Name_____ Teacher_____

GOSFORD HIGH SCHOOL

2015

HIGHER SCHOOL CERTIFICATE

ASSESSMENT TASK 1

MATHEMATICS – EXTENSION 1

Time Allowed - 60 minutes plus 5 minutes reading time

- Write using a black or blue pen. Black pen is preferred.
- Board approved calculators may be used.
- Answers to the multiple choice are to be done on the answer sheet provided.
- Start each section on a **new** page. **Each section** will be collected **separately.**
- In Sections 2-4, show relevant mathematical reasoning and/or calculations.

Section 1 Multiple choice	**4** questions worth **1 mark each**.	**/4**
Section 2	Parametric treatment of the parabola	**/14**
Section 3	Polynomials	**/16**
Section 4	Miscellaneous review	**/12**
TOTAL		**/46**

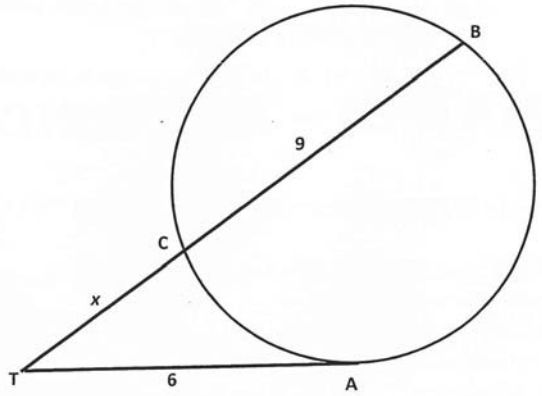
Section 1 Objective response questions (1 mark each)

Answer each question on the multiple choice answer sheet provided.

1) What is the sum of the zeros of $(x) = 2x^3 + 8x^2 - x + 4$?

(A) 4 **(B)** -4 **(C)** 0 **(D)** -8

2) Line *TA* is a tangent to the circle at *A* and *TB* is a secant meeting the circle at *B* and *C*. Given that *TA* = 6, *CB* = 9 and *TC* = *x*, what is the value of *x*?



(A) -12 **(B)** 2 **(C)** 3 **(D)** 4

3) The polynomial $P(x)$ is monic and of degree 5. It has a single zero at $x = -2$ and a double zero at $x = 1$ Its other two zeros are not real.

Which of the following equations **best** represents $P(x)$?

(A) $(x^2 - 1)(x + 2)(x - a)(x - b), where\ b^2 - 4c < 0$

(B) $(x - 1)(x + 2)^2(x + a)(x + b), where\ b^2 - 4c \geq 0$

(C) $(x + 2)(x - 1)^2(x^2 + bx + c), where\ b^2 - 4c \geq 0$

(D) $(x + 2)(x - 1)^2(x^2 + bx + c), where\ b^2 - 4c < 0$

4) Evaluate

$$\lim_{x \to \infty} \frac{3x^2 + 1}{7 + 4x^2}$$

(A) $\frac{3}{7}$ **(B)** $\frac{3}{4}$ **(C)** $\frac{1}{7}$ **(D)** $\frac{1}{4}$

MARKS

Section 2 Parametric treatment of the parabola (Start a new page)

1) The point $P(2ap, ap^2)$ lies on the parabola $x^2 = 4ay$

(a) **Derive** the equation of the **tangent** and the **normal** at the point *P* 3

(b) The tangent and the normal at ***P*** meet the axis of the parabola $x^2 = 4ay$ at the points ***T*** and ***N*** respectively. Show that $(TN)^2 = 4a^2(1 + p^2)^2$ 4

2) The variable point $(6t, 4t^2)$ lies on a parabola. Find the coordinates of the focus of this parabola. 2

3) Find the equation of the chord of contact of the tangents to the parabola $x^2 = 4y$ from the external point (4,-6) 2

4) The points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ lie on a parabola with focus S. PQ is a focal chord with equation $y - \frac{1}{2}(p + q)x + apq = 0$. Show that

(a) $pq = -1$ 1

(b) If $p + q = 4$ and using part (a), show that the midpoint of ***PQ*** is $(4a, 9a)$ 2

Section 3 Polynomials (Start a new page)

1) If α, β, γ are the roots of the cubic equation $x^3 - 3x + 2 = 0$ find

(i) $\alpha + \beta + \gamma$ 1

(ii) $\alpha\beta + \alpha\gamma + \beta\gamma$ 1

(iii) $\alpha\beta\gamma$ 1

(iv) $\alpha^2 + \beta^2 + \gamma^2$ 2

2) The polynomial $P(x) = x^3 - 6x^2 + \boldsymbol{k}x + 14$ has a root at x=1.

(i) Determine the value of k 1

(ii) For this value of k, express $P(x)$ as a product of its linear factors 3

(iii) For this value of k, solve $P(x) > 0$ 1

3) When $P(x)$ is divided by $x^2 - 4$ the remainder is $2x + 3$. Find the remainder when $P(x)$ is divided by $x - 2$ 2

4) The equation $x^3 - mx + 2 = 0$ has two equal roots.

(i) Write down expressions for the sum of the roots (1 at a time) and the product of the roots. 2

(ii) Hence find the value of m 2

Section 4 Miscellaneous Topics (Start a new page)

1) Sketch showing all important features $y = \frac{x+1}{x^2+3x+2}$ 4

2) Using the graph from part 1) or otherwise, solve $\frac{x+1}{x^2+3x+2} > 0$ 2

3) The acute angle between the lines $y = (m + 2)x$ and $y = mx$ is 45°.

(i) Show that $\left|\frac{2}{m^2+2m+1}\right| = 1$ 1

(ii) Hence find any values of m 3

4) The point (3, -4) divides the interval AB externally in the ratio 3:2. If the coordinates of A are (6,5), then find the coordinates of B. 2

Section 1

1) $-\frac{b}{a} = \frac{-8}{2}$

$= -4$ (B)

2) $(9+x), x = 36$

$x^2 + 9x - 36 = 0$

$(x+12)(x-3) = 0$

$x = -12$ or $x = 3$

↓ can't be negative

$\therefore x = 3$ only (C)

3) (D)

4) $\lim_{x \to \infty} \dfrac{\frac{3x^2}{x^2} + \frac{1}{x^2}}{\frac{7}{x^2} + \frac{4x^2}{x^2}}$

$= \lim_{x \to \infty} \dfrac{3 + \frac{1}{x^2}}{\frac{7}{x^2} + 4}$

$= \frac{3}{4}$ (B)

Section 2

1)(a) $y = \frac{x^2}{4a}$

$\frac{dy}{dx} = \frac{2x}{4a}$

$= \frac{x}{2a}$ at $x = 2ap$

$m_T = \frac{2ap}{2a}$ ✓ $\therefore m_N = -\frac{1}{p}$

$= p$ ✓

Equation of tangent is $y - ap^2 = p(x - 2ap)$

$y - ap^2 = px - 2ap^2$

$y = px - ap^2$ ✓

Equation of tangent is $y - ap^2 = -\frac{1}{p}(x - 2ap)$

$py - ap^3 = -x + 2ap$ ✓

(b) Axis of parabola is $x = 0$

$y = ap^2$ ✓ $\therefore$ T is $(0, -ap^2)$ ✓

For normal $py - ap^3 = 2ap$

$py = 2ap + ap^3$

$y = 2a + ap^2$ ✓

$\therefore$ N is $(0, 2a + ap^2)$

$\therefore (TN)^2 = (0-0)^2 + (2a + ap^2 + ap^2)^2$ ✓

$= (2a + 2ap^2)^2$

$= [2a(1+p^2)]^2$ ✓

$= 4a^2(1 + 2p^2 + p^4)$ $= 4a^2(1+p^2)^2$

2) $x = 6t \rightarrow t = \frac{x}{6}$

$y = 4t^2 \therefore y = \frac{4x^2}{36}$

$y = \frac{x^2}{9}$

$\therefore x^2 = 9y$ ✓ $4a = 9$

$a = \frac{9}{4}$

Focus is $(0, \frac{9}{4})$ ✓

3) Using $xx_1 = 2a(y+y_1)$ ✓ where $a = 1$

$4x = 2(y-6)$

$4x - 2y + 12 = 0$

$\therefore 2x - y + 6 = 0$ ✓

4)(a) Using $y - \frac{1}{2}(p+q)x + apq = 0$

Focal chord if passess thrugh $(0, a)$

$a + apq = 0$

$apq = -a$

$pq = -1$ ✓

(b) Midpoint PQ is $\left(\frac{2ap+2aq}{2}, \frac{ap^2+aq^2}{2}\right)$ ✓

$= \left(a(p+q), \frac{a(p^2+q^2)}{2}\right)$

$= \left(a(p+q), a\frac{[(p+q)^2 - 2pq]}{2}\right)$

but $p+q = 4$ and $pq = -1$

$\therefore$ midpoint $PQ = \left(4a, a\frac{(16+2)}{2}\right)$

$= (4a, 9a)$ ✓

Section 3

1) $x^3 + 0x^2 - 3x + 2 = 0$

(i) $-\frac{b}{a} = 0$

(ii) $\frac{c}{a} = \frac{-3}{1}$

$= -3$

(iii) $-\frac{d}{a} = -2$

(iv) $(\alpha+\beta+\gamma)^2 - 2(\alpha\beta+\alpha\gamma+\beta\gamma)$

$= \left(-\frac{b}{a}\right)^2 - 2\left(\frac{c}{a}\right)$

$= 0 - 2 \times -3$

$= 6$

2) (i) $0 = 1 - 6 + k + 14$

$k = -9$

(ii) $P(x) = x^3 - 6x^2 - 9x + 14$

$$x-1 \overline{)\, x^3-6x^2-9x+14}$$

$x^2-5x-14$

x^3-x^2

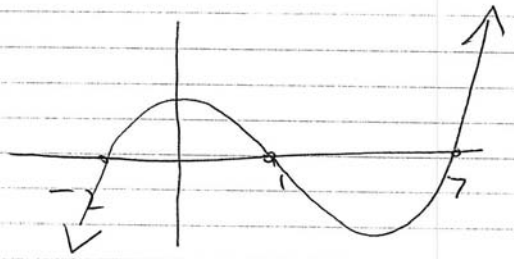
$-5x^2-9x$

$-5x^2+5x$

$-14x+14$

$P(x) = (x-1)(x-7)(x+2)$

(iii)



$x > 7$ or $-2 < x < 1$

3) $P(x) = (x^2-4)\cdot Q(x) + 2x + 3$

$P(2) = 2(2)+3$

$= 7$

∴ remainder is 7.

4)(i) $\alpha^2\beta = -2$ $\quad 2\alpha+\beta = 0$

$2\alpha = -\beta$

$-2\alpha = \beta$

(ii) $\alpha^2(-2\alpha) = -2$

$-2\alpha^3 = -2$

$\alpha^3 = 1$

$\alpha = 1$ $\quad$ sub $x = 1$

$1 - m + 2 = 0$

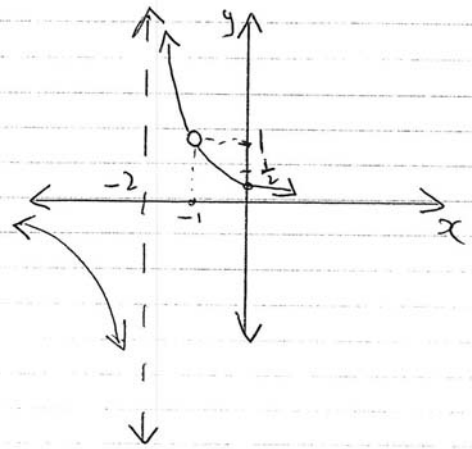
$m = 3$

Section 4

1) $y = \dfrac{x+1}{(x+2)(x+1)}$ $\quad x \neq -2$ or -1

$y = \dfrac{1}{(x+2)}$ $\quad \lim\limits_{x\to -1} \dfrac{1}{x+2}$

$= 1$



2) $x > -2$, $x \neq -1$

3)(i) $\tan 45° = \left|\dfrac{m+2-m}{1+m(m+2)}\right|$

$1 = \left|\dfrac{2}{1+m^2+2m}\right|$

(ii) $\pm 1 = \dfrac{2}{1+m^2+2m}$

$m^2+2m+1 = 2$ $\quad$ OR $\quad -m^2-2m-1 = 2$

$m^2 + 2m - 1 = 0$ OR $m^2 + 2m + 3 = 0$

$(m+3)(m+1) = 0$

$m = -3$ or $m = -1$

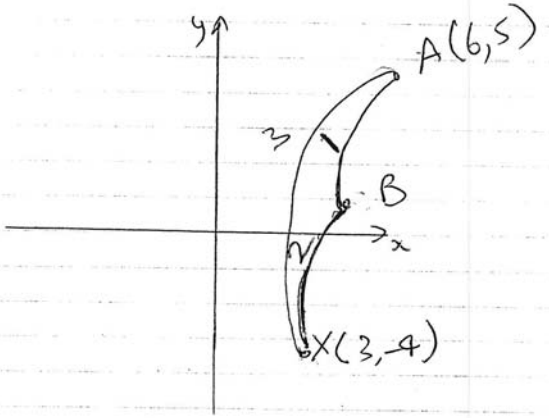
$m = \frac{-2 \pm \sqrt{4 - 4 \times 1 \times -1}}{2}$

$m = \frac{-2 \pm \sqrt{8}}{2}$

$m = \frac{-2 \pm 2\sqrt{2}}{2}$

$m = -1 \pm \sqrt{2}$

4)



A(6,5) X(3,-4)

1 2

$\left(\frac{(2)(6) + (1)(3)}{3}, \frac{(2)(5) + (1)(-4)}{3} \right)$

B is $(5, 2)$