Caringbah High School
Year 12 2011
Extension 1 Mathematics
HSC Assessment Task 1
December 2010

Time Allowed: 60 minutes

Instructions:
All questions may be attempted.
All necessary working should be shown.
Approved calculators may be used.
Start each question on a new page
Marks may not be awarded for partial or incomplete answers

Question 1: (10 marks)

(a) Find the remainder when the polynomial $P(x) = x^3 - 2x^2 - 9x + 10$ is divided by $x - 2$ **1**

(b) Write down the equation of the horizontal and vertical asymptotes of: $y = \dfrac{x}{x+3}$ **2**

(c) The equation $x^3 - x^2 + x - 2 = 0$ has a root which is approximately 1. Use one application of Newton's Method to find a better approximation. **2**

(d) A monic polynomial $P(x)$ is of degree 4 and has a single root at $x = -3$ and a double root at $x = 1$. Write an expression for $P(x)$ **2**

(e) When a council offers free reflective house numbers, 20% of residents install them in the first month. In the second month the number of residents who install them is only 20% of those who installed them in the first month. In the third month the number installed is 20% of the number installed in the second month. Beyond the third month, the pattern is continued. What percentage of residents eventually installs them? **2**

(f) For what values of x is the curve $y = x^3 - 6x^2 + 4x - 1$ concave down? **1**

Question 2:   (10 marks)

(a) The equation $2x^3 - 5x^2 + 6x - 4 = 0$ has roots α, β and γ. Without solving the equation, evaluate:

(i) $\alpha + \beta + \gamma$ **1**

(ii) $\alpha\beta + \alpha\gamma + \beta\gamma$ **1**

(iii) $\alpha\beta\gamma$ **1**

(iv) $\dfrac{1}{\alpha} + \dfrac{1}{\beta} + \dfrac{1}{\gamma}$ **1**

(v) $\alpha^2 + \beta^2 + \gamma^2$ **1**

(b) The sum of the first *n* terms of the series 30 + 26 + 22 + ... is 120.   **2**
     Find the value(s) of *n*.

(c) A sequence has as its *nth* term $T_n = 5^{n-1} - 3n$

(i) Find the first 3 terms of the sequence   **1**

(ii) Hence find the an expression for S_n **2**

Question 3:   (10 marks)

(a) For the polynomial $P(x) = x^4 - 8x^3 + 15x^2 + 4x - 20$

(i) Factorise $P(x)$ given that it has a double root at $x = 2$ **3**

(ii) Hence draw a neat sketch of $P(x)$ **2**

Question 3: cont.

(b)   Blue metal is distributed by a lorry operating from a base at the beginning of a road. The first load is deposited 500m from the base, and the truck then returns to the base to collect the next load. Subsequent loads are deposited 150m further down the road.

  (i)   Write an expression for the distance travelled to deposit the *nth* load.   **2**

  (ii)   If the truck travels 52km by the end of the day, how many deposits has it made?   **3**

Question 4:   (10 marks)

For the curve $y = \dfrac{x}{(x+1)^2}$

  (i)   Find the equation of any vertical asymptotes.   **1**

  (ii)   Show $\dfrac{dy}{dx} = \dfrac{1-x}{(x+1)^3}$ **1**

(iii)   Find any stationary points and determine their nature.   **2**

(iv)   Find any intercepts on the x-axis.   **1**

 (v)   Given $\dfrac{d^2y}{dx^2} = \dfrac{2x-4}{(x+1)^4}$, find any points of inflexion.   **1**

(vi)   As $x \to \infty^+$, what does the curve $y = \dfrac{x}{(x+1)^2}$ approach?   **1**

(vii)   Find y when $x = -2$ **1**

(viii) Hence sketch the curve showing illustrating all the features found above.   **2**

Question 5: (10 marks)

(a) Marc borrows $250 000. Interest is charged on the amount owing at the beginning of each month at the rate of 8.4% p.a., compounded monthly. Marc intends to repay this loan in 8 years by making 16 payments of $M, one payment at the end of every 6 months.

(i) Given that the amount owing at the end of the first 6 months, after Marc has made his first payment can be expressed as: $250000 \times 1.007^6 - M$ write an expression for the amount owing at the end of the first year.   **1**

(ii) Hence write an expression, in terms of M, for the amount owing at the end of 8 years ie 96 months   **1**

(iii) If the loan is fully repaid at the end of the 8 years, find the value of the payment, $M. (Give your answer correct to the nearest dollar)   **3**

(iv) What is the simple interest rate that was charged on this loan?   **1**

(b) Find two values of m such that the roots of the equation $x^3 + 2x^2 + mx - 16 = 0$ are α, β and $\alpha\beta$.   **4**

End of Exam