NORTH SYDNEY GIRLS HIGH SCHOOL

HSC MATHEMATICS EXTENSION 1 ASSESSMENT TASK

2005 – TERM 4

Time Allowed: 1 hour + 2 minutes reading time

Instructions:

- Start each question on a new page
- Write on one side of the paper only, work down the page and do not work in columns
- Leave a margin on the left hand side of the page
- Show all necessary working
- Marks may not be awarded for untidy or poorly arranged work
- Diagrams are not drawn to scale
- There are six questions
- Marks are as indicated

This task is worth 10% of the HSC Assessment Mark

Name / Number:______________________________________

Question 1: **(11 marks)** **Marks**

a) Differentiate with respect to x:

i) $y = e^{2-x}$ 1

ii) $y = \dfrac{\log_e x}{x}$ 2

b) What is the domain of $\log_e(2-x)$? 1

c) Write down a primitive function of

i) $\dfrac{4x+1}{x}$ 1

ii) e^{3x+2} 1

d) Find, in degrees and minutes, the angle subtended by an arc of length 10 cm at the centre of a circle radius 6 cm. 2

e) A chord of length 1 metre subtends an angle of 1·46 radians at the centre of a circle. Find, correct to 2 decimal places, the area of the smaller segment cut off by the chord. 3

Question 2: **(9 marks)** **Start a new page**

a) Find the value of m if $\int_m^3 3x^2 dx = 19$. 3

b) i) Find the x-coordinates of the points of intersection of the curves $y = x^2$ and $y = 8 - x^2$. 1

ii) Shade on a number plane, the region bounded by the curves given in part i). 2

iii) Find the area of this region. 3

Question 3: **(9 marks)** **Start a new page**

a) Write down a primitive function of:

i) $(2x+1)^5$ 1

ii) $(3x+4)(x-2)$ 1

b) Use Simpson's Rule with 5 function values to find $\int_2^6 x\sqrt{x-1}\,dx$ correct to 2 decimal places. 2

c) Given that the gradient function of a curve is $\dfrac{dy}{dx} = x^2 - 4x - 1$ and that the curve passes through the point $(-1, 2)$, find the equation of the curve. 2

d) Find the volume of the solid of revolution formed by rotating the portion of the curve $y = x^{\frac{2}{3}}$ from $0 \le x \le 8$ about the y-axis. 3

Question 4: **(11 marks)** **Start a new page** **Marks**

The curve $y = 2x^3 + ax^2 + bx + 3$ has stationary points when $x = 1$ and $x = -2$.

a) Find the values of a and b. 4

b) Determine the nature of these stationary points. 3

c) Write down the y-coordinate of each of the stationary points. 1

d) Find, if any, the coordinates of the point(s) of inflexion. 2

e) Sketch the curve. 1

Question 5: **(10 marks)** **Start a new page**

a) The function $f(x)$ is defined by $f(x) = 12x - x^3$. Find the set of values of x for which the curve is both increasing and concave up. 4

b) A cone has a perpendicular height h cm, radius r cm and slant height 6 cm.

i) Show that the volume of the cone is given by

$$V = 12\pi h - \frac{1}{3}\pi h^3.$$ 2

ii) Show that the volume will be a maximum when $h = 2\sqrt{3}$ cm. 4

Question 6: **(10 marks)** **Start a new page**

a) Find $\int_0^1 \frac{x^2}{x^3+1}dx$ in exact form. 2

b) Find the equation of the tangent to the curve $y = e^{x^2}$ at the point $(1, e)$. 3

c) Find $\int \frac{e^{2x}-1}{e^x-1}dx$. 2

d) If $f(x) = \log\left(\frac{1+x}{1-x}\right)$ show that $f\left(\frac{2x}{1+x^2}\right) = 2f(x)$. 3

End of Test

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n<0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x>0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a>0, \quad -a<x<a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x>a>0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x>0$