BAULKHAM HILLS HIGH SCHOOL

MATHEMATICS EXTENSION 1 ASSESSMENT

December 2011

Time allowed: 50 minutes
plus 5 minutes reading time

STUDENT NUMBER :___________________

TEACHER'S NAME:___________________

QUESTION	MARK
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
TOTAL	
PERCENTAGE	

<u>Topics Tested:</u> Polynomials and Induction

Extension-1 Mathematics

December 2011

Time: 50 minutes + 5 minutes reading time

DIRECTIONS

- Full working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Use black or blue pen only (*not pencils*) to write your solutions.
- No liquid paper is to be used. If a correction is to be made, one line is to be ruled through the incorrect answer.
- At the end of the exam, staple your answers in order behind the cover sheet provided, and your questions on the back
- Approved Maths aids and calculators may be used

1.	Determine whether each statement is true or false for $n = 1$. a) $5 + 10 + x + \cdots + 5(2^{n-1}) = 5(2^n - 1)$ b) $\sum_{n=1}^{r} 64(-2)^{1-n} = 32(1 - 2^{-r})$	2
2.	Determine which of the following expressions are polynomials a) $x^4 - 2x + 9$ b) $4x^3 - 5x + \log x$	 1 1
3.	What is the degree of the polynomial $P(x) = (4x - 6)(3x^4 - 2x) + 12$	1
4.	A polynomial of degree 3 has zeros at -1,2 and 3. Its leading coefficient is 4. What is the constant term.	2
5.	a) Without using calculus, sketch the graph $f(x) = x^3 - 5x^2 - 14x$ b) Hence (or otherwise) solve $\dfrac{x^2 - 4x - 14}{x} \geq 1$	2 2
6.	If α, β and γ are the roots of the polynomial $x^3 + 2x^2 - x - 5 = 0$ Find a) $\alpha + \beta + \gamma$ b) $\alpha\beta + \beta\gamma + \alpha\beta$ c) $\alpha\beta\gamma$ d) $\dfrac{1}{\alpha} + \dfrac{1}{\beta} + \dfrac{1}{\gamma}$ e) $(\alpha + 1)(\beta + 1)(\gamma + 1)$	 1 1 1 2 2
7.	Using mathematical induction prove that for integers $n \geq 1$ $$\sum_{r=1}^{n} r \times 2^{r-1} = (n - 1)2^n + 1$$	3
8.	Find the roots of the following polynomial $4x^3 - 4x^2 - 29x + 15 = 0$ given that one root is the difference between the other two roots.	3
9.	Use the principle of mathematical induction to prove that $5^n + 12n - 1$ is a multiple of 16 for all positive integers n	3
10.	The polynomial $P(x) = 4x^3 + ax^2 + 3x + b$ is divisible by $(x + 2)$ and $(x - 1)$ Find a and b.	3

11.	Given that $(x-5)$ is a factor of $P(x) = -x^3 - x^2 + 21x + 45$ a) Express $P(x)$ in factored form b) Without the aid of calculus, sketch $P(x)$ showing main features.	 3 2
12.	The polynomial $f(x)$ • Has degree 37 • A remainder of 1 when divided by $(x-1)$ • A remainder of 3 when divided by $(x-3)$ • A remainder of 21 when divided by $(x-5)$ Find the remainder when $f(x)$ is divided by $(x-1)(x-3)(x-5)$	3
~ END OF EXAM ~		

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1. a) T ✓

b) F ✓

2 a) Yes ✓

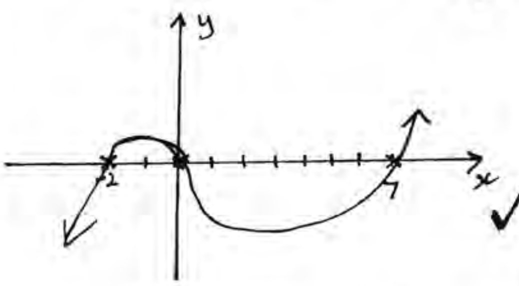
b) NO ✓

3. 5 ✓

4. 24 ✓✓

$4(x+1)(x-2)(x-3)$

5 a) $f(x) = x(x-7)(x+2)$ ✓



✓

b) $\frac{x^2-4x-14}{x} - 1 \geq 0$

$x^2\left(\frac{x^2-5x-14}{x}\right) \geq 0 \times x^2$

$\therefore x(x^2-5x-14) \geq 0$ ✓✓

$\therefore -2 \leq x < 0,\ x \geq 7$

6. $x^3+2x^2-x-5=0$

a) sum: $-\frac{b}{a} = -2$ ✓

b) sum×2: $\frac{c}{a} = -1$ ✓

c) product: $\frac{-d}{a} = 5$ ✓

d) $\frac{\beta\gamma+\alpha\beta+\alpha\gamma}{\alpha\beta\gamma} = \frac{-1}{5}$ ✓✓

e) $\alpha\beta\gamma + \alpha\gamma + \beta\gamma + \gamma + \alpha\beta + \alpha + \beta$ ✓

$= 5 - 1 - 2 + 1$

$= 2 + 1 = 3$ ✓

✓ structure

(7) $\sum_{r=1}^{n} r \times 2^{r-1} = (n-1)2^n + 1$ ✓✓ step 3.

$S_n = 2^0 + 2\times2 + 3\times2^2 + \dots n\times2^{n-1} = (n-1)2^n + 1$

Test true for n=1

$LHS = 2^0 = 1$ $RHS = 0\times2^1+1 = 1$ $\therefore$ true for $n=1$.

$LHS = RHS$

assume true for n=k.

$S_K = 1 + 2\times2 + 3\times2^2 + \dots K\times2^{K-1} = (K-1)2^K + 1$

prove true for n=k.

ie $S_K + T_{K+1} = S_{K+1}$

prove $(K-1)2^K + 1 + (K+1)2^K = (K)2^{K+1} + 1$

$LHS = K.2^K - 2^K + 1 + K.2^K + 2^K$

$= 2.K.2^K + 1$

$= (K)2^{K+1} + 1$

$= RHS$.

$\therefore$ if true for n=k now true for n=k+1

Since true for n=1 then true for n=1+1=2

and n=2+1=3 and so on by M.I for all integers $n \geq 1$

(8) $4x^3 - 4x^2 - 29x + 15 = 0$

let $\gamma = \alpha - \beta$

sum: $\alpha + \beta + \alpha - \beta = -\frac{b}{a}$

$2\alpha = 1$ ✓

$\alpha = \frac{1}{2}$

product: $\alpha\beta(\alpha-\beta) = \frac{-15}{4}$

$\frac{1}{2}\beta(\frac{1}{2}-\beta) = -\frac{15}{4}$ ✓

$2\beta^2 - \beta - 15 = 0$

$(2\beta+5)(\beta-3) = 0$

$\beta = \frac{-5}{2}$ $\beta = 3$

$\therefore$ check.

$\gamma = \frac{1}{2} - -\frac{5}{2} = 3$ $\gamma = \frac{1}{2} - 3 = -\frac{5}{2}$

$\therefore$ zeros or roots are $\frac{1}{2}, -\frac{5}{2}, 3$. ✓

9. $5^n + 12n - 1$ is a multiple of 16 ✓✓✓

test true for $n = 1$

$5^1 + 12 - 1 = 16$ which is divisible by 16

assume true for $n = k$

$\frac{5^k + 12k - 1}{16} = m$ (where m is some integer)

$\therefore 5^k + 12k - 1 = 16m$

or $5^k = 16m - 12k + 1$ ... ①

prove true for $n = k+1$

prove $5^{k+1} + 12(k+1) - 1 = 16N$ (where N is some integer)

now $LHS = 5^k . 5 + 12k + 12 - 1$

using ① $= (16m - 12k + 1)5 + 12k + 11$

$= 5 \times 16m - 60k + 5 + 12k + 11$

$= 5 \times 16m - 48k + 16$

$= 16(5m - 3k + 1)$

which is a multiple of 16 for $N = 5m - 3k + 1$

$\therefore$ if true for $n = k$ now true for $n = k+1$

Since true for $n = 1$ now true for $n = 1+1 = 2$, $n = 2+1 = 3$ and so on by MI for all $n \geq 1$.

10. $P(x) = 4x^3 + ax^2 + 3x + b$

$P(-2) = 4 \times (-2)^3 + a \times 4 - 6 + b = 0$

$4a + b = 38$ ---①

$P(1) = 4 + a + 3 + b = 0$ ✓

$a + b = -7$ -②

① - ② $3a = 45$

$a = 15$ ✓

sub back into ② $15 + b = -7$

$b = -22$

$\therefore a = 15$, $b = -22$. ✓

3

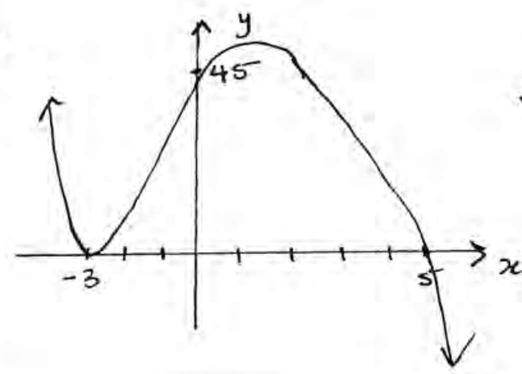
11.

$$\begin{array}{r} -x^2 - 6x - 9 \\ x-5 \overline{) -x^3 - x^2 + 21x + 45} \\ -x^3 + 5x^2 \\ \hline -6x^2 + 21x \\ -6x^2 + 30x \\ \hline -9x + 45 \\ -9x + 45 \\ \hline 0 \end{array}$$

✓✓ working

$\therefore P(x) = (x-5)(-x^2 - 6x - 9)$

$= -(x-5)(x+3)^2$ ✓

b) 

✓✓

12.

$P(x) = (x-1)(x-3)(x-5)Q(x) + R(x)$

$R(x)$ could be $ax^2 + bx + c$ ✓ to have degrees less than $A(x)$

$\therefore R(1) = a + b + c = 1$ ---① ✓

$R(3) = 9a + 3b + c = 3$ ---②

$R(5) = 25a + 5b + c = 21$ ---③

②-① $8a + 2b = 2$ ---④

③-② $16a + 2b = 18$ ---⑤

⑤-④ $8a = 16$

$a = 2$

sub into ④ $16 + 2b = 2$

$2b = -14$

$b = -7$

sub into ① $2 - 7 + c = 1$

$c = 6$

remainder $R(x) = 2x^2 - 7x + 6$ ✓

3