Name:	
Class:	

YEAR 12

ASSESSMENT TEST 1
TERM 4, 2013

MATHEMATICS EXTENSION 1

Time Allowed – 90 Minutes
(Plus 5 minutes Reading Time)

- *All* questions may be attempted
- Department of Education approved calculators and templates are permitted
- In every Question, show all relevant mathematical reasoning and/or calculations.
- Marks may not be awarded for careless or badly arranged work
- No grid paper is to be used unless provided with the examination paper

The answers to all questions are to be returned in separate bundles clearly labeled Question 1, Question 2, etc.
Each question must show (in the top right hand corner) your Candidate Number.

QUESTION 1 (10 Marks) **COMMENCE A NEW PAGE** **MARKS**

(a) Find: $\int x^2 e^{x^3} dx$ **1**

(b) Find: $\lim_{x \to 0} \frac{1 - \cos 2x}{x^2}$ **3**

(c) Find the exact value of : $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \tan^{-1}(\tan\frac{7\pi}{8})$ **2**

(d) (i) Prove $\cos(A + B) - \cos(A - B) = -2 \sin A \sin B$ **2**

(ii) Hence find the exact value of $\cos 75^0 - \cos 15^0$ **2**

QUESTION 2 (10 Marks) **COMMENCE A NEW PAGE**

(a) Differentiate $\cos^{-1}(2x)$. **1**

(b) Use Simpson's Rule with 3 function values to estimate **2**

$$\int_0^{\frac{1}{2}} \frac{1}{2}\cos^{-1}(1 - 2x)\, dx$$

(c) (i) Show that $\frac{5x^2 + 4}{x^2 + 4} = 5 - \frac{16}{x^2 + 4}$ **1**

(ii) Hence, evaluate to 2 decimal places, **2**

$$\int_{-\frac{1}{2}}^{0} \frac{5x^2 + 4}{x^2 + 4}\, dx$$

(d) The vertices of ΔABC are $A(4, -3), B(-9, 7)$ and $C(1, k)$. If the area of ΔABC is 15 square units, find the value(s) of k. **4**

QUESTION 3 (10 Marks) **COMMENCE A NEW PAGE** **MARKS**

(a) Given $f(x) = \frac{x^3-3x^2}{x^2-1}$, $(x^2-1)(x-3) = x^3 - 3x^2 a - x + 3$

(i) Determine all intercepts of $f(x)$. **1**

(ii) Write down the equations for the horizontal and vertical asymptotes, if any. **1**

(iii) Show that the oblique asymptote is given by $y = x - 3$. **1**

(iv) Hence make a neat sketch of $f(x)$, showing all intercepts and turning points. **2**

[You may assume, without proof, the following:

(i) $f'(x) = \frac{x^4-3x^2+6x}{(x^2-1)^2}$

(ii) $x^3 - 3x + 6 = 0$, when $x \approx -2.3$]

(b) Find the area enclosed by the curve of $y = \frac{9}{9+x^2}$, the x-axis and the lines $x = 1$ and $x = -1$. **2**

(c) Using $t = \tan x$, find the general solution in radians to $\sin 2x = \tan x$. **3**

QUESTION 4 (10 Marks) **COMMENCE A NEW PAGE**

(a) Using the substitution $u = 1 + 2x$, find **2**

$$\int \frac{6\,dx}{\sqrt{(1+2x)^3}}$$

(b) Consider $y = \sqrt{3}\sin\theta - 3\cos\theta$

(i) Express y in the form $R\sin(\theta - \alpha)$ where $R > 0$, and $0 \leq \alpha \leq 2\pi$. **1**

(ii) By first finding the maximum and minimum turning points, sketch a neat graph of $y = \sqrt{3}\sin\theta - 3\cos\theta$, for $0 \leq \theta \leq 2\pi$. **3**

(c) Prove the identity: $2\cos^2\theta - 2\cos^2 2\theta = \cos 2\theta - \cos 4\theta$ **2**

(d) Assuming, $\frac{d}{dx}\left[\tan^{-1}\frac{x}{2} + \frac{2x}{x^2+4}\right] = \frac{16}{(x^2+4)^2}$ evaluate $\int_{-2}^{2} \frac{dx}{(x^2+4)^2}$ **2**

QUESTION 5 (10 Marks) **COMMENCE A NEW PAGE** **MARKS**

(a) Prove that $\frac{d}{dx}[\tan^{-1}\frac{x}{\sqrt{1-x^2}}] = \frac{d}{dx}[\sin^{-1}x]$ 3

(b) Evaluate $\int_0^{\sqrt{3}} \frac{dx}{\sqrt{3-x^2}}$ 2

(c) A tank which is initially empty is being filled with water at a rate given by

$$\frac{dV}{dt} = \pi\left(100 - \frac{t^2}{100}\right) L/s$$

Where V is the volume of water in Litres after t seconds.

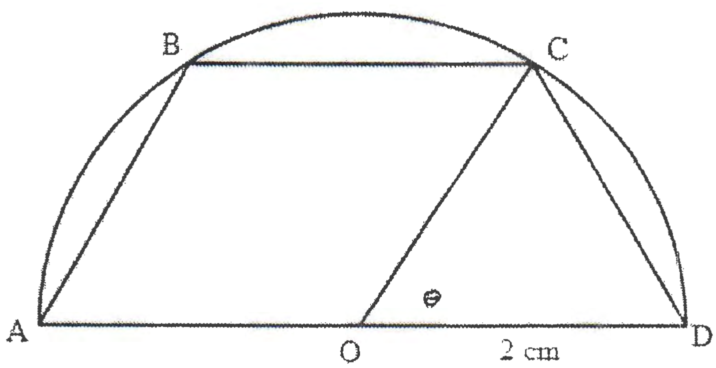
(i) At what rate is the volume of water changing when $t = 10$ seconds? 1

(ii) How long will it take to fill the tank? 1

(iii) Find the maximum volume of water in the tank. 3

QUESTION 6 (10 Marks) **COMMENCE A NEW PAGE**

(a) A trapezium ABCD, is inscribed in a semicircle, centre O and radius 2 cm, so that one side is along the diameter, as shown below. θ is the angle subtended at the centre of the semicircle by one of the non-parallel sides of the trapezium.



(i) Redraw the diagram and derive an expression for the area of the trapezium in terms of θ only. 2

(ii) Find the maximum possible area for the trapezium. Leave answer in exact form. 3

(b) If $y = 2\cos^{-1}\frac{x}{3}$,

(i) State the domain and range of y. **1**

(ii) Hence sketch its graph. **1**

(iii) Express $\cos^2 x$ in terms of $\cos 2x$ **1**

(iv) Hence find the exact volume of the solid formed when the area bounded by the curve $y = 2\cos^{-1}\frac{x}{3}$, the y −axis and the line $y = 2\pi$, is rotated about the y −axis. **2**

QUESTION 7 (10 Marks) COMMENCE A NEW PAGE

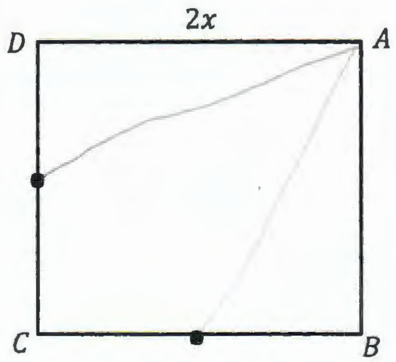
(a) Find $\int(\sin^{-1}x + \cos^{-1}x)\,dx$ **1**

(b) Solve for x: $\cos^{-1}x - \sin^{-1}x = \sin^{-1}(1-x)$ **2**

(c) Find the exact area bounded by the curves $y = 2\sin\frac{\pi x}{4}$ and $x = 2\sin\frac{\pi y}{4}$ for $0 \le y \le 2$. **3**

(d) (i) Prove $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \frac{1}{4}\pi$ **2**

(ii) $ABCD$ is a square field in which a goat is tethered to the corner A by means of a rope. The rope is long enough for the goat to be able to just reach the midpoints of BC and CD. Let $DA = 2x$.



Find the proportion of the area of the field that the goat cannot reach.

Express your answer in the form $a + b\tan^{-1}\left(\frac{1}{3}\right)$, where a and b are rational numbers. **2**

☺ **END OF EXAMINATION** ☺

MATHEMATICS Extension 1 : Question ...1... Page 1 of 2

Suggested Solutions	Marks	Marker's Comments
1a) $\int x^2 e^{x^3}\,dx$ $\qquad \frac{d}{dx}(x^3) = 3x^2$		No half marks
$= \frac{1}{3} e^{x^3} + C$	1	
b) $\lim\limits_{x \to 0} \frac{1 - \cos 2x}{x^2}$ $\qquad \cos 2x = \cos^2 x - \sin^2 x = 1 - \sin^2 x - \sin^2 x = 1 - 2\sin^2 x$		2 marks
$= \lim\limits_{x \to 0} \frac{1 - (1 - 2\sin^2 x)}{x^2}$		
$= \lim\limits_{x \to 0} \frac{2\sin^2 x}{x^2}$		
$= 2\lim\limits_{x \to 0} \frac{\sin x}{x} \cdot \lim\limits_{x \to 0} \frac{\sin x}{x}$		
$= 2 \times 1 \times 1$ $= 2$	3	1 mark for the correct substn.
Alternatively $\lim\limits_{x \to 0} \frac{1 - \cos 2x}{x^2}$		2 marks
$= 2\lim\limits_{x \to 0} \frac{\sin^2 x}{x^2}$		
$= 2 \times 1 \times 1$ or 2×1^2 $= 2$		1 mark for the correct substn.
c) $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \tan^{-1}\left(\tan \frac{7\pi}{8}\right)$		1 mark
$= -\frac{\pi}{3} + \tan^{-1}\left(\tan\left(-\frac{\pi}{8}\right)\right)$ $\qquad -\frac{\pi}{2} < x < \frac{\pi}{2}$		
$= -\frac{\pi}{3} + \tan^{-1}\left(-\tan \frac{\pi}{8}\right)$		½ mark for $-\frac{\pi}{3}$
$= -\frac{\pi}{3} - \frac{\pi}{8}$		1 mark for $-\frac{\pi}{8}$
$= -\frac{11\pi}{24}$	2	½ mark for correct answer
		NB. $\left.\begin{array}{l} -\frac{\pi}{3} + \frac{7\pi}{8} \\ = \frac{13\pi}{24} \end{array}\right\}$ 1mk

Suggested Solutions	Marks	Marker's Comments
1 d) (i) Prove $\cos(A+B) - \cos(A-B) = -2\sin A \sin B$		
Proof: $LHS = \cos A \cos B - \sin A \sin B - (\cos A \cos B + \sin A \sin B)$		1 mark to correctly expand $\cos(A+B)$ & 1 mark to correctly expand $\cos(A-B)$.
$= -2\sin A \sin B$		
$\therefore LHS = RHS$ QED	2	
(ii) Hence, find $\cos 75° - \cos 15°$		
$= \cos(45°+30°) - \cos(45°-30°)$		1 mark for using the outcome in (i) correctly.
$= -2 \sin 45° \sin 30°$		
$= -2 \times \frac{1}{\sqrt{2}} \times \frac{1}{2}$		
$= -\frac{1}{\sqrt{2}}$		1 mark for correct solution
$= -\frac{\sqrt{2}}{2}$ on rationalizing the denominator	2	

Question 2

a) $\frac{d}{dx}\left(\cos^{-1} 2x\right) = \frac{-1}{\sqrt{1-4x^2}} \cdot 2$

$= \frac{-2}{\sqrt{1-4x^2}}$ 1

b) $\int_0^{\frac{1}{2}} \frac{1}{2}\cos^{-1}(1-2x)\,dx \approx \frac{\frac{1}{2}-0}{6}\left[\frac{1}{2}\cos^{-1}0 + 4\times\frac{1}{2}\cos^{-1}\frac{1}{2} + \frac{1}{2}\cos^{-1}0\right]$ 1

$= \frac{1}{12}\left[\pi/4 + 2\times\pi/3 + \frac{1}{2}\times 0\right]$

$= \frac{1}{12}\left(\pi/4 + 2\pi/3\right)$

$= 0.23998\ldots$

$\doteqdot 0.24$ (2dp) 1

c) (i) RHS $= \frac{5x^2+20-16}{x^2+4} = \frac{5x^2+4}{x^2+4} =$ LHS 1

(ii) $\int_{-\frac{1}{2}}^{0} \frac{5x^2+4}{x^2+4}\,dx = \int_{-\frac{1}{2}}^{0}\left(5 - \frac{16}{x^2+4}\right)dx$

$= 5x\Big]_{-\frac{1}{2}}^{0} - 16\times\frac{1}{2}\tan^{-1}\frac{x}{2}\Big]_{-\frac{1}{2}}^{0}$ 1

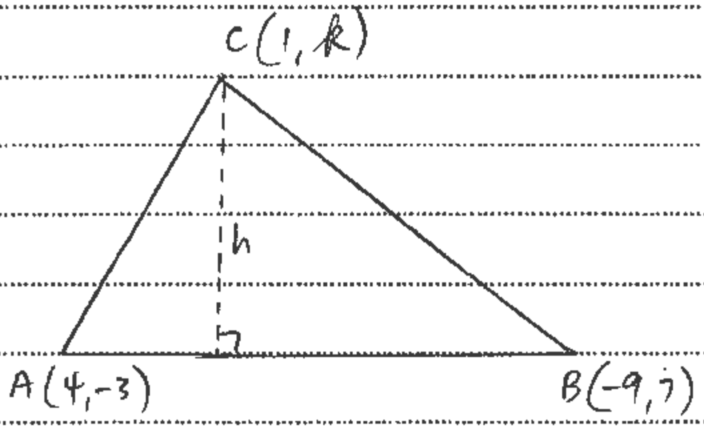
$= \frac{5}{2} - 8\left(\tan^{-1}0 - \tan^{-1}\left(\frac{1}{4}\right)\right)$

$= \frac{5}{2} - 8\left(\tan^{-1}\frac{1}{4}\right)$

$= 0.54017\ldots$

$= 0.54$ (2dp) 1

d)



Equation of AB: $y+3 = \left(\frac{7+3}{-9-4}\right)(x-4)$

$$= -\frac{10}{13}(x-4)$$

$$13y + 39 = -10x + 40$$

$$\therefore \quad 10x + 13y - 1 = 0 \qquad 1$$

$\perp$ar distance from C to AB:

$$h = \frac{|10 \times 1 + 13 \times k - 1|}{\sqrt{10^2 + 13^2}}$$

$$= \frac{|9 + 13k|}{\sqrt{269}} \qquad 1$$

Hence $|ABC| = \frac{1}{2}\sqrt{10^2+13^2} \cdot \frac{|9+13k|}{\sqrt{269}}$ $\qquad 1$

i.e. $15 = \frac{1}{2}|9+13k|$

$\therefore \; 9 + 13k = 30$ or -30

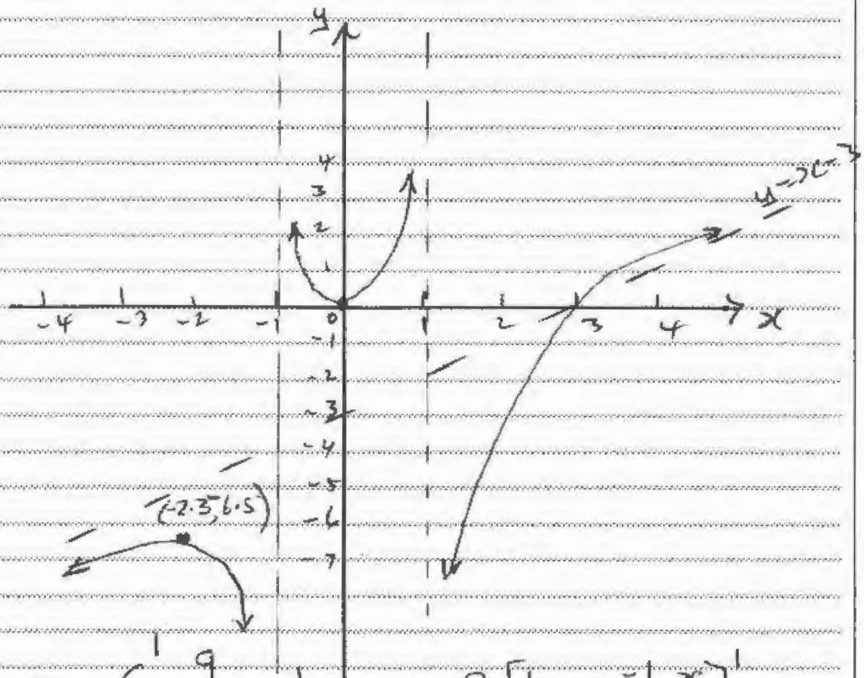
$\Rightarrow \quad k = \frac{21}{13}$ or -3 $\qquad 1$

$\longrightarrow$

MATHEMATICS Extension 1 : Question 3

Suggested Solutions	Marks	Marker's Comments
a) (i) $f(x) = \dfrac{x^3 - 3x^2}{x^2 - 1}$ x intercept $y = 0$ $\quad x^2(x-3) = 0$ $x = 0$ OR $x = 3$ y intercept $x = 0$ $\quad y = 0$.	1	Accept $(0,0)$ and $(3,0)$ $\frac{1}{2}+\frac{1}{2}$ mark each.
(ii) No horizontal asymptote. vertical asymptotes $x = \pm 1$	1	
(iii) $x^2-1 \overline{) x^3 - 3x^2}$ quotient $x - 3$ $x^3 - x$ $-3x^2 + x$ $-3x^2 + 3$ $x - 3$ $\therefore f(x) = (x-3) + \dfrac{x-3}{x^2+1}$ as $x \to \infty$ $\quad \dfrac{x-3}{x^2+1} \to 0$ $\therefore f(x) \to x - 3$ $\quad \therefore$ oblique asymptote Alternatively $f(x) = \dfrac{\dfrac{x^3}{x^2} - \dfrac{3x^2}{x^2}}{\dfrac{x^2}{x^2} - \dfrac{1}{x^2}}$ $= \dfrac{x-3}{1-\frac{1}{x^2}}$ as $x \to \infty$ $\quad \dfrac{1}{x^2} \to 0$ $\quad \therefore 1 - \dfrac{1}{x^2} \to 1$ $\therefore f(x) \to x - 3$ $\therefore$ oblique asymptote	1	Must show complete division for both methods not just cancel out
(iv) stationary point when $f'(x) = 0$ $x \doteqdot -2.3$ $\therefore f(-2.3) \doteqdot -6.5$		No need for further calculations to be shown

MATHEMATICS Extension 2: Question……..

Suggested Solutions	Marks	Marker's Comments
	2	$\frac{1}{2} + \frac{1}{2} + \frac{1}{2}$ for each branch and appropriate asymptote $\frac{1}{2}$ for plotting turning point accurately. and showing coordinates to scale
b) $A = \int_{-1}^{1} \frac{9}{9+x^2}\,dx = \frac{9}{3}\left[\tan^{-1}\frac{x}{3}\right]_{-1}^{1}$ As $\frac{9}{9+x^2}$ is even $= 2\times\left[\tan^{-1}\frac{x}{3}\right]_{0}^{1}$ $= 6\tan^{-1}(1/3)$	2	1 integration Decimal equivalent answer 1.93 accepted. 1 Answer (simplified).
c) $\sin 2x = \tan x$ Let $t = \tan x$ $\quad x = \pi/2$. $\frac{2t}{1+t^2} = t$ $2t = t + t^3$ $0 = t^3 - t$ $0 = t(t^2-1)$ $t = 0$ OR $t = \pm 1$ $\tan x = 0 \quad x = n\pi \quad n \in Z$ $\tan x = \pm 1 \quad x = n\pi \pm \pi/4 \quad n \in Z$ Test $x = \pi/2$ LHS $= \sin\pi = 0$ RHS is undefined $\therefore\ x = \pi/2$ is not a solution	3	1 correct t substitution and simplified quadratic $\frac{1}{2}$ 3 quadratic solutions 1 3 general solutions $\frac{1}{2}$ test/reject $x = \pi/2$.

TERM 4 MATHEMATICS EXTENSION 1 2013
SECTION 4

a) $$\int \frac{6\,dx}{\sqrt{(1+2x)^3}} = \int \frac{6}{\sqrt{u^3}} \times \frac{du}{2}$$

$u = 1+2x$

$\frac{du}{dx} = 2$

$dx = \frac{du}{2}$

$$= 3\int \frac{du}{u^{3/2}}$$

① integral wrt u

$$= 3\left[-2u^{-\frac{1}{2}}\right]$$

$$= \frac{-6}{\sqrt{1+2x}} + C$$

① answer in terms of x

(b) (i) $\sqrt{3}\sin\theta - 3\cos\theta = R\sin\theta\cos\alpha - R\cos\theta\sin\alpha$

$\sqrt{3} = R\cos\alpha \qquad 3 = R\sin\alpha$

$R^2 = 3+9$

$R = \sqrt{12}$

$= 2\sqrt{3}$

$\tan\alpha = \frac{3}{\sqrt{3}}$

$= \sqrt{3}$

$\alpha = \frac{\pi}{3}$

α is in 1^{st} Q

$\frac{1}{2}$ R

$\frac{1}{2}$ α

$\therefore \; y = 2\sqrt{3}\sin\left(\theta - \frac{\pi}{3}\right)$

(ii)

tp's are at $\frac{\pi}{2}$ and

shifed $\frac{\pi}{3}$ to right

$\frac{\pi}{2} + \frac{\pi}{3} = \frac{5\pi}{6}$

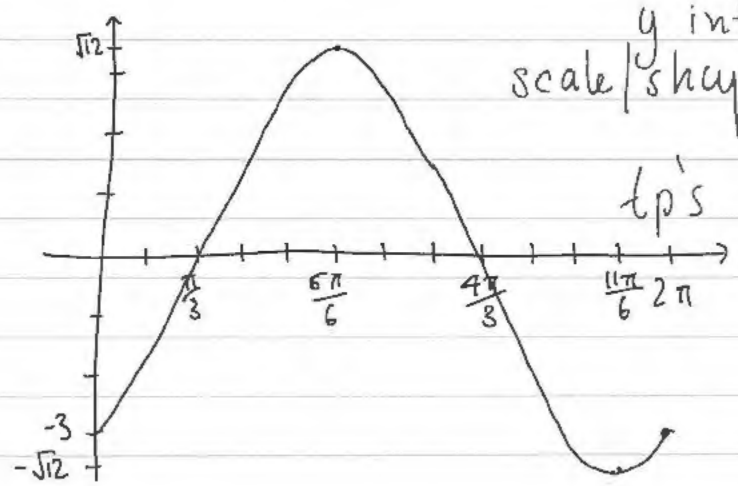
$\frac{3\pi}{2} + \frac{\pi}{3} = \frac{11\pi}{6}$

when $x = 0$

$y = 2\sqrt{3}\sin -\frac{\pi}{3}$

$= 2\sqrt{3} \times \frac{\sqrt{3}}{2}$

$= -3$



amp
x int
y int
scale/shape

$\frac{1}{2}$

tp's — ①

(c) RHS $= \cos 2\theta - \cos 4\theta$

$= 2\cos^2\theta - 1 - (2\cos^2 2\theta - 1)$ ① double angle formulae

$= 2\cos^2\theta - 1 - 2\cos^2 2\theta + 1$

$= 2\cos^2\theta - 2\cos^2 2\theta$ ① answer

(d) $\int_{-2}^{2} \frac{dx}{(x^2+4)^2} = \frac{1}{16}\int_{-2} \frac{16}{(x^2+4)^2}\,dx$

$= \frac{1}{16}\left[\tan^{-1}\frac{x}{2} + \frac{2x}{x^2+4}\right]_{-2}^{2}$ ① primitive

$= \frac{1}{16}\left(\tan^{-1}(1) + \frac{4}{8} - \tan^{-1}(-1) - \frac{(-4)}{8}\right)$

$= \frac{1}{16}\left(\frac{\pi}{4} + \frac{1}{2} - \left(\frac{-\pi}{4}\right) + \frac{1}{2}\right)$

$= \frac{1}{16}\left(\frac{\cancel{2}^{1}\pi}{\cancel{4}_{2}} + 1\right)$

$= \frac{\pi}{32} + \frac{1}{16}$ ① answer

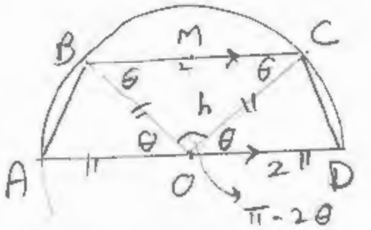
$\boxed{\text{no half marks}}$

MATHEMATICS EXTENSION 1: Question 5

Suggested Solutions	Marks	Marker's Comments
(a) $\frac{d}{dx}\frac{x}{\sqrt{1-x^2}} = \frac{\sqrt{1-x^2}(1) + x^2(1-x^2)^{-\frac{1}{2}}}{1-x^2}$		
$= \frac{1-x^2+x^2}{(1-x^2)^{3/2}}$		
$= (1-x^2)^{-3/2}$	1	Quotient rule
$\therefore LHS = \frac{d}{dx}\tan^{-1}\frac{x}{\sqrt{1-x^2}}$		
$= \frac{(1-x^2)^{-3/2}}{1+\frac{x^2}{1-x^2}}$	1	Chain rule
$= \frac{(1-x^2)^{-1/2}}{(1-x^2)+x^2}$	1	Simplification
$= \frac{1}{\sqrt{1-x^2}}$		* Using triangles to prove $\sin^{-1}x = \tan^{-1}\frac{x}{\sqrt{1-x^2}}$ requires great care with the resultant restrictions
$= \frac{d}{dx}\sin^{-1}x$		
$=$ RHS as required		
(b) $\int_0^{\sqrt{3}}\frac{dx}{\sqrt{3-x^2}} = \left[\sin^{-1}\frac{x}{\sqrt{3}}\right]_0^{\sqrt{3}}$	1	Integrate
$= \sin^{-1}1 - \sin^{-1}0$	1	Evaluate boundaries
$= \frac{\pi}{2}$		
(c) $\frac{dV}{dt} = \pi\left(100 - \frac{t^2}{100}\right)$ L/s		
(i) When $t=10$,		
$\frac{dV}{dt} = \pi\left(100 - \frac{100}{100}\right)$		
$= 99\pi$ L/s	1	Evaluate

MATHEMATICS EXTENSION 1: Question 5		
Suggested Solutions	**Marks**	**Marker's Comments**
(c) Continued.		
(ii) Given no further information, tank is filled when $\frac{dV}{dt} = 0$.		
ie. $\pi\left(100 - \frac{t^2}{100}\right) = 0$		
$\frac{t^2}{100} = 100$		
$t = \pm 100$		
But since $t > 0$, $t = 100$ s only.	1	Solve for $\frac{dV}{dt} = 0$
(iii) $V = 100\pi t - \frac{\pi t^3}{300} + c$	1	Integrate
When $t = 0$, $V = 0$ $\therefore c = 0$.	1	Constant
When $t = 100$, $V = 10000\pi - \frac{1000000\pi}{300}$		
$= \frac{20000\pi}{3}$ L.	1	Evaluate
ALTERNATIVELY		
$V = \int_0^{100} \pi\left(100 - \frac{t^2}{100}\right) dt$	1	Definite integral
$= \left[100\pi t - \frac{\pi t^3}{300}\right]_0^{100}$	1	Integrate
$= \left(10000\pi - \frac{1000000\pi}{300}\right) - 0$		
$= \frac{20000\pi}{3}$ L	1	Evaluate

(i)



no marks for diagram

$\triangle OAB \equiv \triangle ODC$ (SAS) — 1m

Area $\triangle OAB = \frac{1}{2} \cdot 2 \cdot 2 \cdot \sin\theta = 2\sin\theta$

Area $\triangle BOC = \frac{1}{2} \cdot 2 \cdot 2 \cdot \sin(\pi - 2\theta)$

$= 2\sin 2\theta$

some do not simplify $\sin(\pi - 2\theta) = \sin 2\theta$ $-\frac{1}{2}$m

Area Trapezium $ABCD = 2 \times 2\sin\theta + 2\sin 2\theta$

$= 4\sin\theta + 2\sin 2\theta$ — 1m

many students thought the 3 triangles are congruent $\therefore 6\sin\theta$ 1m only

OR $h = 2\sin\theta$ — $\frac{1}{2}$m

$BC = 2\,BM = 2 \cdot 2\cos\theta = 4\cos\theta$ — $\frac{1}{2}$m

Many students thought ABCO is parm

$|ABCD| = \frac{1}{2}(BC + AD) \cdot h$

$= \frac{1}{\cancel{2}}(4\cos\theta + 4)\cancel{2}\sin\theta$

$= 4\cos\theta\sin\theta + 4\sin\theta$ — 1m

or $= 2\sin 2\theta + 4\sin\theta$

ii) $A' = 4\cos\theta + 4\cos 2\theta$ or — 1m

$= -4\sin^2\theta + 4\cos^2\theta + 4\cos\theta$

$\theta = \frac{\pi}{2}$ 0m

SP, $A' = 0$ $\quad \cancel{4}\cos\theta = -\cancel{4}\cos 2\theta$

$\cos\theta = -(2\cos^2\theta - 1)$

$\therefore 2\cos^2\theta + \cos\theta - 1 = 0$

$(2\cos\theta - 1)(\cos\theta + 1) = 0$ $\therefore \theta = \frac{\pi}{3}$ or π — 1m

Many forgot to mention $\theta \neq \pi$ $-\frac{1}{2}$m
students need to mention π but reject it later

but $0 < \theta < \frac{\pi}{2}$ $\therefore \theta = \frac{\pi}{3}$ only

$A'' = -4\sin\theta - 8\sin 2\theta$

$A''\left(\frac{\pi}{3}\right) = -2\sqrt{3} - 4\sqrt{3} = -6\sqrt{3} \doteqdot -10.39 < 0$ — $\frac{1}{2}$m

$\frown$ $\therefore$ rel max at $\frac{\pi}{3}$

Since A is continuous for $0 < \theta < \frac{\pi}{2}$, & only 1 T.P $\theta = \frac{\pi}{3}$ will give absolute max Area

max Area $= \frac{6\sqrt{3}}{2} = 3\sqrt{3}$ units² # — $\frac{1}{2}$m

b) $y = 2\cos^{-1}\frac{x}{3}$

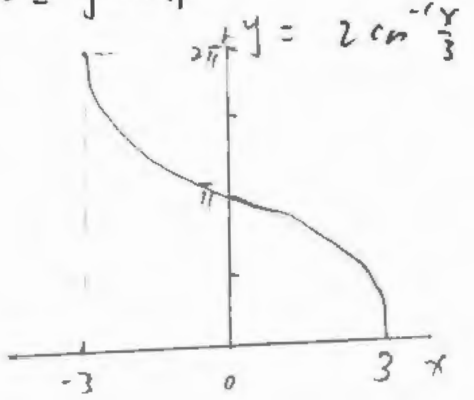
i) D: $-3 \le x \le 3$

R: $0 \le y \le 2\pi$

$\frac{1}{2}$m well done

$\frac{1}{2}$m

ii)



well done

1 m Pay attention to the vertical gradients at end points $-\frac{1}{2}$m

iii) $\cos^2 x = \dfrac{1+\cos 2x}{2}$

1 m

iv) $$vol = \int_{\pi}^{2\pi} \pi x^2\,dy = \pi\int_{\pi}^{2\pi}\left[3\cos\left(\frac{y}{2}\right)\right]^2 dy$$

1 m no half mark

some use limits from 0 to π need to explain -1m

$$vol = 9\pi\int_{\pi}^{2\pi}\frac{1+\cos y}{2}\,dy$$

$$= \frac{9\pi}{2}\left(y + \sin y\right)\Big]_{\pi}^{2\pi}$$

$$= \frac{9\pi}{2}\left(2\pi + \cancel{\sin 2\pi}^{\,0} - \pi - \cancel{\sin\pi}^{\,0}\right)$$

$$= \frac{9\pi^2}{2}\ \text{unit}^3 \ \#$$

1 m

X1 **MATHEMATICS: Question** 9

Suggested Solutions	Marks	Marker's Comments

a) $\sin^{-1}x + \cos^{-1}x = \pi/2$ (radians!!)

$\therefore \int \sin^{-1}x + \cos^{-1}x \, dx = \int \frac{\pi}{2}\,dx$

$= \frac{\pi x}{2} + k$ — **1**

Marker's comments: Marks were not deducted if k omitted.

b) Let $\alpha = \cos^{-1}x$, $\beta = \sin^{-1}x$

Take sine of both sides, noting that $\cos\alpha = x$, $\sin\alpha = \sqrt{1-x^2}$, $0 \le \alpha \le \pi$

$\sin\beta = x$, $\cos\beta = \sqrt{1-x^2}$, $-\frac{\pi}{2} \le \beta \le \pi/2$

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$= \sqrt{1-x^2}\sqrt{1-x^2} - x.x$

$= 1-x^2-x^2$

$= 1-2x^2$

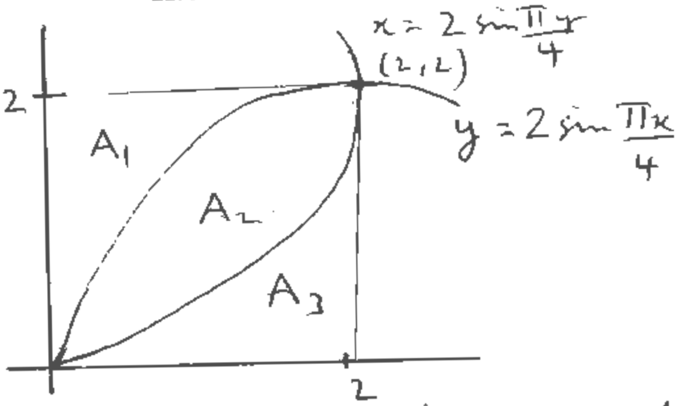
$\sin(\sin^{-1}(1-x)) = 1-x$

$\therefore 1-x = 1-2x^2$ — **1**

$2x^2 - x = 0$

$x = 0$ or $1/2$ — **1**

c)



Marker's comments: Diagrams were scarce and generally too small and useless.

The curves are inverses of each other so $A_1 + A_2 = A_2 + A_3$. ($\therefore A_1 = A_3$)

$\therefore A_2 = 2(A_2+A_3) - (A_1+A_2+A_3)$ — **1**

Marker's comments: Mark for some sensible combination of areas.

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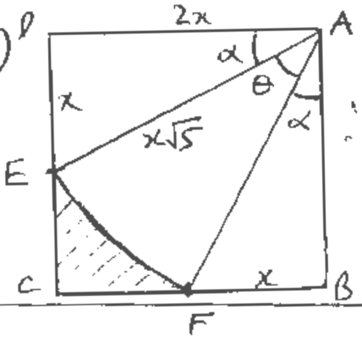
MATHEMATICS: Question 7

Suggested Solutions	Marks	Marker's Comments
c) (cont) $= 2\int_0^2 2\sin\frac{\pi x}{4}\,dx - 4$		
$= 4\left[-\frac{4}{\pi}\cos\frac{\pi x}{4}\right]_0^2 - 4$	1	Mark for a correct integral
$= 4\left(0 + \frac{4}{\pi}\right) - 4$		
$= \frac{16}{\pi} - 4$		
$\therefore$ Common area is $\left(\frac{16}{\pi} - 4\right) u^2$	1	Mark for correct answer.
d) i) Let $\alpha = \tan^{-1}(1/2)$, $0 < \alpha < \pi/4$; $\beta = \tan^{-1}(1/3)$, $0 < \beta < \pi/4$ $\}*$		
$\therefore \tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$		
$= \frac{1/2 + 1/3}{1 - 1/2 \cdot 1/3}$		
$= \frac{5/6}{5/6} = 1$		1 Mark for answer
$\therefore \alpha + \beta = \frac{\pi}{4} + n\pi \quad n \in \mathbb{Z}$	1	1 Mark for correct handling of domains.
But, from $*$, $0 < \alpha + \beta < \pi/2$		
$\therefore n = 0$ and $\tan^{-1}(1/2) + \tan^{-1}(1/3) = \pi/4$	1	
ii) 		
$\alpha = \tan^{-1} \frac{x}{2x} = \tan^{-1} 1/2$		½ for $\tan^{-1}(1/2)$
$\therefore \theta = \frac{\pi}{2} - 2\tan^{-1}(1/2)$		
$= 2\left(\frac{\pi}{4} - \tan^{-1} 1/2\right)$	½	
$= 2\tan^{-1}(1/3)$ (part (i))		

Suggested Solutions	Marks	Marker's Comments
d) ii) (cont) $EA = x\sqrt{5}$ (Pythagoras on ΔADE)		
$\therefore$ Unaccessible area		
$= 4x^2 - \left(\frac{2}{2} \times 2x \times x + \frac{(x\sqrt{5})^2}{2}\theta\right)$		
$= 2x^2 - \frac{5x^2}{2}\left(2\tan^{-1}\left(\frac{1}{3}\right)\right)$		
$= \underline{\underline{2x^2 - 5x^2\tan^{-1}(1/3)}}$	1	
$\therefore$ Proportion of total area is		
$\frac{2x^2 - 5x^2\tan^{-1}(1/3)}{4x^2}$		
$= \underline{\underline{\frac{1}{2} - \frac{5}{4}\tan^{-1}(1/3)}}$ $\left(a = 1/2, b = -\frac{5}{4}\right)$	1/2	Lots of people forgot this last bit.