QUESTION 1 *(13 marks)*

(a) Solve $\log_e x + \log_e (2-x) = 0$. *(2 marks)*

(b) The arc length of a sector of a circle is 6cm. If the circle has radius 3cm, find the area of the sector.

(2 marks)

(c) Given that $\dfrac{1}{x+1} - \dfrac{1}{x+2} = \dfrac{1}{(x+1)(x+2)}$

show that $\displaystyle\int_{1}^{3} \dfrac{dx}{(x+1)(x+2)} = \log_e \dfrac{6}{5}$ *(2 marks)*

(d) The gradient at any point (x,y) on a curve is given by $\dfrac{dy}{dx} = \dfrac{x-6}{x}$. If the curve intersects the line x = 1 at y = 2 , find where the curve cuts the line x = 3. *(3 marks)*

(e) Water evaporates from a lake at a rate $\dfrac{dV}{dt}$ proportional to the volume V of the water remaining. If 50% of the water evaporates in 20 days, find the percentage of the original volume of water remaining after 50 days without rain. Give your answer to the nearest percentage. *(4 marks)*

QUESTION 2 *(13 marks)*   **Start a new page**

(a) Find $\displaystyle\lim_{x \to 0} \dfrac{\sin 2x}{\tan x}$ *(3 marks)*

(b) (i) Write down the expansion of $\cos (\alpha + \beta)$. *(1 mark)*

(ii) If $\sin \alpha = \dfrac{2}{3}$, $\dfrac{\pi}{2} < \alpha < \pi$ and $\cos \beta = \dfrac{3}{4}$, $0 < \beta < \dfrac{\pi}{2}$, find the exact value of $\cos (\alpha + \beta)$. *(3 marks)*

(c) Differentiate $e^{x^2} \sin x$ with respect to x . *(2 marks)*

(d) (i) Find $\dfrac{d}{dx}(x \sin x + \cos x)$. *(2 marks)*

(ii) Hence find the value of $\displaystyle\int_{\frac{\pi}{2}}^{\pi} x\cos x\, dx$. *(2 marks)*

QUESTION 3 *(13 marks)*   **Start a new page**

(a)   Simplify $\cos^{-1} x + \cos^{-1}(-x)$.   *(1 mark)*

(b)   Find the domain and range of the function $y = \dfrac{1}{3}\sin^{-1}\dfrac{x}{2}$ *(2 marks)*

(c)   Show that $\tan^{-1}\dfrac{1}{2} - \tan^{-1}\dfrac{1}{4} = \tan^{-1}\dfrac{2}{9}$ *(3 marks)*

(d)   Find the indefinite integrals

        (i) $\displaystyle\int \dfrac{dx}{x^2+4}$ *(1 mark)*

        (ii) $\displaystyle\int \dfrac{x\,dx}{x^2+4}$ *(1 mark)*

        (iii) $\displaystyle\int \dfrac{dx}{\sqrt{9-4x^2}}$ *(2 marks)*

(e)   If $y = \sin^{-1}\sqrt{x}$ show that $\dfrac{dy}{dx} = \dfrac{1}{\sin 2y}$ *(3 marks)*

END OF ASSESSMENT