SYDNEY TECHNICAL HIGH SCHOOL

HIGHER SCHOOL CERTIFICATE
ASSESSMENT TASK 1

DECEMBER 2013

Mathematics

General Instuctions

- Working time - 70 minutes
- Write using black or blue pen
- Board-approved calculators may be used
- All necessary working should be shown in questions 6 to 13
- Start each question on a new page

Total marks - 53

Section 1 - 5 marks

Attempt Questions 1 – 5.
Allow about 7 minutes for this section.

Section 2 - 48 marks

Attempt Questions 6 – 13.
Allow about 63 minutes for this section.

Name : ____________________

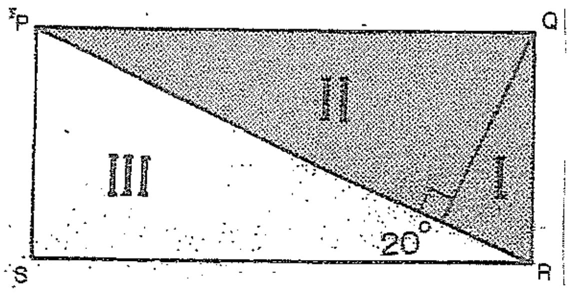
Teacher : ____________________

Section 1

5 marks
Attempt Questions 1-5
Allow about 7 minutes for this section

Use the multiple choice answer sheet in your answer booklet for Questions 1-5.
Do not remove the multiple choice sheet from your answer booklet.

1.



PQRS is a rectangle. Which triangles are similar?

A. I and II only.

B. II and III only.

C. I, II and III.

D. No two triangles are similar.

2. The coordinates of the centre and length of the radius of the circle

$x^2 + y^2 - 4x + 10y - 35 = 0$

are

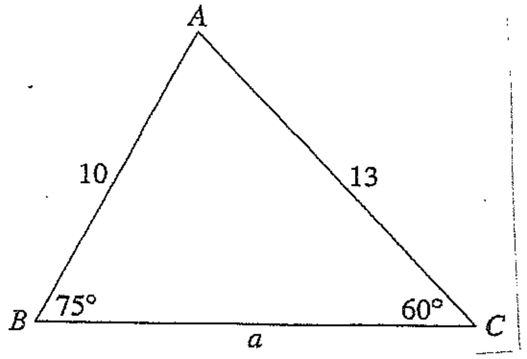
A. centre (2,-5), radius $\sqrt{6}$

B. centre (-2,5), radius $\sqrt{6}$

C. centre (2,-5), radius 8

D. centre (-2,5), radius 8

3.



The value of a is given by

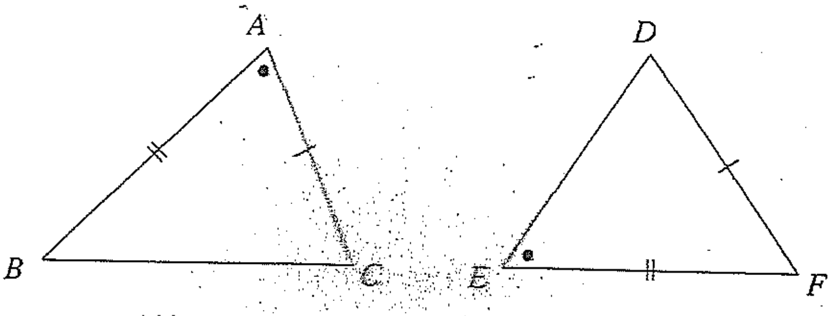
A. $\frac{10 \sin 60^\circ}{\sin 75^\circ}$

B. $\frac{10 \sin 45^\circ}{\sin 60^\circ}$

C. $\frac{10 \sin 60^\circ}{\sin 45^\circ}$

D. $\frac{13 \sin 60^\circ}{\sin 75^\circ}$

4.



$\angle A = \angle E$

AB = EF

AC = DF

If the above triangles are congruent, which of the following statements is correct?

A. The triangles must be scalene.

B. The triangles must be isosceles.

C. The triangles can be either scalene or isosceles.

D. The types of triangle cannot be determined from the information given.

5. If the angles of a pentagon form an Arithmetic Progression, then one of the angles will <u>always</u> be:

A. 120°

B. 90°

C. 108°

D. 105°

Section 2

48 marks
Attempt Questions 6-13

Allow about 63 minutes for this section
Start each question on a new page

Question 6 (6 marks) Start a new page.

a) For the series 3 + 10 + 17 + ...

i) Show that the sum of the first n terms is given by $Sn = \frac{n}{2}(7n-1)$. 2

ii) Find the least number of terms required to give a sum greater than 2000. 2

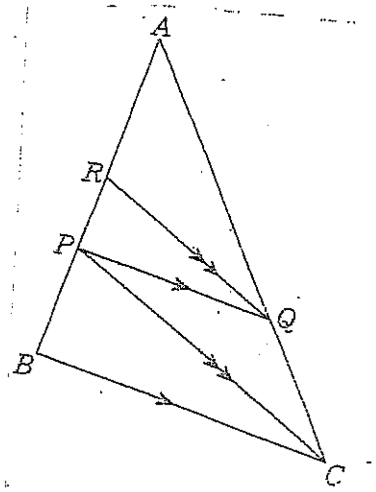
b) Solve $\tan x = -\sqrt{3}$ for $0^\circ \le x \le 360^\circ$. 2

Question 7 (6 marks) Start a new page.

a) i) Find the equation of the tangent to the parabola $x^2 = 12y$ at (6,3). 2

ii) Find the coordinates of the point where this tangent intersects with the directrix. 2

b) In the figure AP = 12cm and PB = 4cm



i) Explain why AQ:QC = 3:1 1

ii) Hence find RP. 1

Question 8 (6 marks) Start a new page.

a) The first three terms of a sequence are 3, -1 and -5. Find the 19th term. 2

b) If α and β are the roots of the quadratic equation $3x^2 + mx + p = 0$.

Find the value of

i) $\alpha + \beta$ 1

ii) $\alpha\beta$ 1

iii) If $\beta = 3\alpha$, show that $m^2 - 16p = 0$. 2

Question 9 (6 marks) Start a new page.

a) i) Find the coordinates of the vertex of the parabola with focus at (-4,-1) and equation of the directrix $y = 3$. 1

ii) Hence write down the equation of this parabola. 2

b) The sponsors of a golf tournament have provided \$232,500 for the total prizes for the first 15 places. The prize for the winner is to be \$26,000 and, from there down, each prize decreases by a constant amount.

Find:

i) the prize for finishing 15th 2

ii) the prize for finishing 2nd 1

Question 10 (6 marks) Start a new page.

a) Show that $5x - 3 - 6x^2$ is negative for all values of x. 2

b) Let A and B be fixed points (-1,0) and (2,0) and let P be the variable point (x,y).

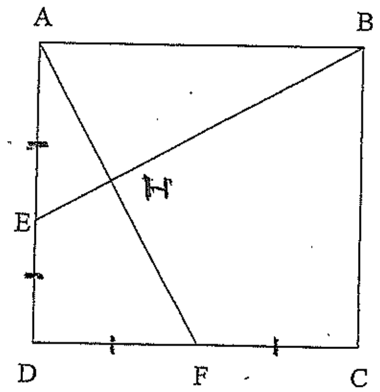
i) Write down expressions for PA^2 and PB^2 in terms of x and y. 1

ii) Suppose that P moves so that PA = 2PB, find the equation of the locus. 2

iii) Describe the locus. 1

Question 11 (6 marks) Start a new page.

a) In the diagram ABCD is a square, E and F are the mid points of the sides AD and DC respectively.



i) Redraw the diagram.

ii) Prove that $\triangle ABE$ is congruent to $\triangle ADF$. 2

iii) Explain why $< AHE = 90^\circ$. 2

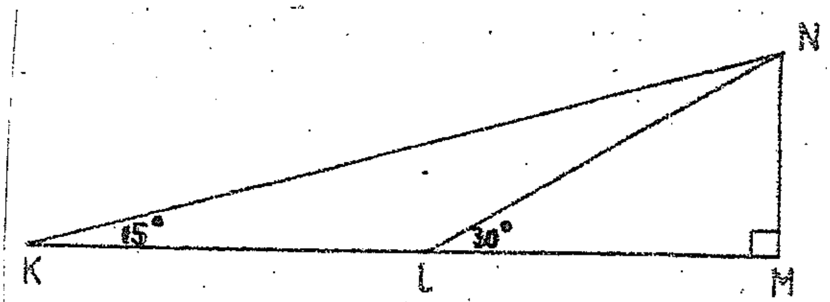
b) If α and β are the roots of the quadratic equation $2x^2 + 6x - 1 = 0$, find the value of 2

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$$

Question 12 (6 marks) Start a new page

a) Solve $x^4 = 32 - 4x^2$ 2

b)



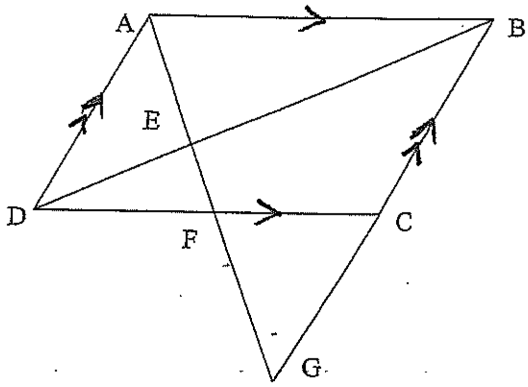
i) Explain why KL=LN 1

ii) If NM = 1, deduce that $\tan 15^\circ = 2 - \sqrt{3}$ 3

Question 13 (6 marks) Start a new page

a) Simplify $\dfrac{1 - \cos^2\alpha}{\sin\alpha \cos\alpha}$ 2

b) In the diagram ABCD is a parallelogram. A line is drawn from A to cut DB, DC and BC produced at E, F and G respectively.



i) Redraw the diagram.

ii) Prove that $\triangle ADE$ is similar to $\triangle BGE$. 2

iii) Hence or otherwise prove 2

$$\frac{AE}{EG} = \frac{AF}{AG}$$

Student Name: ____________ Teacher Name: ____________

H.S.C Assessment Task 1 Dec 2013

2 unit

Section I

1) C

2) C

3) B

4) B

5) C

Section 2

Question 6

a i) $S_n = \frac{n}{2}[2\times3+(n-1)7]$

$= \frac{n}{2}[6+7n-7]$

$= \frac{n}{2}[7n-1]$

ii) $\frac{n}{2}[7n-1] = 2000$

$7n^2 - n = 4000$

$7n^2 - n - 4000 = 0$

$n = \frac{1 \pm \sqrt{1 - 4\times7\times-4000}}{14}$

$n = \frac{1 \pm \sqrt{112001}}{14}$

∴ need 24 terms

b) $\tan x = -\sqrt{3}$

$x = 60°$ (2nd, 4th)

$x = 120°, 300°$

Question 7

a i) $y = \frac{x^2}{12}$

$y' = \frac{x}{6}$

at $(6,3)$ $y' = 1$

∴ equation of tangent

$y - 3 = 1(x-6)$

$y = x - 3$

ii) when $y=-3, x=0$

∴ point $(0,-3)$

b i) The intercepts on all transversals by a family of parallel lines are in the same ratio

ii) $\frac{RP}{AP} = \frac{x}{12-x} = \frac{1}{3}$

∴ $3x = 12 - x$

$4x = 12$

$x = 3$

∴ RP = 3cm

Student Name: ____________ Teacher Name: ____________

Question 8

a) $T_{19} = 3 + 18\times-4$

$= -69$

b) i) $\alpha + \beta = -\frac{m}{3}$

ii) $\alpha\beta = \frac{p}{3}$

iii) $\alpha + 3\alpha = -\frac{m}{3}$

$4\alpha = -\frac{m}{3}$

$\alpha = -\frac{m}{12}$

also $3\alpha^2 = \frac{p}{3}$

$3\left(\frac{-m}{12}\right)^2 = \frac{p}{3}$

$m^2 = \frac{144p}{9}$

$m^2 - 16p = 0$

Question 9

a) i) vertex $= (-4, 1)$

ii) $(x+4)^2 = -4\times2(y-1)$

$(x+4)^2 = -8(y-1)$

b i) $232\,500 = \frac{15}{2}[26000 + L]$

$465\,00 = 390000 + 15L$

$15L = 75000$

$L = 5000$

Prize for 15th is \$5000

ii) $26000 = 5000 + 14d$

$14d = 21000$

$d = 1500$

∴ 2nd prize $= 26000 - 1500$

$= \$24,500$

Question 10

a) $\Delta = 5^2 - 4\times-6\times-3$

$= -47$

$a = -6$ ∴

∴ $a < 0$

∴ expression is negative for all values of x

b i) $PA^2 = (x+1)^2 + y^2$

$PB^2 = (x-2)^2 + y^2$

ii) $PA = 2PB$

$PA^2 = 4PB^2$

$(x+1)^2 + y^2 = 4[(x-2)^2 + y^2]$

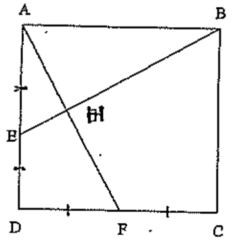
$x^2 + 2x + 1 + y^2 = 4[x^2 - 4x + 4 + y^2]$

$3x^2 + 3y^2 - 18x + 15 = 0$

iii) Locus is a circle

Student Name: ______ Teacher Name: ______

Question 11



a) ii) In $\triangle ABE$ and $\triangle ADF$

$AB = AD$ (sides of square)

$AE = DF$ (data)

$\angle BAE = \angle FDA = 90°$ (angles of square)

$\therefore \triangle ABE \equiv \triangle ADF$ (SAS)

iii)

$\angle AEB + \angle ABE = 90°$ (angle sum of triangle)

but $\angle EAH = \angle ABE$

(corresponding angles of congruent triangles)

$\therefore \angle EAH + \angle AEH = 90°$

$\therefore \angle AHE = 90°$

b) $\alpha + \beta = -3$, $\alpha\beta = -1/2$

Now $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta}$

$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$

$= \frac{(3)^2 - 2 \times -1/2}{-1/2}$

$= -20$

Question 12

a) $x^4 + 4x^2 - 32 = 0$

Let $m = x^2$

$\therefore m^2 + 4m - 32 = 0$

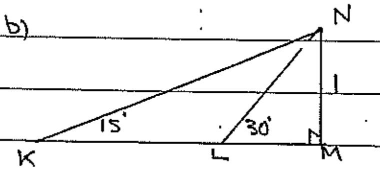
$(m+8)(m-4) = 0$

$m = -8$ or 4

$\therefore x^2 = -8$ or $x^2 = 4$

$\therefore x = 2$ or -2

b)



i) $\angle KNL = 15°$ (exterior $\angle$ of triangle)

$\therefore KL = LN$ (sides opposite equal angles in isosceles $\triangle$)

ii) $\tan 60° = \frac{LM}{1}$

$\therefore LM = \sqrt{3}$

$\text{cosec } 30° = \frac{LN}{1}$

$\therefore LN = 2$

$\therefore KL = 2$

$\tan 15° = \frac{1}{2+\sqrt{3}}$

$= \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$

$= 2-\sqrt{3}$

Student Name: ______ Teacher Name: ______

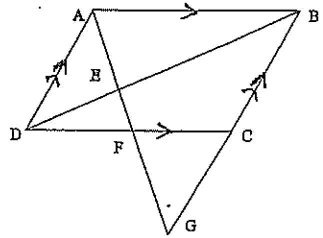
Question 13

a) $\frac{1-\cos^2\alpha}{\sin\alpha\cos\alpha} = \frac{\sin^2\alpha}{\sin\alpha\cos\alpha}$

$= \frac{\sin\alpha}{\cos\alpha}$

$= \tan\alpha$

b)



ii) In $\triangle ADE$ and $\triangle BGE$

$\angle AED = \angle BEG$ (vertically opposite)

$\angle DAE = \angle BGE$ (alternate angles on parallel lines)

$\therefore \triangle ADE \parallel\!| \triangle BGE$ (equiangular)

ii) $\frac{AE}{EG} = \frac{AD}{BG}$ (corresponding sides of similar triangles ~~in same ratio~~)

Now $\triangle ADF \parallel\!| \triangle GBA$ (equiangular)

$\therefore \frac{AD}{BG} = \frac{AF}{AG}$ corresponding sides of similar triangle in same ratio)

$\therefore \frac{AE}{EG} = \frac{AF}{AG}$