# NORTH SYDNEY BOYS HIGH SCHOOL

## 2015 YEAR 12 ASSESSMENT TASK 1

# Mathematics Extension 1

### General Instructions

- Working time – 55 minutes
- **Reading time – 5 minutes**

- Write on the lined paper in the booklet provided
- Write using blue or black pen
- Board approved calculators may be used
- All necessary working should be shown in every question
- Each new question is to be started on a **new page.**

Class Teacher: Please colour

- O Mr Ireland
- O Mr Lam
- O Mr Lin
- O Ms Ziaziaris
- O Mr Berry
- O Mr Zuber

Student Number:__________________________

(To be used by the exam markers only.)

| Question No | 1-5 | 6 | 7 | 8 | 9 | 10 | 11 | Total | Total |
|---|---|---|---|---|---|---|---|---|---|
| Mark | /5 | /8 | /6 | /6 | /6 | /10 | 5/6 | /47 | /100 |

# HSC 2015 ASSESSMENT TASK 1 – MATHEMATICS EXTENSION 1

**QUESTIONS 1 -5 are to be answered on the multiple choice answer sheet provided.**

**QUESTION 1**

$\sum_{r=1}^{25} 2r - 14 =$

(A) 300
(B) 288
(C) 462.5
(D) 450

**QUESTION 2**

The remainder when $x^4 - 3x^2 + 4x + 1$ is divided by $x - 2$ is

(A) -3
(B) 1
(C) 2
(D) 13

**QUESTION 3**

Which of the following is true for all values of $a$?

(A) $\int_{-a}^{a} 2x^4 dx = 0$

(B) $\int_{-a}^{a} 2x^3 dx = 0$

(C) $\int_{-a}^{a} (x^2 - 1) dx = 0$

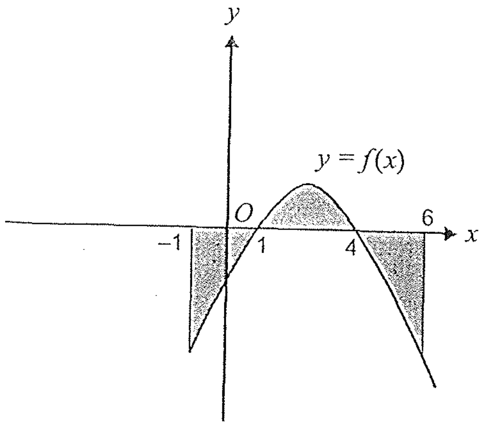
(D) $\int_{-a}^{a} |x| dx = 0$

**QUESTION 4**

Which of the following is TRUE?

(A) The zero polynomial does not have an infinite number of zeros.
(B) If a quartic has two distinct real zeros, then they must be double zeros.
(C) If a quartic polynomial has 4 distinct real zeros then it can be factorised into four real linear factors.
(D) $\sqrt{7}x - 5$ is not a polynomial.

## QUESTION 5




$\int_{-1}^{6} f(x)dx =$

(A) $-\int_{-1}^{0} f(x)dx + \int_{0}^{6} f(x)dx$

(B) $\int_{1}^{4} f(x)dx + 2\int_{4}^{6} f(x)dx$

(C) $-\int_{-1}^{1} f(x)dx + \int_{1}^{4} f(x)dx - \int_{4}^{6} f(x)dx$

(D) $-\int_{1}^{-1} f(x)dx + \int_{1}^{4} f(x)dx - \int_{6}^{4} f(x)dx$

**QUESTIONS 6 – 11 are to be answered in the answer booklet provided.**

**QUESTION 6 (Start a NEW page)**

(a) Find the linear factors of $x^3 + 7x^2 + 14x + 8$ 3

(b) The polynomial $P(x) = x^3 - 4x^2 + kx + 12$ has zeros $\alpha, \beta, \gamma$.

Find (i) $\alpha + \beta + \gamma$ 1

(ii) $\alpha\beta\gamma$ 1

(iii) the third zero given that two of the zeros are equal in magnitude but opposite in sign. 1

(iv) the value of $k$. 2

**QUESTION 7 (Start a NEW page)**

(a) Sketch $y = x^3(x+1)(2-x)^2$ 3

(b) Let $P(x) = (x-1)(x+1)Q(x) + ax + b$

where $Q(x)$ is a polynomial and $R(x) = ax + b$ is the remainder.
Given that $(x+1)$ is a factor of $P(x)$ and when
$P(x)$ is divided by $(x-1)$, the remainder is 6, find the remainder when
$P(x)$ is divided by $(x-1)(x+1)$. 3

**QUESTION 8 (Start a NEW page)**

(a) $\int \left(1+\frac{1}{2}x\right)^5 dx$ 2

(b) $\int \frac{du}{u\sqrt{u}}$ 2

(c) A train moving between two points in a straight line records the following velocities in m/s at intervals of 10s.

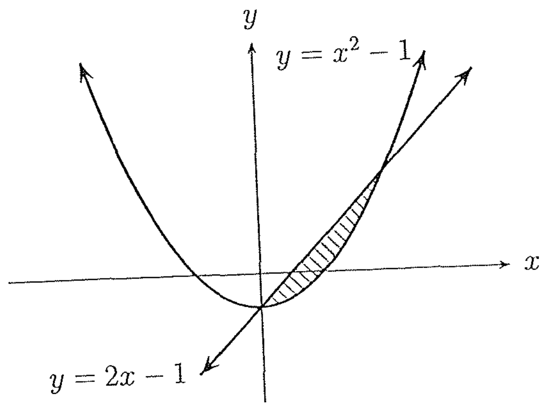
| | | | | | |
|---|---|---|---|---|---|
| t | 0 | 10 | 20 | 30 | 40 |
| v | 0 | 15 | 39 | 45 | 45 |

Use Simpson's Rule and five function values

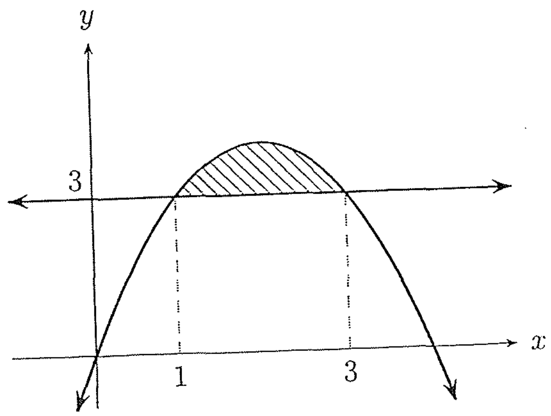
to approximate $\int_0^{40} v\,dt$. 2

**QUESTION 9 (Start a NEW page)**

(a) Find the area enclosed by $y = x^2 - 1$ and $y = 2x - 1$. 3




(b) Find the volume of the solid obtained by rotating about the X-axis the region enclosed by $y = 4x - x^2$ and the line y=3. 3



**QUESTION 10 (Start a NEW page)**

(a) Find the sum of the geometric series $\frac{1}{2}-1+2-4$......... to 15 terms. 2

(b) If $S_n = 3^n + 3n$, find the twelfth term. 2

(c) Find the first term which is less than 0.01 for the geometric series

$333+111+\frac{111}{3}+$.......... 3

(d) (i) Show that the series below is geometric.

$(x)+(1)+\left(\frac{1}{x}\right)+\left(\frac{1}{x^2}\right)+$......... 1

(ii) Find the values of $x$ for which the series above is to have a sum to infinity. 2

**QUESTION 11 (Start a NEW page)**

(a) (i) Use the remainder theorem to find one factor of

$P(x) = x(x+m) - n(n+m)$ 1

(ii) By division or otherwise, find the other factor. 2

(b) (i) Show that the derivative of $x^{m+1}(1-x)^n$ is equal to

$(m+1)x^m(1-x)^n - nx^{m+1}(1-x)^{n-1}$ 1

(ii) Deduce that $\int_0^1 x^m(1-x)^n\,dx = \frac{n}{m+1}\int_0^1 x^{m+1}(1-x)^{n-1}\,dx$ 2

**END OF EXAMINATON**

## STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1} + C, \qquad n \neq -1; \quad x \neq 0 \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x + C, \qquad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax} + C, \qquad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax + C, \qquad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax + C, \qquad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax + C, \qquad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax + C, \qquad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + C, \qquad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a} + C, \qquad a > 0, -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right) + C, \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right) + C$$

NOTE: $\ln x = \log_e x, \; x > 0$

2015 HSC ASSESS TSK 1 – EXT 1 SOLUTIONS

(1) A (2) D (3) B (4) C (5) D

6. (a) $x^3 + 7x^2 + 14x + 8$

Test using factor theorem:

$P(-1) = -1 + 7 - 14 + 8$

$= 0$

$\therefore$ $x+1$ is a factor. ✓

$$\begin{array}{r|l} & x^2 + 6x + 8 \\ \hline x+1 & x^3 + 7x^2 + 14x + 8 \\ & x^3 + x^2 \\ \hline & 6x^2 + 14x \\ & 6x^2 + 6x \\ \hline & 8x + 8 \\ & 8x + 8 \\ \hline & 0 \end{array}$$

(or equivalent working) ✓

$\therefore P(x) = (x+1)(x^2 + 6x + 8)$

$= (x+1)(x+2)(x+4)$. # ✓

(b) $P(x) = x^3 - 4x^2 + kx + 12$ has zeros $\alpha, \beta, \gamma$.

(i) $\alpha + \beta + \gamma = \frac{-(-4)}{1} = \boxed{4}$ ✓

(ii) $\alpha\beta\gamma = \boxed{-12}$ ✓

(iii) Let zeros be $\alpha, -\alpha, \beta$

$\therefore$ $\alpha + (-\alpha) + \beta = 4$ $\therefore$ $\beta = 4$

i.e. 3rd zero is $\boxed{4}$ ✓

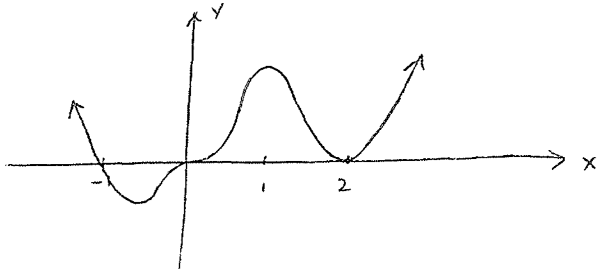
(iv) If one zero is 4, then

$4^3 - 4(4^2) + 4k + 12 = 0$ ✓

$4k = -12$

$\boxed{k = -3}$ ✓

(7) a) $y = x^3(x+1)(2-x)^2$




b) $P(x) = (x-1)(x+1).Q(x) + ax + b$

$P(-1) = 0$

$P(1) = 6$

$0 = -a + b$ — (1)

$6 = a + b$ — (2)

$2b = 6$

$b = 3$

$\therefore a = 3$

$\therefore$ Remainder $= 3x + 3$.

(8) a) $\int (1 + \frac{1}{2}x)^5 \, dx$

$= \frac{(1 + \frac{1}{2}x)^6}{6 \cdot \frac{1}{2}} + C$

$= \frac{(1 + \frac{1}{2}x)^6}{3} + C$

b) $\int u^{-3/2} \, du$

$= \frac{u^{-1/2}}{-\frac{1}{2}} + C$

$= -\frac{2}{\sqrt{u}} + C$

c) $\int_0^{40} v\,dt = \frac{10}{3}\{f(0) + 4[f(10) + f(30)] + 2f(20) + f(40)\}$

$= \frac{10}{3}\{0 + 4(15+45) + 2\times 39 + 45\}$

$= 1210m$

9. a) Points of Intersection

$x^2 - 1 = 2x - 1$

$x^2 - 2x = 0$

$x(x-2) = 0$

$x = 0, 2$

$A = \int_0^2 (2x-1) - (x^2-1)\,dx$

$= \int_0^2 (2x - x^2)\,dx$

$= \left[x^2 - \frac{x^3}{3}\right]_0^2$

$= 4 - \frac{8}{3}$

$= \frac{4}{3}$ sq u

b) $V = \pi\int_1^3 (4x-x^2)^2\,dx - \pi\int_1^3 3^2\,dx$

$= \pi\int_1^3 (16x^2 - 8x^3 + x^4)\,dx - \pi\int_1^3 9\,dx$

$= \pi\left[\frac{16x^3}{3} - 2x^4 + \frac{x^5}{5}\right]_1^3 - \pi\left[9x\right]_1^3$

$= \pi\left[16\times 9 - 2\times 81 + \frac{3^5}{5} - \left(\frac{16}{3} - 2 + \frac{1}{5}\right)\right] - \pi[27-9]$

$= \frac{136\pi}{15}$ c.u.

(10) a) $S_{15} = \frac{a(r^n-1)}{r-1}$

$= \frac{\frac{1}{2}(-2^{15}-1)}{-3}$

$= 5461.5$

b) $T_{12} = S_{12} - S_{11}$

$= (3^{12} + 36) - (3^{11} + 33)$

$= 354297$

c) $T_n < 0.01$

$ar^{n-1} < 0.01$

$333\left(\frac{1}{3}\right)^{n-1} < 0.01$

$\frac{1}{3}^{n-1} < \frac{0.01}{333}$

$(n-1)\log\frac{1}{3} < \log\left(\frac{0.01}{333}\right)$

$n - 1 < \frac{\log\left(\frac{0.01}{333}\right)}{\log\frac{1}{3}}$

$n < \frac{\log\left(\frac{0.01}{333}\right)}{\log\frac{1}{3}} + 1$

$n > 10.47$

$\therefore n = 11$

$\therefore T_{11} = 333\left(\frac{1}{3}\right)^{10}$

$= 0.0056$

d) (i) If geometric

$$\frac{T_2}{T_1} = \frac{T_3}{T_2}$$

$LHS = \frac{1}{x}$

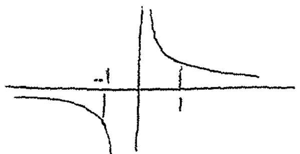
$RHS = \frac{\frac{1}{x}}{1}$

$= \frac{1}{x}$

$LHS = RHS$

(ii) Limiting Sum exist when

$-1 < \frac{1}{x} < 1$




True for $x > 1$, $x < -1$

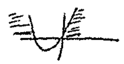
OR algebraically

$\frac{1}{x} < 1$ and $\frac{1}{x} > -1$

$x < x^2$     $x > -x^2$

$x - x^2 < 0$     $x + x^2 > 0$

$x(1-x) < 0$     $x(1+x) > 0$



$x > 1$, $x < -1$

(11) a) (i) $P(n) = n(n+m) - n(n+m)$

$= n^2 + mn - n^2 - mn$

$= 0$

$\therefore x - n$ is a factor.

(ii)

$$\begin{array}{r} x + m + n \\ x - n \overline{) \, x^2 + mx - n^2 - mn} \\ \underline{x^2 - nx \qquad\qquad} \\ (m+n)x - n^2 - mn \\ \underline{(m+n)x - n^2 - mn} \\ 0 \end{array}$$

$\therefore P(x) = (x-n)(x+m+n)$

OR $x^2 + mx - n^2 - mn$

$= (x-n)(x+n) + m(x-n)$

$= (x-n)(x+n+m)$

b) (i) Using Product Rule,

$$\frac{d}{dx} x^{m+1}(1-x)^n = (1-x)^n . (m+1)x^m + x^{m+1} . n(1-x)^{n-1}(-1)$$

$$= (m+1)x^m(1-x)^n - nx^{m+1}(1-x)^{n-1}$$

(ii) From part (i)

$$\int [(m+1)x^m(1-x)^n - nx^{m+1}(1-x)^{n-1}]\,dx = x^{m+1}(1-x)^n$$

ie. $\int (m+1)x^m(1-x)^n dx - \int nx^{m+1}(1-x)^{n-1}dx = x^{m+1}(1-x)^n$

$$\therefore (m+1)\int_0^1 x^m(1-x)^n dx - n\int_0^1 x^{m+1}(1-x)^{n-1}dx = [x^{m+1}(1-x)^n]_0^1$$

$$(m+1)\int_0^1 x^m(1-x)^n dx = n\int_0^1 x^m(1-x)^{n-1}dx + [x^{m+1}(1-x)^n]_0^1$$

$$= n\int_0^1 x^m(1-x)^{n-1}dx + [1(1-1)^n - 0]$$

$$= n\int_0^1 x^m(1-x)^{n-1}dx + 0$$

$$\therefore \int_0^1 x^m(1-x)^n dx = \frac{n}{m+1}\int_0^1 x^m(1-x)^{n-1}dx.$$