TRINITY GRAMMAR SCHOOL

Mathematics Department

E	H								

(NESA Student Number | Year 12 only)

Suggested solutions.

2019

Year 12

Mathematics

HSC ASSESSMENT TASK 1

Date of Assessment Task: Thursday, 15 November 2018

General Instructions

- Reading time – not applicable to this Task
- Working time – 45 minutes
- Write using black pen
- NESA approved calculators may be used
- A formula and data sheet is provided
- In Questions 6 – 7, show relevant mathematical reasoning and/or calculations
- Write your NESA Student Number (Year 12 HSC) or Name (Year 11 or 10) and your Class teacher on the question paper and on any answer sheets or writing booklets used to write your responses to the questions submitted
- If you do not attempt a question you must submit an answer sheet or writing booklet for that question clearly indicating N/A and your NESA Student Number or Name

Total marks: 35

Section I – 5 marks (pages 2 – 4)

- Attempt Questions 1 – 5
- Allow about 5 minutes for this section

Section II – 30 marks (pages 5 – 7)

- Attempt Questions 6 – 7
- Allow about 40 minutes for this section

- **HSC Assessment Weighting:** 20%

D	B	D	A	C
Q1	Q2	Q3	Q4	Q5

Section I

5 marks
Attempt Questions 1 – 5
Use the multiple-choice answer sheet for Questions 1 – 5.

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1 If $y=(x^2+3)^5$ then $\dfrac{dy}{dx}=$

$5(x^2+3)(2x)$
$= 10x(x^2+3)$

A. $2x$

B. $5(x^2+3)^4$

C. $2x(x^2+3)^4$

D. $10x(x^2+3)^4$

2 Ken decided to use differentiation from first principles to differentiate the expression $2x-x^2$. A correct expression, that Ken ought to have used in order to differentiate this expression is

let $f(x) = 2x - x^2$
$f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h)-f(x)}{h}$
$= \lim\limits_{h \to 0} \left[\dfrac{2(x+h)-(x+h)^2-2x+x^2}{h}\right]$

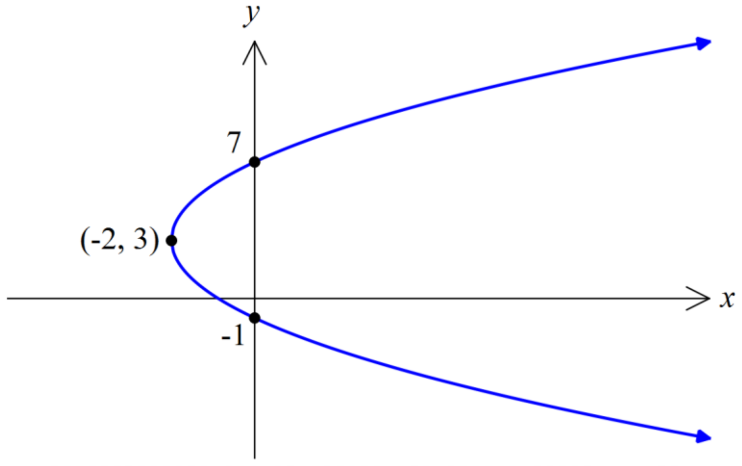
A. $\lim\limits_{h\to 0}\left(\dfrac{2(x+h)-(x+h)^2}{h}\right)$

B. $\lim\limits_{h\to 0}\left(\dfrac{x^2-2x-(x+h)^2+2(x+h)}{h}\right)$

C. $\lim\limits_{h\to 0}\left(\dfrac{x^2-2x-(x+h)^2-2(x-h)}{h}\right)$

D. $\lim\limits_{h\to 0}\left(\dfrac{x^2-2x+(x+h)^2-2(x+h)}{h}\right)$

3 The diagram below is of a parabola with vertex at $(-2, 3)$ and it has y-intercepts at -1, and 7 respectively.



The Cartesian equation of the parabola is given by

A. $(x+2)^2 = 4(y-3)$

B. $(x+2)^2 = 8(y-3)$

C. $(y-3)^2 = 4(x+2)$

D. $(y-3)^2 = 8(x+2)$

$(y-k)^2 = 4a(x-h)$

$(y-3)^2 = 4a(x+2)$

sub $(0,7)$

$4^2 = 4a(2)$

$\therefore \; a = 2$

4 The quadratic equation $2kx^2 - 4kx + 1 = 0$ has two equal roots. The value(s) of k is

A. $\frac{1}{2}$ only

B. 0 and $\frac{1}{2}$

C. 2 only

D. 0 and 2

Want $k > 0$ for a quadratic equation to exist and $\Delta = 0$ for equal roots.

$\therefore \; (-4k)^2 - 4(2k)(1) = 0$

$16k^2 - 8k = 0$

$8k(2k-1) = 0$

$\Rightarrow k = ½$ only.

Turn over

5 A quadratic equation with **integer** coefficients for which the sum **and** product of its roots is -3 and $-\frac{1}{2}$ respectively, is

Let $\alpha+\beta=-3$

$\alpha\beta=-\frac{1}{2}$

A. $2x^2-3x-1=0$

B. $2x^2-3x-1=0$

C. $2x^2+6x-1=0$

D. $2x^2-6x-1=0$

$\therefore x^2-(\alpha+\beta)x+\alpha\beta=0$

$x^2-(-3x)-\frac{1}{2}=0$

$x^2+3x-\frac{1}{2}=0$

$2x^2+6x-1=0$

Section II

30 marks
Attempt Questions 6 – 7
Allow about 40 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 6 – 7, your responses should include relevant mathematical reasoning and/or calculations.

Question 6 (15 marks) Use a SEPARATE writing booklet.

(a) Find:

(i) $\frac{d}{dx}\left(3+2x^3\right)$; $= 6x^2$ ✓✓ **2**

(ii) $\frac{d}{dx}\left(\frac{3x+5}{x-2}\right)$ using the quotient rule; **2**

$= \frac{(x-2)(3)-(3x+5)(1)}{(x-2)^2}$ ✓

$= \frac{-11}{(x-2)^2}$ ✓ (should simplify)

(iii) $\frac{d}{dx}\left(x\sqrt{1+x}\right)$ using the product rule. **2**

$= (\sqrt{1+x})(1) + (x)\left[\frac{1}{2}(1+x)^{-1/2}(1)\right]$ ✓✓

$= \sqrt{1+x} + \frac{x}{2\sqrt{1+x}}$ (should simplify)

(b) Evaluate $\lim_{x\to 4}\left(\frac{x^3-64}{x-4}\right)$. **3**

$= \lim_{x\to 4} \frac{(x-4)(x^2+4x+16)}{(x-4)}$ ✓✓ must have correct factorisation

$= 4^2+4(4)+16 = 48$ ✓ as $x \to 4$.

(c) Find the value of x for which $f(x)=x^2-2x+1$ has zero gradient. **2**

$f'(x) = 2x-2$ ✓

want $2x-2=0$ (zero gradient)

$\therefore x = 1$ ✓

Question 6 continues on page 6

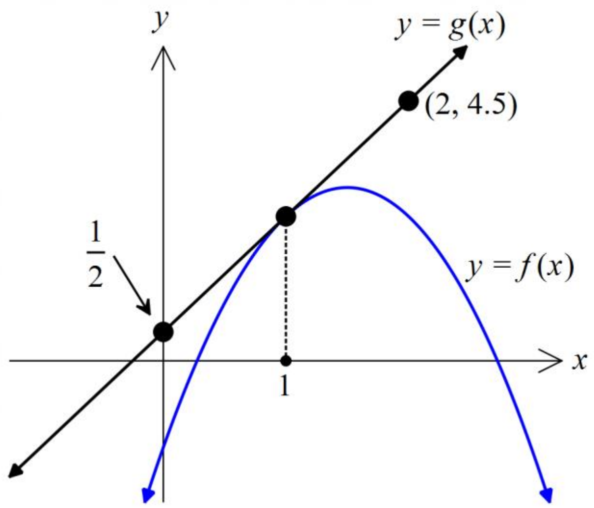
(can use $x=-b/2a$).

[There are several other methods] e.g. $f(x)=(x-1)^2$ $\therefore x=1$ is a zero as well. $\Delta=0$ & $(1,0)$ is the vertex.

Turn over

Question 6 (continued)

(d) Consider the diagram below where the graph of $y = g(x)$ is a straight line and tangent to the graph of $y = f(x)$ at the point where $x = 1$. The graph of $y = g(x)$ intersects the y-axis at $\left(0, \frac{1}{2}\right)$ and passes through the point (2, 4.5).



(i) Find the value of $f'(1)$. $= \dfrac{4.5 - \frac{1}{2}}{2 - 0} = 2$ **2**

(ii) Evaluate $f(1)$. **2**

$g(x) = 2x + b$

but $b = \frac{1}{2}$ (y-int)

$\therefore g(x) = 2x + \frac{1}{2}$

End of Question 6

$\therefore f(1) = g(1) = 2 \times 1 + \frac{1}{2}$

$= 2.5$ //

[There may be other methods]

Question 7 (15 marks) Use a SEPARATE writing booklet.

(a) The roots of a quadratic equation $x^2 - 3x - 7 = 0$ are α and β.

Without finding the actual roots,

(i) Write down the value of $\alpha + \beta$. $= 3 \quad (-b/a)$ ✓ **1**

(ii) Write down the value of $\alpha\beta$. $= -7 \quad (c/a)$ ✓ **1**

(iii) Evaluate $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$. $= \dfrac{\alpha+\beta}{\alpha\beta} = -\dfrac{3}{7}$ ✓ **1**

(iv) Evaluate $a^2\beta + \beta^2\alpha$. $= \alpha\beta(\alpha+\beta) = (-7)(3)$ $= -21.$ ✓ **1**

(v) Evaluate $\alpha^2 + \beta^2$. **1**

$= (\alpha+\beta)^2 - 2(\alpha\beta)$

$= 3^2 - 2(-7)$

$= 9 + 14 = 23$ ✓ must have correct answer

(b) Find the value(s) of k for which the expression $kx^2 + 3x + k - 2$ is positive definite. **3**

Want $k > 0$ and $\Delta < 0$ ✓ i.e. $9 - 4k(k-2) < 0$

$9 - 4k^2 + 8k < 0$

$4k^2 - 8k - 9 > 0$ ✓

roots are $k = \dfrac{8 \pm \sqrt{64 - 4(4)(-9)}}{8} = \dfrac{8 \pm \sqrt{208}}{8}$

$= \dfrac{8 \pm 4\sqrt{13}}{8}$

$= 1 \pm \sqrt{13}/2$

$1 - \sqrt{13}/2 \qquad 0 \qquad 1 + \dfrac{\sqrt{13}}{2} \quad k$

$\therefore k > 1 + \dfrac{\sqrt{13}}{2}$ only ✓

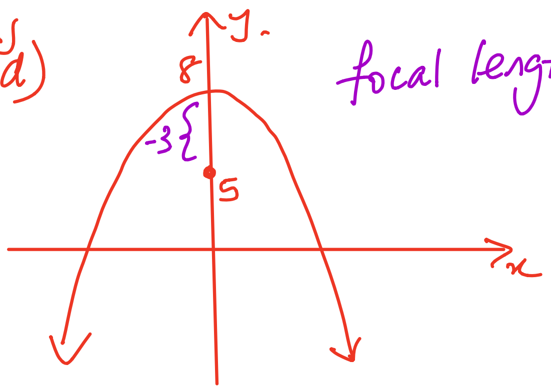
(Since $k > 0$ as well)

(c) Solve for x if $6^{2x} - 37(6^x) + 36 = 0$. **2**

(d) A parabola has its focus at (0, 5) and its vertex at (0, 8). Find the focal length **and** the equation of the directrix of this parabola. **2**

(e) Find the equation of the locus of a point $P(x, y)$ such that AP and PB are perpendicular, given $A(3, 2)$ and $B(4, -1)$. Describe the locus of P. **3**

d)



focal length $= |a| = |-3|$ $= 3$ ✓

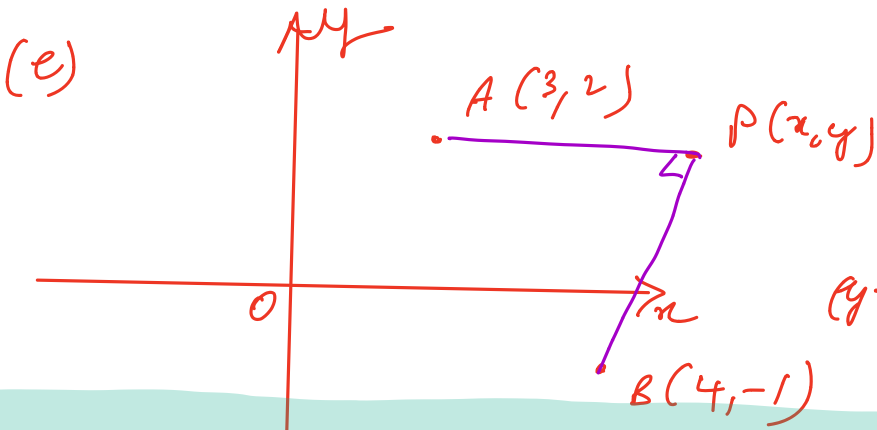
End of Question 7

End of Task

Directrix $y = 8 + 3$

i.e. $y = 11$ ✓

(e)



Want $m_{AP} \times m_{PB} = -1$

$$\left(\frac{y-2}{x-3}\right)\left(\frac{y+1}{x-4}\right) = -1$$

$$(y-2)(y+1) = -(x-3)(x-4)$$

$$y^2 - y - 2 = -x^2 + 7x - 12$$

$$x^2 - 7x + y^2 - y = -10$$

$$x^2 - 7x + \left(-\tfrac{7}{2}\right)^2 + y^2 - y + \left(-\tfrac{1}{2}\right)^2$$

$$= -10 + \left(-\tfrac{7}{2}\right)^2 + \left(-\tfrac{1}{2}\right)^2$$

$$= -10 + \frac{49}{4} + \frac{1}{4}$$

$$= 5/2$$

$$\left(x - \tfrac{7}{2}\right)^2 + \left(y - \tfrac{1}{2}\right)^2 = 5/2$$

Centre $\left(\frac{7}{2}, \frac{1}{2}\right)$

$r = \sqrt{5/2} \quad \left(= \frac{\sqrt{10}}{2}\right)$

Alternatively AB is a diameter of a circle.

midpt is $M\left(\frac{3+4}{2}, \frac{2-1}{2}\right) = \left(\frac{7}{2}, \frac{1}{2}\right)$

$$AM = \sqrt{\left(3 - \tfrac{7}{2}\right)^2 + \left(2 - \tfrac{1}{2}\right)^2}$$

$$\therefore \left(x - \tfrac{7}{2}\right)^2 + \left(y - \tfrac{1}{2}\right)^2 = (AM)^2$$

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