Name: _____

Teacher/Class: _____

SYDNEY TECHNICAL HIGH SCHOOL

HSC ASSESSMENT TASK 1

DECEMBER 2008

MATHEMATICS

Time Allowed: **70 minutes**

Instructions:

- Write your name and class at the top of each page.
- All necessary working must be shown. Marks may be deducted for careless or badly arranged work.
- Marks indicated are a guide only and may be varied if necessary.
- Start **each** question on a **new** page.
- Diagrams unless otherwise stated are not to scale.

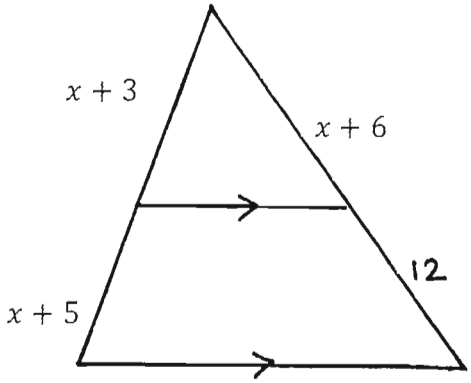
Question 1	Question 2	Question 3	Question 4	Question 5	Question 6	Question 7	Question 8	TOTAL
/6	/6	/6	/6	/7	/7	/6	/7	/51

QUESTION 1 (6 Marks) MARKS

a) i) Sketch $y = x^2 - 4x - 5$ 1

ii) Hence or otherwise solve $x^2 - 4x - 5 > 0$ 1

b)



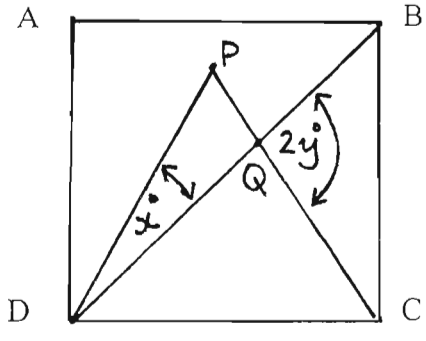
Find 2 possible values for x
(no reason required) 2

c) Find k if the equation
$x^2 - (k - 5)x + (k - 7) = 0$ has one root equal to -2. 2

QUESTION 2 (Start a new page) (6 marks)

a) Solve $x^4 - 5x^2 + 4 = 0$ 2

b) 2



ABCD is a square.
PDC is an equilateral triangle.
Find x *and* y
(no reason required)

MARKS

c) Find the centre and the radius of the circle $x^2 + y^2 + 8x - 2y - 8 = 0$ 2

QUESTION 3 (Start a new page) (6 marks)

a) The first 3 terms of a sequence are 5, 9 and 13 1

i) Find the 46^{th} term

ii) Is 147 a term of the sequence? 2

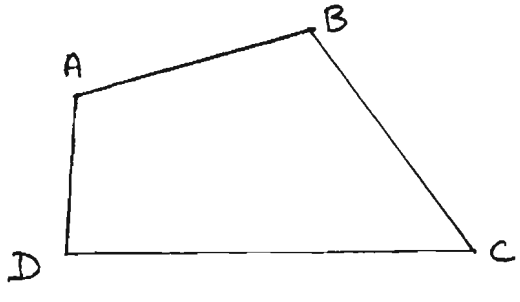
Use appropriate working to explain your answer

b) Evaluate $\sum_{k=3}^{12} (3k-1)$ 2

c) Find k if the product of the roots of $x^2 - (k-4)x + 3k = 0$ is 18. 1

QUESTION 4 (Start a new page) (6 marks)

a) ABCD is a quadrilateral in which the diagonal BD bisects the angles at B and D.



i) Copy this quadrilateral onto your answer sheet and add the additional information.

ii) By completing an appropriate congruence proof explain why AB = BC and AD = DC. 3

iii) What type of quadrilateral is ABCD? 1

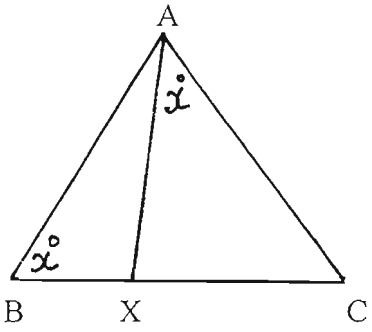
MARKS

b) Find the equation of the parabola with vertex (-2, 3), axis parallel to the y axis and passing through the point (-6, -5). 2

QUESTION 5 (Start a new page) (7 marks)

a) The third term of a geometric series is ¾ and the seventh term is 12. Find the first three terms. 3

b) 3



If $\angle CAX = \angle ABC = x^0$

i) Prove $\Delta CAX \parallel\!\parallel \Delta CBA$

ii) Hence explain why $CA^2 = CX \cdot CB$ 1

QUESTION 6 (Start a new page) (7 marks)

a) i) Find the first and last multiples of seven between 400 and 600. 1

ii) Hence find the sum of all the multiples of seven between 400 and 600. 2

b) i) Find the equation of the directrix of the parabola $x^2 = -12y$ 1

ii) Find the equation of the tangent at the point (-6, -3) on the parabola $x^2 = -12y$ 2

iii) If this tangent meets the directrix at T, find the co-ordinates of T. 1

QUESTION 7 (Start a new page) (6 marks) MARKS

a) For what values of k will the quadratic equation $(k-2)x^2$ - $2kx$-1 =0 have real and different roots. 3

b) Find the sum of the powers of 2 from 1 to 1024 inclusive. 3
(ie $2^0 + 2^1 + 2^2$.....)

QUESTION 8 (Start a new page) (7 marks)

a) i) Express $0.\dot{3}\dot{4}$ as an infinite series by writing the first 4 terms. 1

ii) By using the sum to infinity formula of the above series write $0.\dot{3}\dot{4}$ as a fraction. 2

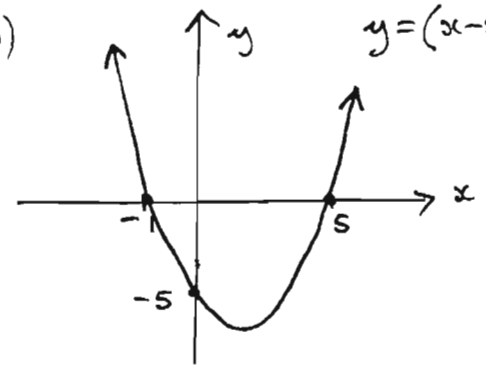
b) i) Let A and B be the fixed points (-1, 0) and (2, 0) and P be the variable point (x, y). 3
If PA = 2PB find the locus of $P\ (x, y)$.

ii) Name the geometric shape traced out by the locus of $P(x, y)$. 1

STHS YEAR 11 2UNIT H.S.C TASK 1 2008

Question 1

a) i) $y = (x-5)(x+1)$



ii) $x > 5$, $x < -1$

b) $\frac{x+3}{x+5} = \frac{x+6}{12}$

$12x + 36 = x^2 + 11x + 30$

$0 = x^2 - x - 6$

$(x-3)(x+2) = 0$

$x = 3, -2$

c) $4 - (k-5)x - 2 + (k-7) = 0$

$4 + 2k - 10 + k - 7 = 0$

$3k - 13 = 0$

$k = 13/3$

Question 2

a) Let $u = x^2$ $\therefore$ $u^2 - 5u + 4 = 0$

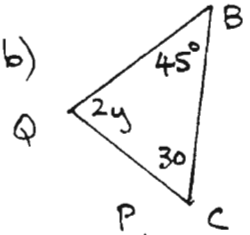
$(u-4)(u-1) = 0$

$u = 4, \; u = 1$

$\therefore u^2 = 4 \qquad u^2 = 1$

$u = \pm 2 \qquad u = \pm 1$

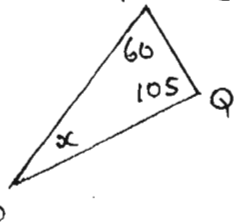
b)



$2y + 45 + 30 = 180$

$2y = 105$

$y = 52\frac{1}{2}$



$x + 60 + 105 = 180$

$x = 15$

c) $x^2 + 8x + 16 + y^2 - 2y + 1 = 8 + 1$

$(x+4)^2 + (y-1)^2 = 25$

Centre $(-4, 1)$ Radius 5

Question 3

a) $5, 9, 13 \ldots$

$a = 5 \quad d = 4$ A.P.

i) $T_{46} = 5 + (46-1) \times 4$

$T_{46} = 185$

ii) $147 = 5 + (n-1) \times 4$

$142 = 4n - 4$

$146 = 4n$

$36\frac{1}{2} = n$

n not a +ve integer $\therefore$

147 not a term

b)

T_1	T_2	T_3		T_{10}
8 ,	11 ,	14		35
$k=3$	$k=4$	$k=5$		$k=12$

$S_{10} = \frac{10}{2}(8 + 35)$

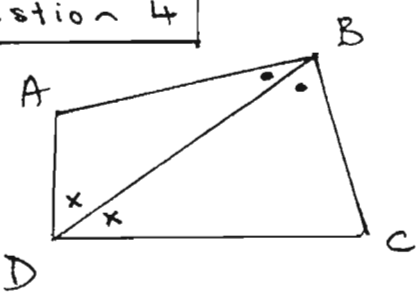
$S_{10} = 215$

c) $\frac{c}{a} = 18$ $\therefore$ $\frac{3k}{1} = 18$

$k = 6$

Question 4

a) i)



ii) In Δ's ABD, CBD

$A\hat{B}D = C\hat{B}D$ (DB bisects $A\hat{B}C$)

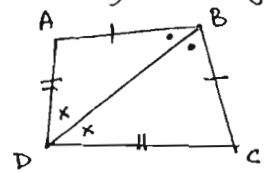
$A\hat{D}B = C\hat{D}B$ (")

DB is common

$\therefore \Delta ABD \equiv \Delta CBD$ (AAS)

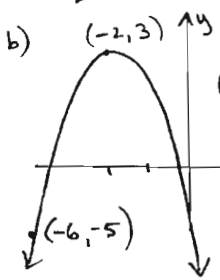
$AB = BC$, $AD = DC$ } corresponding sides in congruent triangles

iii)



ABCD is a kite

b)



Form required $(x+2)^2 = -4a(y-3)$

sub pt $(-6,-5)$

$16 = -4a(-8)$

$16 = 32a$

$\frac{1}{2} = a$

$\therefore$ eqn $(x+2)^2 = -2(y-3)$

Question 5

a) $T_3 = \frac{3}{4}$ $\quad T_7 = 12$

$ar^2 = \frac{3}{4}$ — ① $\quad ar^6 = 12$ — ②

② ÷ ① $\quad \frac{ar^6}{ar^2} = \frac{12}{3/4}$

$r^4 = 16$

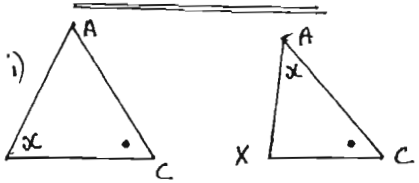
$r = \pm 2$

$r = 2 \quad a = 3/16$

GP: $\frac{3}{16}, \frac{3}{8}, \frac{3}{4}, \ldots$

or $\frac{3}{16}, -\frac{3}{8}, \frac{3}{4}, \ldots$

b) i)



In Δ's CAX, CBA

$\hat{C}$ is common

$C\hat{A}X = A\hat{B}C = x$ (given)

$\therefore \Delta CAX \,|||\, \Delta CBA$ (equiangular)

ii) $\frac{CA}{CX} = \frac{CB}{CA}$ (corsp sides similar Δ's)

$(CA)^2 = CX.CB$

Question 6

a)

i) 406, ... , 595

ii) $T_1 = 406 \quad d = 7$

$595 = 406 + (n-1) \times 7$

$189 = 7n - 7$

$n = 28 \quad \therefore T_{28} = 595 = l$

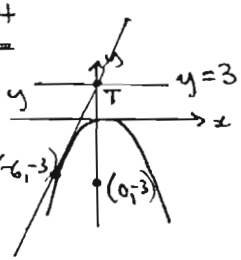
$S_{28} = \frac{28}{2}(406 + 595)$

$S_{28} = 14014$

b) i) $x^2 = -12y$

$4a = 12$

$a = 3$



$y = 3$ directrix

ii) $y = \frac{x^2}{-12}$

$\frac{dy}{dx} = \frac{2x}{-12} = \frac{x}{-6}$

$m_T = 1 \quad$ at $(-6,-3)$

eqn tang: $y + 3 = 1(x + 6)$

$0 = x - y + 3$

iii) $T(0, 3)$

Question 7

a) Real diff roots if $\Delta > 0$

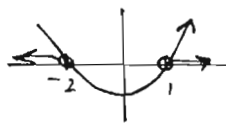
$\Delta = (-2k)^2 - 4(k-2)\times 1$

$\Delta = 4k^2 + 4k - 8$

$\therefore 4k^2 + 4k - 8 > 0$

$k^2 + k - 2 > 0$

$(k+2)(k-1) > 0$



$\therefore k > 1, \; k < -2$

b) $2^0 + 2^1 + 2^2 + \ldots . 2^{10}$

GP $a = 1 \quad r = 2 \quad n = 11$

$S_{11} = \frac{1(2^{11} - 1)}{2 - 1}$

$S_{11} = 2047$

Question 8

a) i) $\frac{3}{10} + \left[\frac{4}{100} + \frac{4}{1000} + \frac{4}{10000} \ldots\right]$

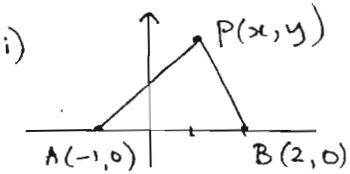
G.P $a = \frac{4}{100} \quad r = \frac{1}{10}$

ii) $0.3\dot{4} = \frac{3}{10} + \frac{4/100}{1 - 1/10}$

$= \frac{3}{10} + \frac{2}{45}$

$= \frac{31}{90}$

b) i)



$PA = 2PB$

$\sqrt{(x+1)^2 + (y-0)^2} = 2\sqrt{(x-2)^2 + (y-0)^2}$

$x^2 + 2x + 1 + y^2 = 4(x^2 - 4x + 4 + y^2)$

$x^2 + 2x + 1 + y^2 = 4x^2 - 16x + 16 + 4y^2$

$0 = 3x^2 - 18x + 3y^2 + 15$

$0 = x^2 - 6x + y^2 + 5$

ii) circle