Student Name:.......................... Number: .... Teacher:..............................

NEWINGTON COLLEGE

IN FIDE SCIENTIAM

2014
HSC Assessment 1

Year 12 Mathematics Extension 1

General Instructions:

- Date of task – Monday 24th November (Wk 8B)
- Working time – 45 mins
- Weighting - 15%
- Board-approved calculators may be used.
- Attempt all questions, start each question in a new booklet.
- Show all relevant mathematical reasoning and/or calculations.

Outcome	Marks
Section 1 – Multiple choice	/4
Section 2 – Differentiation and Integration	/11
Section 3 – Area, Volume and Curve Sketching	/15
Total	/30

Outcomes to be assessed:

PE5 Determines derivatives which require the application of more than one rule of differentiation.

HE5 Applies the chain rule to problems appropriate techniques from the study of series to solve problems

Section 1: Multiple Choice (4 Marks)

1) Which expression is equal to $\int \frac{4x}{3x^2+1}dx$?

(A) $\frac{2}{3}\ln\left(3x^2+1\right)+c$

(B) $\frac{3}{2}\ln\left(3x^2+1\right)+c$

(C) $4\ln\left(3x^2+1\right)+c$

(D) $\ln\left(\frac{4}{6x}\right)+c$

2) If $f(x)=2^x$ then $f\,'(x)$ is equal to:

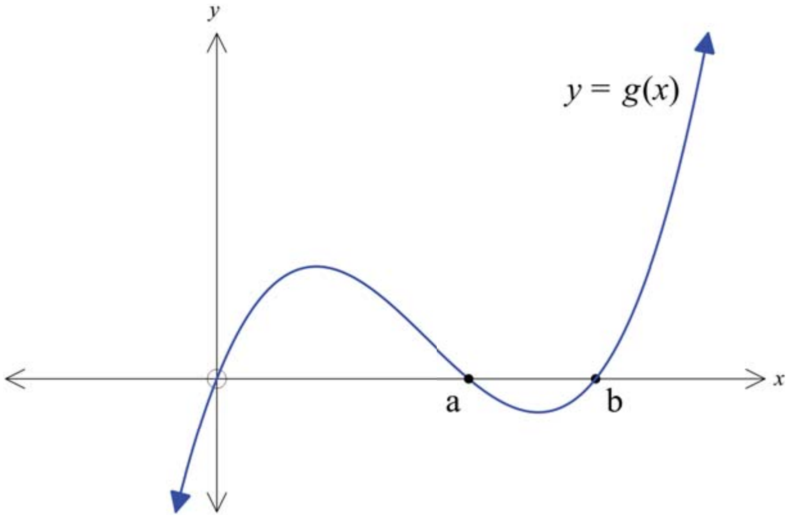
(A) $\ln 4^x$

(B) $\ln 2\times 2^x$

(C) $x\times 2^x$

(D) $\ln 2^x\times 2^x$

3) The graph of the function $y = g\left(x\right)$ is shown below.



Which expression **DOES NOT** correctly describe the area bounded by the $y=g(x)$ and the x axis, between $x=0$ and $x=b$?

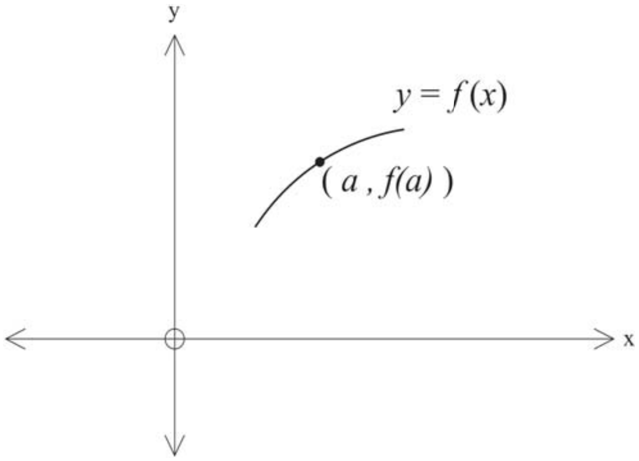
(A) $\int_0^a g(x)dx-\int_b^a g(x)dx$

(B) $\left|\int_0^a g(x)dx\right|+\int_b^a g(x)dx$

(C) $\left|\int_0^a g(x)dx+\int_a^b g(x)dx\right|$

(D) $\left|\int_a^0 g(x)dx\right|+\left|\int_a^b g(x)dx\right|$

4) A section of the graph of $f(x)$ is shown below.



(A)	(B)	(C)	(D)
$f'(x)<0$ $f''(x)<0$	$f'(x)>0$ $f''(x)<0$	$f'(x)<0$ $f''(x)>0$	$f'(x)>0$ $f''(x)>0$

Section 2: Differentiation and Integration (11 Marks)

1) (a) Find $\int \frac{4x^2 - 3x}{x}\,dx$ 2

(b) Find $\frac{d}{dx}\left(\frac{e^x}{x^2}\right)$ 2

(c) Find $\int z.\sqrt[3]{z^2+1}\,dz$ by using the substitution $u = z^2 + 1$ 3

(d) (i) Show that $\frac{5x-4}{x-2} = 5 + \frac{6}{x-2}$ 1

(ii) Hence, or otherwise, evaluate 3

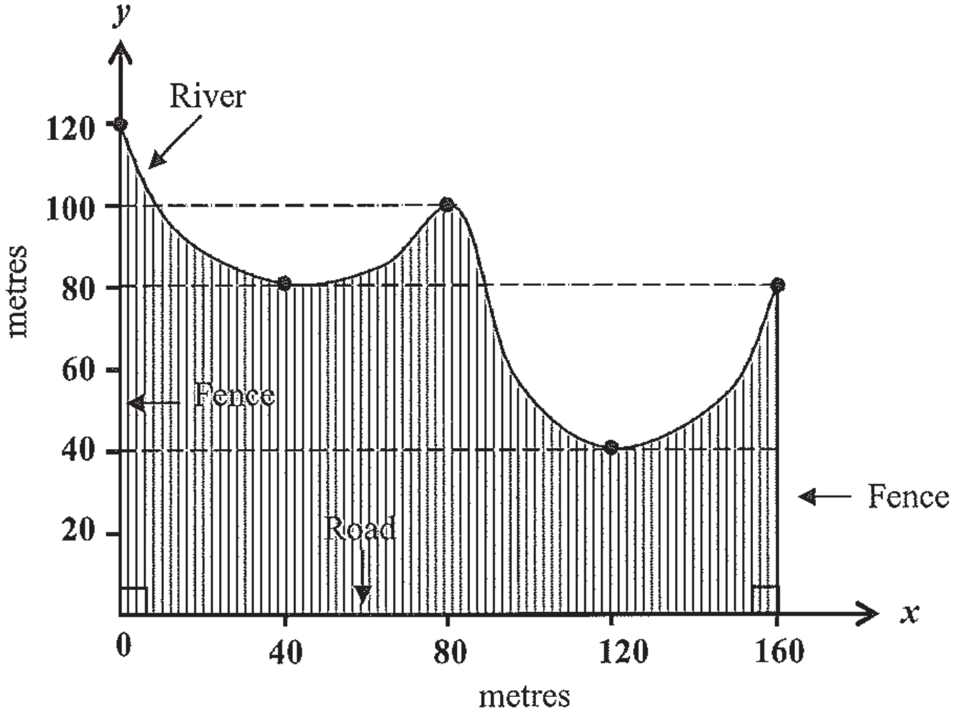
$$\int_3^4 \frac{10x-8}{x-2}\,dx$$

Section 3: Area, Volume and Curve Sketching (15 Marks)

1) Find the area bounded by the curves $f(x) = x^2$ and $g(x) = 3x - 2$ 3

2) Find the volume of the solid generated when $y = e^x - \frac{1}{e^x}$ is rotated about the x-axis between $x = 0$ and $x = 0.5$. Leave your answer in simplest exact form. 3

3) A paddock is bounded by 3 straight sides and a river, as shown by the shaded area in the scale diagram below.



(i) By reading distances from the diagram and using the **trapezoidal rule** with 5 function values, find the area of the paddock. 2

(ii) Does the trapezoidal rule give an under-estimate or an over-estimate for the actual area? Justify your answer 1

4) For the curve $y = \dfrac{x}{x^2+1}$,

i) Show that there are turning points at $\left(1, \dfrac{1}{2}\right)$ and $\left(-1, -\dfrac{1}{2}\right)$ and determine their nature. 6

ii) Show that points of inflexion occur when $x = 0$ and $x = \pm\sqrt{3}$.

iii) State the equation of the horizontal asymptote.

iv) Sketch the curve, labeling all important features.

END OF EXAMINATION

Student Name:.......................... Number: .... Teacher:..............................

Year 12 Extension 1
Section 1 – Multiple Choice Answer Sheet

Completely fill the response oval representing the most correct answer.

1 A ○ B ○ C ○ D ○

2 A ○ B ○ C ○ D ○

3 A ○ B ○ C ○ D ○

4 A ○ B ○ C ○ D ○

HSC ASSESSMENT 1 – YEAR 12 EXT 1

Section 1

1. $\frac{d}{dx}\frac{4x}{3x^2+1}$

$= 4.\frac{d}{dx}\frac{x}{3x^2+1}$

$= \frac{2}{3}\frac{d}{dx}\frac{6x}{3x^2+1}$

$= \frac{2}{3}\ln(3x^2+1)$

(A)

2. $y = 2^x$

$\ln y = x\ln 2$

$x = \frac{\ln y}{\ln 2}$

$\frac{dx}{dy} = \frac{1}{y\ln 2}$

$\frac{dy}{dx} = y\ln 2$

$= \ln 2 . 2^x$

(B)

3. (C)

4. (B)

Section 2

1. a) $\int \frac{4x^2-3x}{x}\,dx$

$= \int 4x-3\,dx$

$= \frac{4x^2}{2} - 3x + c$

$= 2x^2 - 3x + c$

b) $\frac{d}{dx}\left(\frac{e^x}{x^2}\right)$ $u = e^x$ $v = x^2$
$u' = e^x$ $v' = 2x$

$$= \frac{x^2 . e^x - e^x . 2x}{x^4}$$

$$= \frac{xe^x - 2e^x}{x^3}$$

$$= \frac{e^x(x-2)}{x^3}$$ //

c) $\int z(z^2+1)^{\frac{1}{3}}\, dz$ $u = z^2 + 1$
$\frac{du}{dz} = 2z$
$dz = \frac{du}{2z}$

$$= \int z(u)^{\frac{1}{3}} . \frac{du}{2z}$$

$$= \frac{1}{2}\int u^{1/3}\, du$$

$$= \frac{1}{2}\left[\frac{3}{4}u^{4/3}\right] + c$$

$$= \frac{3}{8}(z^2+1)^{4/3}.$$ //

d) i) $\frac{5x-4}{x-2} = \frac{5x - 10 + 6}{x-2}$

$$= \frac{5x-10}{x-2} + \frac{6}{x-2}$$

$$= \frac{5(x-2)}{x-2} + \frac{6}{x-2}$$

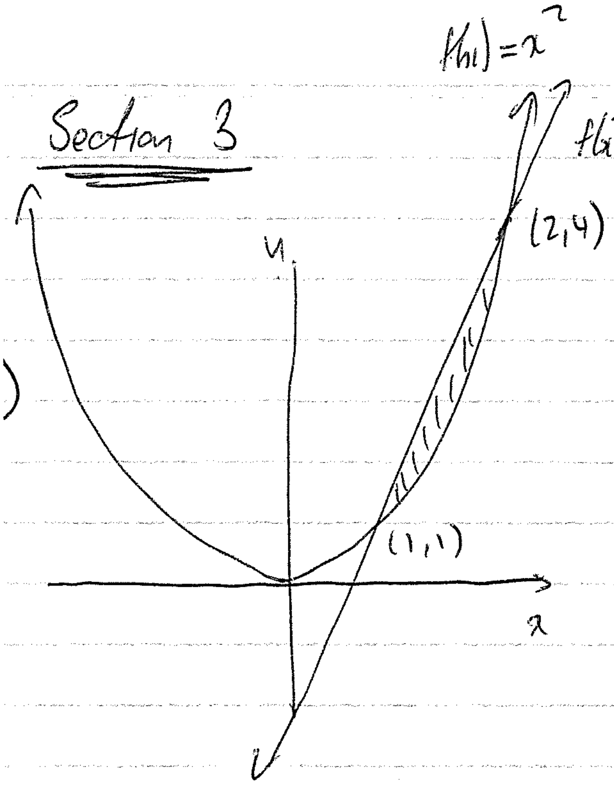
$$= 5 + \frac{6}{x-2}$$ //

ii) $= \int_3^4 5 + \frac{6}{x-2}\, dx$

$$= \left[5x + 6\ln(x-2)\right]_3^4$$

$$= (20 + 6\ln 2) - (15 + 6\ln 1)$$

$$= 5 + 6\ln 2$$ //



Section 3

1) Intersection:

$$x^2 = 3x - 2$$

$$x^2 - 3x + 2 = 0$$

$$(x-2)(x-1) = 0$$

$$x = 2 \quad x = 1$$

$$y = 4 \quad x = 1$$

$$\text{Area} = \int_1^2 (3x-2) - (x^2)\,dx$$

$$= \left[\frac{3x^2}{2} - 2x - \frac{x^3}{3}\right]_1^2$$

$$= \left(\frac{12}{2} - 4 - \frac{8}{3}\right) - \left(\frac{3}{2} - 2 - \frac{1}{3}\right)$$

$$= \frac{1}{6}\ u^2$$

2) $$V = \pi \int f(x)^2\,dx$$

$$\left(e^x - \frac{1}{e^x}\right)^2 =$$

$$V = \pi \int_0^{\frac{1}{2}} \left(e^x - \frac{1}{e^x}\right)^2 dx$$

$$= \pi \int_0^{\frac{1}{2}} \left(e^{2x} - 2 + e^{-2x}\right) dx$$

$$= \pi \left[\frac{e^{2x}}{2} - 2x - \frac{e^{-2x}}{2}\right]_0^{\frac{1}{2}}$$

$$= \pi \left[\left(\frac{e}{2} - 1 - \frac{1}{2e}\right) - \left(\frac{1}{2} - \frac{1}{2}\right)\right]$$

$$= \pi \left(\frac{e}{2} - 1 - \frac{1}{2e}\right)\ u^3$$

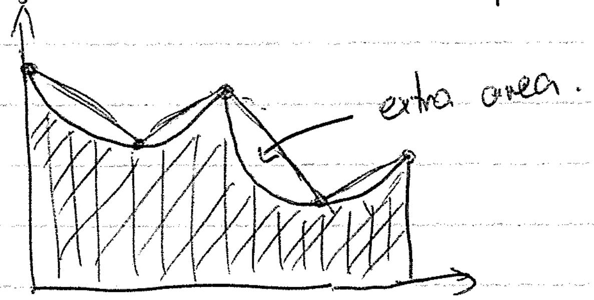
//

3) i) $h = 40$

$$A \approx \frac{40}{2}\left(1\times120 + 2\times80 + 2\times100 + 2\times40 + 1\times160\right)$$

$$\approx 14400\ m^2$$

ii) This would be an over estimation as graph is concave up.



4.) i) $y = \dfrac{x}{x^2+1}$ $\quad u = x \quad v = x^2+1$

$\quad u' = 1 \quad v' = 2x$

$$y' = \frac{(x^2+1).1 - x.2x}{(x^2+1)^2}$$

$$= \frac{x^2+1-2x^2}{(x^2+1)^2}$$

$$= \frac{1-x^2}{(x^2+1)^2}$$

Turning pts at $y' = 0$

$$0 = \frac{1-x^2}{(x^2+1)^2}$$

$$0 = (1-x)(1+x)$$

$x = 1 \qquad x = -1$

$y = \frac{1}{2} \qquad y = -\frac{1}{2}$

x	-2	-1	0	1	2
y'	$-\frac{3}{25}$	0	1	0	$-\frac{3}{25}$
	\	—	/	—	\

max   min   ← By table or y''

ii) Inflexion at $y''=0$

$$y' = \frac{1-x^2}{(x^2+1)^2} \qquad u = 1-x^2 \qquad v = (x^2+1)^2$$
$$u' = -2x \qquad v' = 2(x^2+1)(2x)$$

$$y'' = \frac{(x^2+1)(-2x) - (1-x^2)(2(x^2+1)(2x))}{(x^2+1)^2} = 0$$

$$2x(x^2+1)[-(x^2+1) - 2(1-x^2)] = 0$$

$$2x(x^2+1)(x^2-3) = 0$$

$x=0$ $\qquad$ $x = \pm\sqrt{3}$ $\qquad$ $\left(y = \pm\frac{1}{8}\right)$

iii) $y = \dfrac{x}{x^2+1}$ $\qquad$ $\lim\limits_{x\to\infty} \dfrac{x}{x^2+1} = \lim\limits_{x\to\infty} \dfrac{x/x^2}{x^2/x^2 + 1/x^2}$

$$= \lim_{x\to\infty} \frac{1/x}{1 + 1/x^2}$$

$$= 0$$

Horizontal asymptote is $y=0$

iv)