James Ruse AHS Year 11 Extension 1 Mathematics Term4 2000

- **Time allowed 85 mins**
- Show all necessary working
- Start a new page for each question

Question 1 Start a new page. **Marks**

(a) Write down the exact value of:

(i) $\text{Sec}\dfrac{\pi}{4}$ **1**

(ii) $\sin^2 2+\cos^2 2$ **1**

(b) Evaluate exactly

(i) $\sin^{-1}(-\dfrac{1}{2})$ **1**

(ii) $\cos(\cos^{-1}\dfrac{3}{5}+\sin^{-1}\dfrac{4}{5})$ **2**

(c) Write down the primitive function of

(i) $\dfrac{x}{\sqrt{x}}$ **1**

(ii) $\dfrac{x^4+1}{x^2}$ **2**

(iii) $\dfrac{4}{9+x^2}$ **2**

Question 2 Start a new page.

(a) (i) Show that the point of intersection of $x^2=4y$ and $y^2=4x$ is (4,4) **1**

(ii) The area enclosed by the parabolas $x^2=4y$ and $y^2=4x$ rotates about the x axis. **3**

Calculate the volume of the solid so formed.

(b) The vertex A of the parallelogram ABCD is the point (1,5) and the side CD lies along the line x+y=10. One of the diagonals lies along the line 2x+y=12.

(i) Draw a diagram illustrating the above information. **1**

(ii) Find, using algebra the co-ordinates of B and D **5**

.

Question 3 Start a new page.

(a) For the general sine curve, with equation $y = a\sin(bx + c)$ a, b, c constants

(i) Write down the period and the amplitude. **2**

(ii) The graph of $y = a\sin(bx + c)$ is the same as $y = a\sin(bx)$ with a certain displacement. What is that displacement? **1**

(b) (i) On the same diagram, draw freehand sketches of the graphs **2**

$y = \sin 2x$ and $y = \sin 3x$ for $0 \le x \le \pi$

(ii) From the graph determine how many roots of the equation $\sin 2x = \sin 3x$, **1**

lie in the interval $0 \le x \le \pi$

(c) If $\sin(x + y) = 2\sin x$, prove that $\tan x = \dfrac{\sin y}{2 - \cos y}$ **4**

Question 4 Start a new page.

(i) Use Simpson's rule with 3 ordinates to find an approximate value of **3**

$\displaystyle\int_0^1 \frac{x}{\sqrt{x^2+1}}dx$ to 2 decimal places.

(ii) By differentiating $\sqrt{x^2+1}$, show that it is a primitive of $\dfrac{x}{\sqrt{x^2+1}}$ **2**

(iii) Hence show that $\displaystyle\int_0^1 \frac{x}{\sqrt{x^2+1}}dx = \sqrt{2} - 1$ **2**

(iv) Deduce that $\displaystyle\int_0^1 \frac{dx}{\sqrt{x^2+1}} > \sqrt{2} - 1$ **3**

Question 5 Start a new page**.**

P $(2p, p^2)$ and Q $(2q, q^2)$ lie on the parabola $x^2 = 4y$

(a) The chord through P and Q is given by $y - \frac{(p+q)x}{2} + pq = 0$ and passes through (0,2). **2**

Show that $pq = -2$

(b) Prove that the equation of the normal at P is $x + py = p^3 + 2p$ **4**

(c) The normals to the parabola at P and Q intersect at T. As the chord PQ moves about (0,2), show that T lies on the parabola $x^2 = 4(y-4)$ **4**

Question 6 Start a new page.

A picture 3m high is placed on a wall with the base of the picture 1 m above the level of an observer's eye. The observer stands x m from the wall.

(i) Show that the angle of vision α subtended by the picture to the eye of the observer is given by $\alpha = \tan^{-1}\frac{4}{x} - \tan^{-1}\frac{1}{x}$. **2**

(ii) Determine how far from the wall the observer should stand in order to maximize the angle of vision α. (Answer to be fully justified) **8**

END OF PAPER