Prove ΔABC is similar to ΔBDE (3)

iii) If $BD = 5cm$, $BE = 4cm$ and $AC = 6cm$ find the length of EC (1)

QUESTION 6 (7 Marks) (start a new page) **(MARKS)**

a) Insert three numbers between 5 and 80 so as to form five numbers in a geometric sequence . (3)

b) The first n terms of an arithmetic sequence have a sum given by $S_n = 25n - 2n^2$

i) Find the first term and the second term (2)

ii) Find the common difference (1)

iii) Find the expression for the nth term (1)

QUESTION 7 (7 Marks) (start a new page) **(MARKS)**

a) A man places \$1500 at the beginning of each year into a superannuation fund, for 30 years. Interest on investments in the fund compounds at 12% p.a.

i) Find the amount he has in the fund at the end of 30 years (3)

ii) If this amount in part i) is taken as a lump sum and taxed at the rate of 30% for each dollar over \$50,000 , how much will he receive after tax. (2)

b) For the parabola $y = x^2 + 4x + 5$ find

i) the co-ordinates of the vertex (1)

ii) the co-ordinates of the focus (1)

QUESTION 8 (7 Marks) (start a new page) **(MARKS)**

a) A is the point $(8,0)$ and 0 is the origin. P is the variable point (x, y)

i) If P moves so that $PO = 3PA$, show that the locus of P is given by

$$x^2 + y^2 = 9\left[(x-8)^2 + y^2\right] \quad (2)$$

ii) Show that this locus is a circle by finding its centre and radius (2)

b) i) Find the sum of the geometric series

$$x^4 + x^3y + x^2y^2 + xy^3 + y^4 \quad (2)$$

ii) Hence factorise $x^5 - y^5$ (1)

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QUESTION 1

a) $2x^2+5x-3 = (2x-1)(x+3)$

b) $x^2-4x>0$

$x(x-4)>0$

$\therefore x>4$ and $x<0$

c) $\sum_{r=2}^{5} \frac{1}{r+1} = \frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}$

$= \frac{19}{20}$

d) $a=1 \quad b=-3 \quad c=-7$

$\therefore$ i) $\alpha+\beta = 3$

ii) $\alpha\beta = -7$

e) $(x+2)(x-3)=0$

QUESTION 2

a) sub $x=-2$ into $4x^2+x+k=0$

$4(4)-2+k=0$

$k=-14$

b) $a=\frac{13}{5} \quad r=\frac{1}{5}$

$\therefore S_\infty = \frac{\frac{13}{5}}{1-\frac{1}{5}}$

$= 3\frac{1}{4}$

c) $-8, -1, 6 \ldots$ AP: $a=-8$

$d=7$

i) $T_{29} = -8+28\times7 = 188$

ii) $S_{29} = \frac{29}{2}(-8+188)$

$= 2610$

iii) $167 = -8+(n-1)\times7$

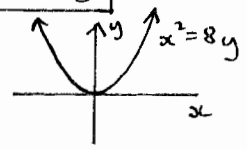
$167 = -8+7n-7$

$182 = 7n$

$\therefore n = 26 \quad \therefore T_{26}=167$

QUESTION 3

a) i)



ii) $4a=8 \quad \therefore a=2$

Focus $(0,2)$

iii) Directrix $y=-2$

iv) $x^2=8y$

$\therefore y = \frac{x^2}{8}$

$\frac{dy}{dx} = \frac{2x}{8} = \frac{x}{4}$

$\therefore m_T = -2$ at $(-8,8)$

eqn. tang: $y-8=-2(x+8)$

$y-8=-2x-16$

$2x+y+8=0$

b) $b=-4 \quad a=3$

$-4ac = 96$

$-12c = 96$

$c=-8$

$\therefore$ equation $3x^2-4x-8=0$

(or any multiple of this)

QUESTION 4

a) $(4x-3)^2 = (x+1)^2+(3x)^2$

$16x^2-24x+9 = x^2+2x+1+9x^2$

$6x^2-26x+8=0$

$3x^2-13x+4=0$

$(3x-1)(x-4)=0$

$x=\frac{1}{3} \quad x=4$

only valid answer $x=4$

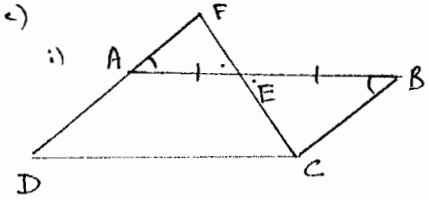
(since $4\times\frac{1}{3}-3<0$; hypot. -ve)

b) $A = 500\left(1+\frac{12\frac{1}{4}}{100}\right)^{20}$

$A = 500(1.03)^{20}$

$A = \$\ 903.06$

c)



i) In Δ's AFE and BCE

AE = EB (given)

$F\hat{A}E = E\hat{D}C$ (alternate angles DF || CB, sides of parallelogram)

$F\hat{E}A = B\hat{E}C$ (vertically opposite angles)

$\therefore \Delta AFE \equiv \Delta BCE$ (AAS)

iii) AF = BC (corresp sides in congruent triangles)

AD = BC (opp. sides of parm.)

$\therefore$ AD = AF

QUESTION 5

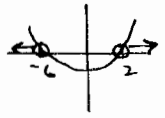
a) $\Delta>0$

$k^2-4\times1\times(3-k)>0$

$k^2-12+4k>0$

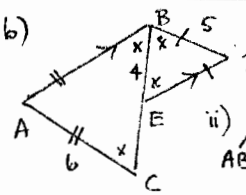
$k^2+4k-12>0$

$(k+6)(k-2)>0$



$k<-6$ and $k>2$

b)



ii) In Δ ABC, B

$A\hat{B}C = B\hat{E}D$ (alt. a

AB

[since $D\hat{B}E = D\hat{E}B$ & $A\hat{B}E = B\hat{C}A$] opp a in iso tria

$D\hat{B}E = A\hat{C}B$ (base an isoscel

$\therefore \Delta ABC \,|||\, \Delta BDE$ equian

iii) $\frac{5}{6} = \frac{4}{BC}$ (corsp. sides similar tri

$BC = 4.8$

$\therefore EC = 0.8$

QUESTION 6

a) 5 ___ ___ ___ 80 C

$a=5 \quad T_5=80$

$80 = 5.r^4$

$r^4 = 16$

$r = \pm2$

$\therefore$ GP 5, 10, 20, 40, 80

OR 5, -10, 20, -40, 80

b) $S_n = 25n-2n^2$

i) $S_1 = T_1 = 23$

$S_2 = 50-8 = 42 \quad \therefore T$

$\therefore T_1 = 23 \quad T_2 = 19$

ii) $d=-4$

iii) $T_n = 23+(n-1)\times-4$

$T_n = 27-4n$

QUESTION 7

a) i)

$$A = 1500(1.12)^{30} + 1500(1.12)^{29} + \cdots + 1500(1.12)^{1}$$

$$= 1500\left(1.12^{1} + 1.12^{2} + \cdots + 1.12^{30}\right)$$

[G.P. $a = 1.12$ $r = 1.12$ $n = 30$]

$$A = 1500 \cdot \left[\frac{1.12\left(1.12^{30} - 1\right)}{1.12 - 1}\right]$$

$$A = \$405438.91$$

ii) TAX = $\cdot 3 \times 355438.91$

$= \$106631.67$

$\therefore$ after TAX super $= \$298807.24$

b) i) $x^2 + 4x + 4 = y - 5 + 4$

$(x+2)^2 = y - 1$

vertex $(-2, 1)$

ii) $4a = 1 \quad \therefore a = \frac{1}{4}$

focus $\left(-2, 1\frac{1}{4}\right)$

QUESTION 8

a) i) $PO = 3PA$

$$\sqrt{(x-0)^2 + (y-0)^2} = 3\sqrt{(x-8)^2 + (y-0)^2}$$

$$x^2 + y^2 = 9\left[(x-8)^2 + y^2\right]$$

ii) $x^2 + y^2 = 9\left(x^2 - 16x + 64 + y^2\right)$

$x^2 + y^2 = 9x^2 - 144x + 576 + 9y^2$

$-576 = 8x^2 - 144x + 8y^2$

$-72 = x^2 - 18x + y^2$

$-72 + 81 = (x^2 - 18x + 81) + y^2$

$9 = (x-9)^2 + y^2$

b) $x^4 + x^3y + x^2y^2 + xy^3 + y^4$

i) $a = x^4 \quad r = \frac{y}{x} \quad n = 5$

$$S_5 = \frac{x^4\left[\left(\frac{y}{x}\right)^5 - 1\right]}{\frac{y}{x} - 1}$$

$$S_5 = x^4\left[\frac{y^5 - x^5}{x^5}\right] \div \left(\frac{y-x}{x}\right)$$

$$= \frac{x^4(y^5 - x^5)}{x^5} \times \frac{x}{(y-x)}$$

$$= \frac{-(x^5 - y^5)}{-(x-y)}$$

$$S_5 = \frac{x^5 - y^5}{x - y}$$

ii) $x^4 + x^3y + x^2y^2 + xy^3 + x^4$

$$= \frac{x^5 - y^5}{x - y}$$

$\therefore x^5 - y^5 = (x-y)\left[x^4 + x^3y + x^2y^2 + xy^3 + y^4\right]$