QUESTION 1 (9 MARKS)

Marks

(a) (i) If $2\tan^2 x - \sec^2 x = 2$, find the exact values of $\tan x$ **2**

(ii) Hence, find the values of x in the domain $0 \leq x \leq 2\pi$ **2**

(b) The gradient of a function is given by $\frac{dy}{dx} = \sqrt{x} - \frac{1}{\sqrt{x}}$ and it passes through the point (4,3). Find the equation of the function. **3**

(c) Solve for x: $\frac{x}{x+3} > 1$ **2**

QUESTION 2 (9 Marks) – START A NEW PAGE

Marks

(a) Using the substitution $u^2 = x$, where $u > 0$, evaluate $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$ **3**

(b) If $\sin^{-1} t$, $\cos^{-1} t$, and $\sin^{-1}(1-t)$ are acute

(i) Show that $\sin\left(\sin^{-1} t - \cos^{-1} t\right) = 2t^2 - 1$. **2**

(ii) Hence, solve the equation: $\sin^{-1} t - \cos^{-1} t = \sin^{-1}(1-t)$ **2**

(c) Find $\lim_{x \to 0} \frac{\sin 3x}{\tan 4x}$ **2**

QUESTION 3 (9 Marks) – START A NEW PAGE

Marks

(a) Use Simpson's rule with five function values to estimate the volume of the solid formed by rotating the curve $y = \frac{2}{1+x^2}$ about the x-axis between 2 and –2. **3**

(b) The flow rate of water first into and then out of a horse trough is R, where

$$R = 3\pi t(8-t)\text{litres/second.}$$

(i) When does water cease to flow into the horse trough **and** how much water has flowed into the trough in this time? Leave your answer in exact form. **3**

(ii) The trough was initially empty. Find how long the trough takes to empty again, **and** the rate at which the trough was then losing water. **3**

QUESTION 4 (9 Marks) – START A NEW PAGE

Marks

(a) If (x_0, y_0) and (x_1, y_1) are two points on the line whose equation is $y = mx + k$, show that the distance between (x_0, y_0) and (x_1, y_1) in terms of x_0 and x_1 is $|x_0 - x_1|\sqrt{1+m^2}$ units. **4**

(b) The area bounded by the curve $y = \frac{x}{\sqrt{1+x}}$, the line $x = 1$ and the x axis is rotated about the x axis. Find the volume of the generated solid of revolution. **5**

QUESTION 5 (9 Marks) – START A NEW PAGE

Marks

(a) A function is defined by $f(x) = x - \frac{1}{x}$, $x > 0$.

(i) Show that $f(x)$ has an inverse and that this inverse is given by **3**

$$g(x) = \frac{x}{2} + \sqrt{1 + \frac{x^2}{4}}.$$

(ii) Sketch the curves $y = f(x)$ and $y = g(x)$ on the same set of axes.

State the domain and range of $g(x)$. **3**

(b) The shaded area enclosed by the curves $y = \cos(x)$ and $y = f(x) = k\left(x - \frac{\pi}{2}\right)^2$ and the y –axis is 4 square units. **3**

Find the exact value of k.

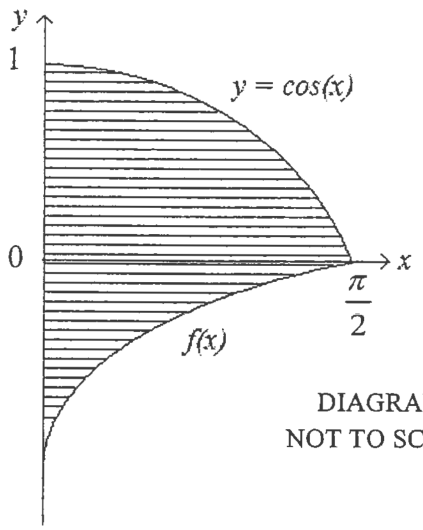


DIAGRAM
NOT TO SCALE

QUESTION 6 (9 Marks) – START A NEW PAGE

Marks

(a) (i) Differentiate $2x\tan^{-1}(2x) - \log_e\sqrt{1+4x^2}$ with respect to x. **2**

(ii) Hence, evaluate $\int_0^{\frac{1}{2}} tan^{-1}(2x)\,dx$ in exact terms. **2**

(b) By finding the intercepts with the x and y-axes and any stationary points and by determining their nature, sketch the curve **5**

$$y = e^x cosx \text{ for } 0 \le x \le \frac{\pi}{2}, \text{ neatly.}$$

QUESTION 7 (9 Marks) – START A NEW PAGE **Marks**

(a) Write the general solution in radians for $\cos 2x = -\sin 3x$ **3**

(b) The Agriculture Faculty at James Ruse have a budget of \$M to spend on constructing a rectangular lucerne paddock $ABCD$ as shown in the diagram below. The side AB runs along the edge of the school dam and costs \$N per metre to fence. The remaining three sides of the paddock cost \$R per metre to fence.

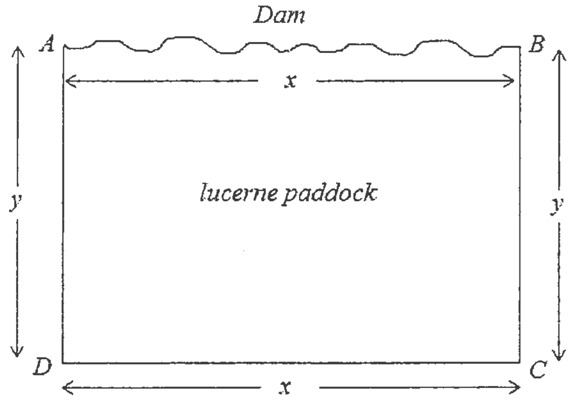


DIAGRAM
NOT TO SCALE

(i) Find in terms of m, n and r the width y of the paddock. **2**

(ii) Hence, find the area of the paddock in terms of x, m, n and r. **1**

(iii) Find the length (x) of the paddock in order to maximise the area of lucerne planted. **3**

END OF PAPER

Suggested Solutions	Marks	Marker's Comments
a) i) $2\tan^2 x - (1+\tan^2 x) = 2$		
$\tan^2 x = 3$		
$\tan x = \pm\sqrt{3}$	2	
ii) $\tan x = \sqrt{3} \Rightarrow x = \pi/3$ or $4\pi/3$		
$\tan x = -\sqrt{3} \Rightarrow x = 2\pi/3$ or $5\pi/3$		
$\therefore x = \pi/3, 2\pi/3, 4\pi/3$, or $5\pi/3$	2	
b) $\frac{dy}{dx} = x^{1/2} - x^{-1/2}$		Many people did not read the question properly
$y = \frac{2x^{3/2}}{3} - 2x^{1/2} + c$	1	
When $x=4$, $y=3$		
$\therefore 3 = \frac{16}{3} - 4 + c$		
$c = 5/3$	1	
Eqn is $y = \frac{2x^{3/2}}{3} - 2x^{1/2} + \frac{5}{3}$	1	
) Multiply by $(x+3)^2$ (>0) $(x \neq -3)$		
$x(x+3) > (x+3)^2$	1	
$x^2+3x > x^2+6x+9$		
$3x < -9$		
$x < -3$	1	

MATHEMATICS Extension 1 : Question 2		
Suggested Solutions	**Marks**	**Marker's Comments**
(a) $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$ Let $u^2 = x$, $u>0$ $\frac{dx}{du} = 2u$ $dx = 2u\,du$ when $x=1$, $u=1$ $x=4$, $u=2$ $u>0$	①	Change Limit values
$\int_1^2 \frac{e^u}{u}.2u\,du$ $2\int_1^2 e^u\,du$	①	Substitution
$2\left[e^u\right]_1^2 = 2(e^2-e)$	①	Integration and answer.
(b) (i) $\sin^{-1}t$, $\cos^{-1}t$ $\sin^{-1}(1-t)$ are acute. $-\frac{\pi}{2} < \sin^{-1}t < \pi/_2$ ∴ $-1<t<1$ $0 < \cos^{-1}t < \frac{\pi}{2}$ ∴ $0<t<1$ $-\frac{\pi}{2} < \sin^{-1}(1-t) < \pi/_2$ ∴ $0<t<2$ $\}$ $0<t<1$		* Note. acute angles can be negative
Let $\alpha = \sin^{-1}t$ ∴ $\sin\alpha = t$ $-1<t<1$ $\beta = \cos^{-1}t$ ∴ $\cos\beta = t$ $0<t<1$ ∴ $0<t<1$		
$\cos^2\alpha + \sin^2\alpha = 1$ $\cos\alpha = +\sqrt{1-t^2}$ $-\pi/_2 < \alpha < \pi/_2$ ∴ $\cos\alpha > 0$ $\cos^2\beta + \sin^2\beta = 1$ $\sin\beta = +\sqrt{1-t^2}$ $0<\beta<\pi/_2$ ∴ $\sin\beta > 0$		
$\sin(\sin^{-1}t - \cos^{-1}t)$ $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$	$\frac{1}{2}$	Expansion
$= t\times t - \sqrt{1-t^2}\times\sqrt{1-t^2}$	①	substitution
$= t^2 - 1 + t^2$ $= 2t^2 - 1$ $0<t<1$.	$\frac{1}{2}$	answer.
		$-\frac{1}{2}$ if $0<t<1$ not mentioned and part (ii) not completed.

MATHEMATICS Extension 1 : Question 2

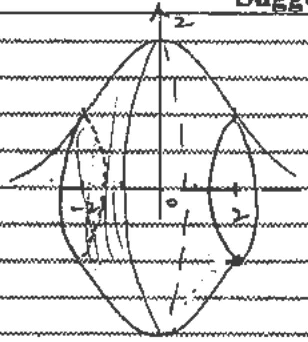
Suggested Solutions	Marks	Marker's Comments
(b) (ii) $\sin^{-1} t - \cos^{-1} t = \sin^{-1}(1-t)$		
consider		
$\sin(\sin^{-1} t - \cos^{-1} t) = \sin(\sin^{-1}(1-t))$		
$2t^2 - 1 = n\pi + (-1)^n (1-t) \quad n \in Z$		
But $0 < t < 1$ and $\sin^{-1}(1-t)$ is acute.		
$\therefore 2t^2 - 1 = 1 - t$		
$2t^2 + t - 2 = 0$	$\frac{1}{2}$	quadratic equation
$t = \frac{-1 \pm \sqrt{1-4(2)(-2)}}{4}$	$\frac{1}{2}$	solution
$t = \frac{\sqrt{17}-1}{4}$ as $0 < t < 1$	$\frac{1}{2}$	one solution
only	$\frac{1}{2}$	for restriction $0 < t < 1$
(c) $\lim_{x \to 0} \frac{\sin 3x}{\tan 4x} = \frac{3}{4} \lim_{x \to 0} \frac{\sin 3x}{3x} \times \frac{4x}{\tan 4x}$	$\frac{1}{2}$	must show "x's"
$= \frac{3}{4} \times 1 \times 1$	$\frac{1}{2}$	Must show "1"'s
$= \frac{3}{4}$	1	Answer

Y12 ASSESSMENT TEST 1, TERM 4, 2011

MATHEMATICS Extension 1 : Question 3

Suggested Solutions	Marks	Marker's Comments
		* students are NOT reading the Q.

(a)



$$\text{VOLUME} = \pi\int_{-2}^{2} y^2\,dx$$

$$= \pi\int_{-2}^{2}\left(\frac{2}{1+x^2}\right)^2 dx \quad \left(\tfrac{1}{2}\right)$$

$$= 2\pi\int_{0}^{2}\frac{4}{(1+x^2)^2}\,dx \quad \text{(even function)}$$

$h = \frac{2-(-2)}{4} = \frac{2-0}{2} = 1$

	x	-2	-1	0	1	2
$y^2 =$	$\frac{4}{(1+x^2)^2}$	$\frac{4}{25}$	1	4	1	$\frac{4}{25}$
		Y_1	Y_2	Y_3	Y_4	Y_5

$\frac{1}{2} + \left(\frac{1}{2}\right)$

$$\therefore V = \pi\int_{-2}^{2} y^2\,dx \doteq \pi\times\frac{h}{3}\left[Y_1+Y_5+4(Y_2+Y_4)+2Y_3\right] \quad 1 \quad \text{or equiv. formula}$$

$$= \pi\times\frac{1}{3}\left[\frac{4}{25}+\frac{4}{25}+4(1+1)+2\times4\right]$$

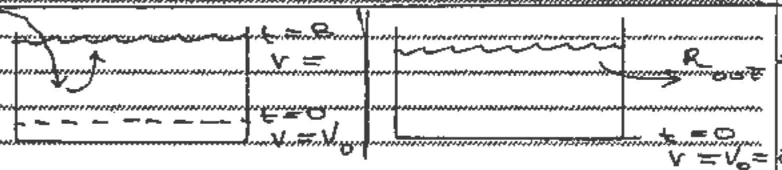
$$= \frac{\pi}{3}\left[\frac{8}{25}+8+8\right] \qquad \boxed{3}$$

$$\therefore \underline{\text{VOLUME}} = \frac{408\pi}{75} \Big/ \frac{136\pi}{25} \Big/ 5{\cdot}44\pi \text{ units}^3 \quad 1$$

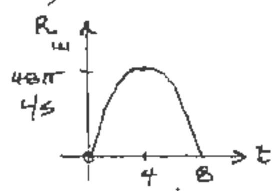
$-\frac{1}{2}$ if not "volume = ... u^3"
$-\frac{1}{2}$ for 17.09...

Rate in $= 3\pi t(8-t)$

b) (i)



$R_{out} = -3\pi t(8-t)$



cease flow when $R_{in} = \frac{dV}{dt} = 3\pi t(8-t) = 0 \quad \frac{1}{2}$

$\therefore t = 0$ or $t = 8$

but $t \neq 0$ $\therefore$ cease flow after 8 seconds $\quad 1$

$$\text{VOLUME flowed in} = \int_0^8 R\,dt = \int_0^8 3\pi(8t - t^2)\,dt$$

$$= 3\pi\left[4t^2 - \tfrac{1}{3}t^3\right]_0^8 \quad \tfrac{1}{2}$$

$$= 3\pi\times\left(256 - \tfrac{512}{3} - 0\right] \qquad \boxed{3}$$

$$\underline{\therefore \text{VOLUME in} = 256\pi\text{ L}} \quad 1\tfrac{1}{2}$$

OR $V = \int 3\pi(8t - t^2)\,dt = 3\pi\left(4t^2 - \tfrac{1}{3}t^3\right) + C$
$= 24\pi t^2 - \pi t^3 + C$

$t = 0 \quad V = V_0\ (\geq 0) \quad \therefore C = V_0$

$\therefore V(t) = 24\pi t^2 - \pi t^3 + V_0$

$\therefore$ volume flowed in $V(8) - V(0) = 256\pi$ L.

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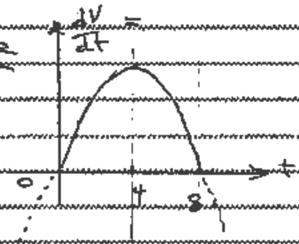
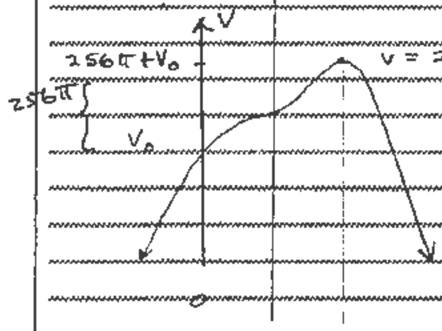
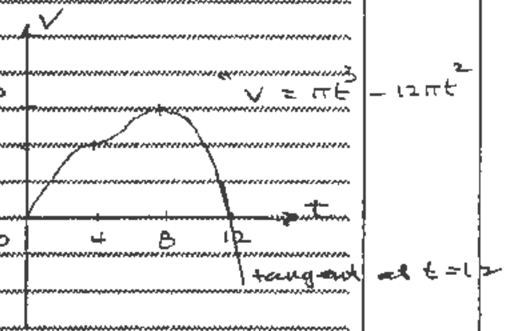
Suggested Solutions	Marks	Marker's Comments
Q 3(b) (ii) $R_{OUT} = -3\pi t(8-t)$		
<u>Data:</u> $t=0 \quad V = V_0 = 0$		
$R = \frac{dV}{dt} = -3\pi t(8-t) = 3\pi t^2 - 24\pi t$		
$\therefore V = \int 3\pi t^2 - 24\pi t \, dt =$		
$V = \pi t^3 - 12\pi t^2 + C$		
ie $V = \pi t^3 - 12\pi t^2 + V_0$	$C = V_0 \equiv 0$ (data)	
$\therefore V = \pi t^3 - 12\pi t^2$ ✓	1	
Empty when $V=0$		
$\therefore \pi t^2(t-12) = 0$		
$\therefore t=0$ or $t=12$ but $t>0$		
$\therefore$ <u>after 12sec</u> no volume in trough	1	$\boxed{3}$
$\therefore$ Rate $= -3\pi \times 12(8-12) = 144\pi$		
$\therefore$ • flow rate <u>out</u> $= 144\pi$ L/s		$-\frac{1}{2}$ if wording incorrect
• trough <u>losing</u> at 144π L/s	1	
• "Rate is -144π L/s"		

<u>Note</u>







b(i) b(ii)

Relative sketch

MATHEMATICS Extension 1 : Question 4		
Suggested Solutions	**Marks Awarded**	**Marker's Comments**

Sub (x_0, y_0) into $y = mx + k$.

$y_0 = mx_0 + k$ —— eq 1
Sub (x_1, y_1) into equation
$y_1 = mx_1 + k$ —— eq 2. ✓

① − ②
$y_0 - y_1 = m(x_0 - x_1)$ —— eq 3 ✓

ow $d = \sqrt{(x_0-x_1)^2 + (y_0-y_1)^2}$ —— eq 4

Sub ③ into ④

$$d = \sqrt{(x_0-x_1)^2 + (y_0-y_1)^2}$$
$$= \sqrt{(x_0-x_1)^2 + m^2(x_0-x_1)^2} \checkmark$$
$$= \sqrt{(1+m)^2\,(x_0-x_1)^2}$$

as $\sqrt{(x_0-x_1)^2} = |x_0-x_1|$ reason compulsory ($\frac{1}{2}$ mark)

$$= \cancel{\sqrt{(x_0-x_1)^2}\cdot\sqrt{1+m^2}}$$
$$= |x_0-x_1|\sqrt{1+m^2}$$

ternatively

) Some used $m = \dfrac{y_0-y_1}{x_0-x_1}$

$\therefore y_0 - y_1 = m(x_0-x_1)$

) Some divided the distance formula.

$$d = \sqrt{(x_0-x_1)^2 + (y_0-y_1)^2}$$

by $\sqrt{(x_0-x_1)^2}$ and times by the same amount

$$d = \sqrt{\frac{(x_0-x_1)^2}{(x_0-x_1)^2} + \frac{y_0-y_1}{(x_0-x_1)^2}} \times \sqrt{(x_0-x_1)^2}$$

MATHEMATICS Extension 1 : Question 4

Suggested Solutions	Marks Awarded	Marker's Comments

Suggested Solutions

b) . $V = \pi \int_0^1 \frac{x^2}{1+x}\,dx$ ✓

$= \pi \int_0^1 \left((x-1) + \frac{1}{x+1} \right) dx$ ✓

$= \pi \left[\frac{x^2}{2} - x + \ln(x+1) \right]_0^1$ ✓

$= \pi \left[\left(\frac{1}{2} - 1 + \ln 2\right) - (0 - 0 + \ln 1) \right]$ ✓

$= \pi \left[\ln 2 - \frac{1}{2} \right]$ unit^3. ✓

Alternatively
Method 2 .

Let $u = x+1$

$x = u - 1$

$x^2 = u^2 - 2u + 1$

when $x = 0 \Rightarrow u = 1$

$x = 1 \Rightarrow u = 2$.

$\therefore V = \pi \int_1^2 \frac{u^2 - 2u + 1}{u}\,du$.

Method 3 .

$V = \pi \int \frac{x^2}{1+x} = \int_0^1 \frac{x^2+2x+1}{x+1} - \frac{2x+1}{x+1}\,dx$

$= \int_0^1 x + 1\,dx - \int \frac{2x+2}{x+1}dx + \int \frac{1}{x+1}\,dx$

OR .

$V = \pi \int \frac{x^2 dx}{1+x} = \int \frac{x^2 - 1 + 1\,dx}{1+x} = \int \frac{x^2 - 1\,dx}{1+x} + \int \frac{1}{1+x}\,dx$

Some used $\tan^{-1}$

Note
instead of getting $\int \frac{x^2}{1+x}\,dx$ you have taken $\int \frac{x^2}{1+x^2}\,dx$

Here the maximum marks you got was 3 marks.

Marker's Comments

- show multiplied by π and squared y
- limits $\int_0^1$

* if no dx or limit lost $(-\frac{1}{2})$

Long division

$$\begin{array}{r} x - 1 \\ x+1 \overline{\smash{)}\, x^2 + 0x + 0} \\ \underline{x^2 + x} \\ 0 - x \\ \underline{-x - 1} \\ 1 \end{array}$$

$\therefore \frac{x^2}{x+1} = (x-1) + \frac{1}{x+1}$

Some gave:

-0.19314718π unit^3

or

$0.60678 9763$ unit^3

- accepted both.

Note.

* If no π and not squared. and simplified their ~~answers~~ working to get answer — got maximum of 2 marks

* Marks taken off for simplifying their working making question easier.

15

(a) (i) $f(x) = x - \frac{1}{x}$ where $x > 0$

$D_f = \{x : x > 0\}$

$R_f = \{y : y \in \mathbb{R}\}$

let $y = f(x)$

$\therefore y = x - \frac{1}{x}$

$R_{f^{-1}} = \{y : y > 0\}$

inverse function: $x = y - \frac{1}{y}$ ½ $\therefore y > 0$

Range is the ½ domain of the original functi

$yx = y^2 - 1$

$\therefore 0 = y^2 - yx - 1$ ½ $D_{f^{-1}} = \{x : x \in \mathbb{R}\}$

$\therefore y = \frac{x \pm \sqrt{x^2 - 4(1)(-1)}}{2}$ ½

$y = \frac{x}{2} \pm \frac{\sqrt{x^2+4}}{2}$ ½

(3)

$\therefore y = \frac{x}{2} + \frac{\sqrt{x^2+4}}{2}$ as $y > 0$ and $\sqrt{x^2+4} > x$

$y = \frac{x}{2} + \frac{1}{2} \cdot 2\sqrt{\frac{x^2}{4} + 1}$ ½

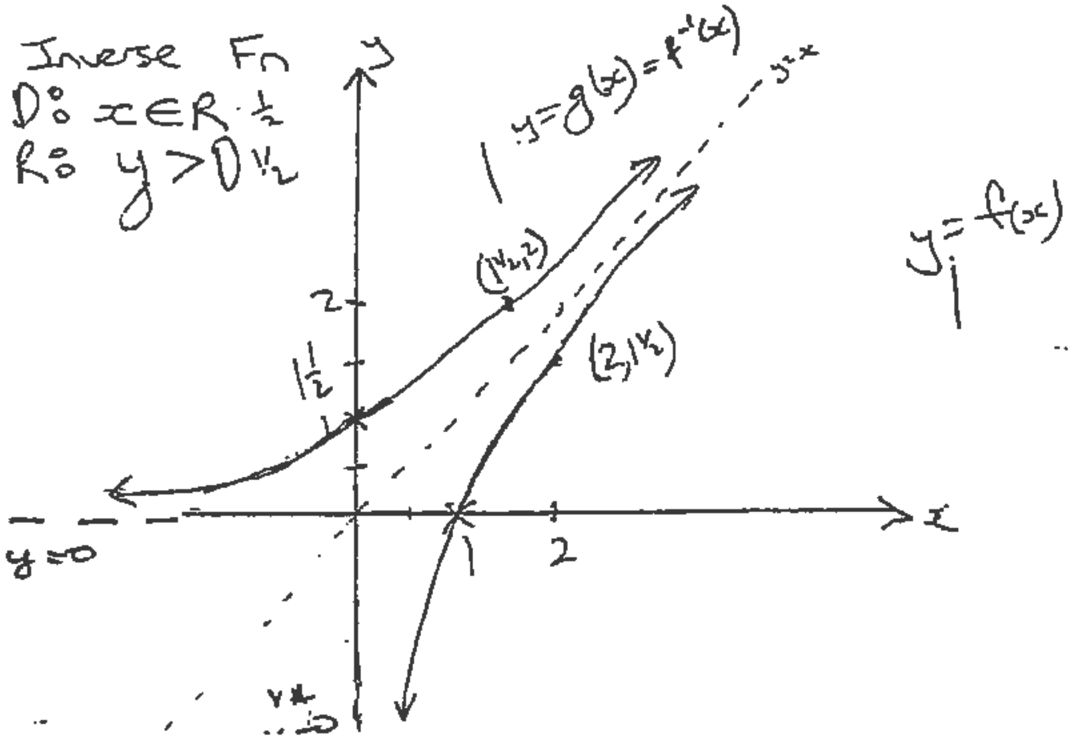
$y = \frac{x}{2} + \sqrt{\frac{x^2}{4} + 1}$

$\therefore g(x) = \frac{x}{2} + \sqrt{\frac{x^2}{4} + 1}$

Inverse Fn

D: $x \in R$ ½

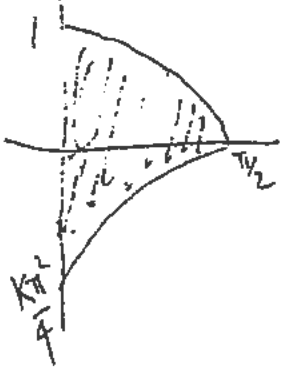
R: $y > 0$ ½



(3)

$$\text{Area} = \int_0^{\pi/2} \cos x\,dx - \int_0^{\pi/2} k(x-\pi/2)\,dx$$

(b)



$$\text{Area} = \int_0^{\pi/2} \cos x\,dx \quad - \quad k\int_0^{\pi/2} \left(x-\tfrac{\pi}{2}\right)^2 dx \qquad \tfrac{1}{2}$$

$$\therefore 4 = \left[\sin x\right]_0^{\pi/2} - \frac{k}{3}\left[\left(x-\tfrac{\pi}{2}\right)^3\right]_0^{\pi/2} \qquad \tfrac{1}{2}$$

$$4 = \sin\tfrac{\pi}{2} - \sin 0 - \frac{k}{3}\left[\left(\tfrac{\pi}{2}-\tfrac{\pi}{2}\right)^3 - \left(0-\tfrac{\pi}{2}\right.\right.$$

$$4 = 1 - \frac{k}{3}\left[\frac{\pi^3}{8}\right] \qquad \tfrac{1}{2}$$

$$9 = -k\left(\frac{\pi^3}{8}\right)$$

3

$$\frac{-72}{\pi^3} = k \qquad \tfrac{1}{2}$$

$$\therefore\ k = \frac{-72}{\pi^3}$$

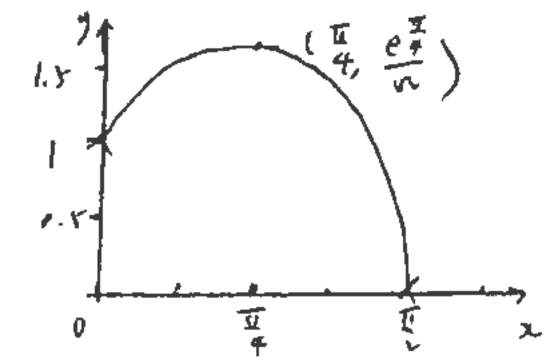
* ½ off for each mistake.

Alternative method

$$\text{Area} = \int_0^{\pi/2} \cos x\,dx + \left| k\int_0^{\pi/2} \left(x-\tfrac{\pi}{2}\right)^2 dx \right|$$

very similar to above but when they remove the absolute value signs they need to take the negative case which nearly all students did not do!!

2011 Yr11 Yr12 Ext 1 **MATHEMATICS: Question** 6

Suggested Solutions	Marks	Marker's Comments
a i) $y' = \underbrace{\dfrac{2x \cdot 2}{1+4x^2} + 2\tan^{-1}(2x)}_{1\,m} - \underbrace{\dfrac{8x}{1+4x^2} \times \dfrac{1}{2}}_{1\,m}$	1+1	no half mark
$y' = 2\tan^{-1}(2x)$		if final answer is wrong $-1m$
ii) $= \frac{1}{2}\left[2x\tan^{-1}(2x) - \ln\sqrt{1+4x^2}\right]_0^{\frac{1}{2}}$	1	forget "$\frac{1}{2}$" in front $-1m$
$= \frac{1}{2}\tan^{-1}1 - \frac{1}{4}\ln 2 + \cancel{\frac{\ln 1}{4}}^{0}$		
$= \frac{\pi}{8} - \frac{1}{4}\ln 2$ #	1	
b) $y = e^x\cos x \quad y' = e^x(\cos x - \sin x)$	$\frac{1}{2}$	
SP $y' = 0 \quad \tan x = 1 \quad 0 \le x \le \frac{\pi}{2}$ $x = \frac{\pi}{4} \quad y = e^{\frac{\pi}{4}} \cdot \frac{1}{\sqrt{2}}$	$\frac{1}{2}+\frac{1}{2}$	Some forget y value.
$y'' = e^x(-\sin x - \cos x) + e^x(\cos x - \sin x)$ $= -2e^x\sin x$	$\frac{1}{2}$	
$y''(\frac{\pi}{4}) = -\sqrt{2}e^{\frac{\pi}{4}} < 0 \quad \therefore$ concave down		
rel max at $\left(\frac{\pi}{4}, \frac{e^{\frac{\pi}{4}}}{\sqrt{2}}\right)$	$\frac{1}{2}$	
Intercepts $(0, 1)$, $(\frac{\pi}{2}, 0)$	$\frac{1}{2}+\frac{1}{2}$	Forget label axes/origin $-\frac{1}{2}m$
	$1\frac{1}{2}$	$\frac{e^{\frac{\pi}{4}}}{\sqrt{2}} \doteqdot 1.551$ 1 m for shape (with labelled TP, Intercepts) $\frac{1}{2}$ m for Correct scale (only with correct shape)

MATHEMATICS Extension 1 : Question 7

Suggested Solutions | **Marks** | **Marker's Comments**

Q7 (a) $\cos 2x = -\sin 3x$

$= \sin(-3x)$ ODD function

$= \cos\left[\frac{\pi}{2} - (-3x)\right]$

$\cos 2x = \cos\left(\frac{\pi}{2} + 3x\right)$

$\therefore \quad 2x = 2m\pi \pm \left(\frac{\pi}{2} + 3x\right)$ — 1

$2x = 2m\pi + \frac{\pi}{2} + 3x$ or $2x = 2m\pi - \frac{\pi}{2} - 3x$ — $\boxed{3}$

$\therefore \quad x = \begin{cases} -(4m+1)\frac{\pi}{2} \\ (4m-1)\frac{\pi}{10} \end{cases} \quad m \in \mathbb{Z}$ — 1 each

• IF $\sin\left(\frac{\pi}{2} - 2x\right) = \sin(-3x)$ from $\cos 2x = -\sin 3x$

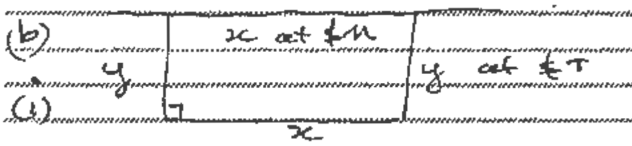
$x = \dfrac{(2m-1)\pi}{2[3(-1)^m - 2]}$ or $\dfrac{[2m + (-1)^m]\pi}{2[2(-1)^m - 3]}$

• IF $-\cos 2x = \sin 3x$

$= \cos\left(\frac{\pi}{2} - 3x\right)$

$\therefore \quad x = \begin{cases} (4m-1)\frac{\pi}{2} \\ (3-4m)\frac{\pi}{10} \end{cases} \quad m \in \mathbb{Z}$

(b) (i)



$m = ux + vx + 2ry$ — 1

$m = (u+v)x + 2ry$

$\therefore \quad y = \dfrac{m - (u+v)x}{2r}$ — 1 — $\boxed{2}$

(ii) $A = xy$ —— (1)

$y = \dfrac{m - (u+v)x}{2r}$ —— (2)

$\therefore \quad A(x) = x\left[\dfrac{m - (u+v)x}{2r}\right]$ — 1 — $\boxed{1}$

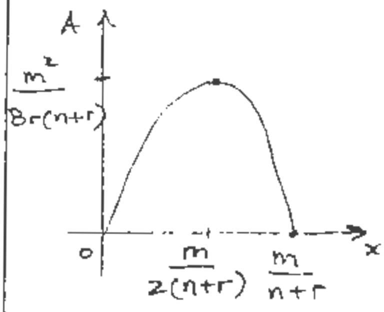
$A(x) = \dfrac{m}{2r}x - \dfrac{(u+v)}{2r}x^2$

(iii) PTO.

Suggested Solutions	Marks	Marker's Comments

Q 7 (b) (iii) $A(x) = \frac{m}{2r}x - \frac{(n+r)}{2r}x^2$

$0 \le x \le \frac{m}{n+r}$



METHOD 1 From Quadratic Polynomials

As $a = -\frac{(n+r)}{2r} < 0$

$\therefore$ concave downwards

Hence a maximum value will exist

at $x = -\frac{b}{2a}$ or $\frac{\alpha+\beta}{2}$

i.e. $x = \frac{m}{2(n+r)}$

$\therefore y = \frac{m}{2r} - \frac{n+r}{2r} \times \frac{m}{2(n+r)} = \frac{m}{4r}$

$A = \frac{m}{2(n+r)} \times \frac{m}{4r} = \frac{m^2}{8r(n+r)}$

$\therefore$ abs max Area when $x = \frac{m}{2(n+r)}$

$a = -\frac{(n+r)}{2r}$; $b = \frac{m}{2r}$

$\alpha = 0$; $\beta = \frac{m}{n+r}$

METHOD 2 $A = \frac{m}{2r}x - \frac{(n+r)}{2r}x^2$

$\frac{dA}{dx} = \frac{m}{2r} - \frac{(n+r)x}{r}$ 1

$\frac{d^2A}{dx^2} = -\frac{(n+r)}{r}$

For possible max/min values of A to occur $\frac{dA}{dx} = 0$

$\therefore \frac{m}{2r} - \frac{(n+r)x}{r} = 0$

$\therefore x = \frac{m}{2(n+r)}$ 1

$Y = \frac{m}{4r}$, $A = \frac{m^2}{8r(n+r)}$ $\boxed{3}$

TEST nature

As $\frac{d^2A}{dx^2} = -\frac{(n+r)}{r} < 0$; r and n > 0

$\therefore$ concave downwards and since a SP at $x = \frac{m}{2(n+r)}$

$\therefore$ a Rel. min TP at $x = \frac{m}{2(n+r)}$ 1

Since continuous and no other SPs for $x > 0$

$\therefore$ the abs. max. Area occurs when $x = \frac{m}{2(n+r)}$

(OR)

x	$\frac{m}{4(n+r)}$	$\frac{m}{2(n+r)}$	$\frac{m}{(n+r)}$
$\frac{dA}{dx}$	$\frac{m}{4r}$	0	$-\frac{m}{2r}$
	+		−
	/	—	\

$\therefore$ a Rel. min TP at $x = \frac{m}{2(n+r)}$

etc. see above