Question 1 (14 marks)

(a) Solve $\log_e x + \log_e(2-x) = 0$ **(3 marks)**

(b) The arc length of a sector of a circle is 6 cm. If the circle has a radius of 3 cm, find the area of the sector. **(2 marks)**

(c) Find the size of the acute angle between the lines whose equations are

$y = 2\sqrt{3}x - \sqrt{6}$ and

$7y = \sqrt{3}x + \sqrt{2}$ **(3 marks)**

(d) (i) Write down the expansion of $\cos(A+B)$. **(1 mark)**

(ii) If $\sin A = \frac{2}{3}$ and $\frac{\pi}{2} < A < \pi$ and $\cos B = \frac{3}{4}$ and $0 < B < \frac{\pi}{2}$

Find the exact value of $\cos(A+B)$ **(3 marks)**

(e) Find $\frac{dy}{dx}$ if $y = e^x \sin x$ **(2 marks)**

Question 2 (14 marks) (START QUESTION ON A NEW PAGE)

(a) (i) Show that $\frac{\sin 2x}{\tan x} = 2\cos^2 x$ **(3 marks)**

(ii) Hence find $\lim_{x\to 0}\frac{\sin 2x}{\tan x}$ **(1 mark)**

(b) Given that $\frac{1}{x+1} - \frac{1}{x+2} = \frac{1}{(x+1)(x+2)}$

Show that $\int_1^3 \frac{dx}{(x+1)(x+2)} = \log_e \frac{6}{5}$ **(3 marks)**

(c) The gradient at any point (x,y) on a curve is given by $\frac{dy}{dx} = 1 - \frac{6}{x}$. Find the equation of the curve if it passes through the point (1,2). **(2 marks)**

Question 2 continued.

(d) (i) Find $\frac{d}{dx}(x\sin x+\cos x)$ **(2 marks)**

(ii) Hence find the value of $\int_{\frac{\pi}{2}}^{\pi} x\cos x dx$ **(3 marks)**

Question 3 (14 marks) (START QUESTION ON A NEW PAGE)

(a) P $(2at, at^2)$ is a point on the parabola $x^2 = 4ay$

(i) Show that the equation of the tangent to this parabola at P is $y = tx - at^2$ **(3 marks)**

(ii) If this tangent meets the x axis at T, find the co-ordinates of T. **(1 mark)**

(iii) M is the mid-point of PT. Find the co-ordinates of M. **(1 mark)**

(iv) Find the equation of the locus of the point M. **(2 marks)**

(b) (i) Write down an expansion for $\cos 2x$ in terms of $\cos^2 x$ **(1 mark)**

(ii) Hence find a primitive function of $\frac{\cos 2x}{\cos^2 x}$ **(2 marks)**

(c) Find $\int \frac{x}{1-x^2} dx$ **(2 marks)**

(d) By using the substitution $u = \sqrt{x}$ or otherwise, find $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$ **(2 marks)**

END OF PAPER

Solution

Q1 (14)

(a)
$$\log_e x(2-x) = 0 \quad (1)$$
$$\therefore x(2-x) = 1 \quad (1)$$
$$x^2 - 2x + 1 = 0$$
$$(x-1)^2 = 0$$
$$x = 1 \quad (1)$$

(b)
$$6 = 3\theta$$
$$\therefore \theta = 2 \text{ radians} \quad (1)$$
$$\therefore A = \tfrac{1}{2} \times 3^2 \times 2$$
$$= 9\text{cm}^2 \quad (1)$$

(c)
$$\left.\begin{aligned} m_1 &= 2\sqrt{3} \\ m_2 &= \frac{\sqrt{3}}{7} \end{aligned}\right\} \quad (1)$$
$$\therefore \tan\theta = \frac{2\sqrt{3} - \frac{\sqrt{3}}{7}}{1 + 2\sqrt{3}\cdot\frac{\sqrt{3}}{7}} \quad (1)$$
$$= \frac{14\sqrt{3} - \sqrt{3}}{7+6}$$
$$= \frac{13\sqrt{3}}{13}$$
$$= \sqrt{3}$$
$$\therefore \theta = 60^\circ \quad (1)$$

(d) (i) $\cos A.\cos B - \sin A.\sin B$ (1)

(ii) If $\sin A = \frac{2}{3}$, $\cos A = -\frac{\sqrt{5}}{3}$ (1)

If $\cos B = \frac{3}{4}$, $\sin B = \frac{\sqrt{7}}{4}$ (1)

$$\therefore \cos(A+B) = -\frac{\sqrt{5}}{3}\cdot\frac{3}{4} - \frac{2}{3}\cdot\frac{\sqrt{7}}{4}$$
$$= \frac{-\sqrt{5}}{4} - \frac{\sqrt{7}}{6} \quad (1)$$

(e)
$$\frac{dy}{dx} = e^x \cos x + \sin x\, e^x \quad (2)$$
$$= e^x(\cos x + \sin x)$$

Q2

(a) (i)
$$LHS = \frac{2\sin x.\cos x}{\frac{\sin x}{\cos x}} \quad (1)$$
$$= \frac{2\sin x \cos^2 x}{\sin x} \quad (1)$$
$$= 2\cos^2 x = RHS \quad (1)$$

(ii)
$$\lim_{x\to 0} \frac{\sin 2x}{\tan x} = \lim_{x\to 0} 2\cos^2 x$$
$$= 2 \times 1 = 2 \quad (1)$$

(b) $\int_1^3 \left(\frac{1}{x+1} - \frac{1}{x+2}\right) dx = \left[\log_e(x+1) - \log_e(x+2)\right]_1^3$ (1)

$= \left[\log_e \frac{(x+1)}{(x+2)}\right]_1^3$

$= \log_e \frac{4}{5} - \log_e \frac{2}{3}$ (1)

$= \log_e \left(\frac{4}{5} \times \frac{3}{2}\right)$

$= \log_e \frac{6}{5}$ (1)

(c) $y = x - 6\log_e x + c$ (1)

$2 = 1 - 6\log_e 1 + c$

$2 = 1 - 0 + c$

$c = 1$ (1)

$\therefore y = x - 6\log_e x + 1$

(d) (i) $\frac{d}{dx}(x\sin x + \cos x) = x\cos x + \sin x . 1 - \sin x$ (1)

$= x\cos x$ (1)

(ii) $\therefore \int_{\frac{\pi}{2}}^{\pi} x\cos x \, dx = \left[x\sin x + \cos x\right]_{\frac{\pi}{2}}^{\pi}$ (1)

$= \pi\sin\pi + \cos\pi - \left(\frac{\pi}{2}\sin\frac{\pi}{2} + \cos\frac{\pi}{2}\right)$

(1)

$= \pi . 0 + (-1) - \frac{\pi}{2} . 1 - 0$

$= -1 - \frac{\pi}{2}$ (1)

Q3 (i) $y = \frac{x^2}{4a}$

$\frac{dy}{dx} = \frac{x}{2a}$ (1)

m_T at P $= \frac{2at}{2a} = t$ (1)

$\therefore$ Equation is $y - at^2 = t(x - 2at)$

$y - at^2 = tx - 2at^2$ (1)

$y = tx - at^2$

(ii) at T, $y=0$ $\therefore tx = at^2$

$x = at$

$\therefore$ T is $(at, 0)$ (1)

(iii) $\therefore$ M is $\left(\frac{2at+at}{2}, \frac{at^2+0}{2}\right)$

ie $\left(\frac{3at}{2}, \frac{at^2}{2}\right)$ (1)

(iv) $\therefore$ Parametric Eqns of the locus are

$x = \frac{3at}{2}$, $y = \frac{at^2}{2}$

$\therefore t = \frac{2x}{3a}$ (1)

$\therefore y = \frac{a \frac{4x^2}{9a^2}}{2}$

or $y = \frac{2x^2}{9a}$ (1)

(b) (i) $2\cos^2 x - 1$ (1)

(ii) $\frac{2\cos^2 x - 1}{\cos^2 x} = 2 - \sec^2 x$ ~~(1)~~

$\therefore$ a Primitive is $2x - \tan x$ (1)

(c) $\int \frac{x}{1-x^2}\,dx = -\frac{1}{2}\int \frac{-2x}{1-x^2}$ (1)

$= -\frac{1}{2}\log_e(1-x^2) + c$ (1) (no penal for leav. out c)

(d) Let $u = x^{\frac{1}{2}}$

$\therefore \frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$

$2du = \frac{dx}{\sqrt{x}}$ (1)

$\therefore \int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = 2\int e^u\,du$

$= 2e^u + c$

$= 2e^{\sqrt{x}} + c$ (1)

otherwise indicates that some will do it by inspection.

no penalty for leaving out c.