SYDNEY GIRLS HIGH SCHOOL

YEAR 12 MATHEMATICS
Extension 1
2012

ASSESSMENT TASK 1

November 2011

Time allowed: 60 minutes +5 min reading

Integration and Locus

Instructions:

- There are Five (5) questions. Questions are of equal value.
- Attempt all questions.
- Show all necessary working. Marks may be deducted for badly arranged work.
- Start each question on a new page. Write on one side of the paper only.
- **Note:** $\int x^n \, dx = \frac{1}{n+1}x^{n+1}$, $n \neq -1$; $x \neq 0$, *if* $n < 0$

Name: ....

Teacher's Name ....

QUESTION ONE (12 marks)

a) Given $f'(x)=3x^2-3x+1$, find the equation of the curve which passes through $(-1,0)$. (2)

b) Find

i) $\int \frac{x^3+2x}{x^3}\,dx$ (2)

ii) $\int x(x-1)^2\,dx$ (2)

c) A and B are the points $(1,3)$ and $(5,9)$ respectively. The point $P(x,y)$ moves so that $m_{PA}\times m_{PB}=-1$. Find the locus of point P. (3)

d) The parabola $x^2=ky$ passes through the point $(-6,3)$. Find

i) The coordinates of the focus (2)

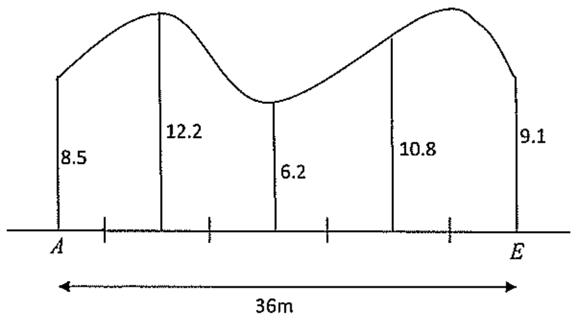
ii) The equation of the directrix (1)

QUESTION TWO (12 marks)

a) Find the area bounded by $y=3x-x^2$ and $y=3-x$. (4)

b) Find $\int \frac{dx}{(5x+3)^2}$ (2)

c) Use Simpson's rule to find an approximation of the area between the road and fence between A and E. (3)



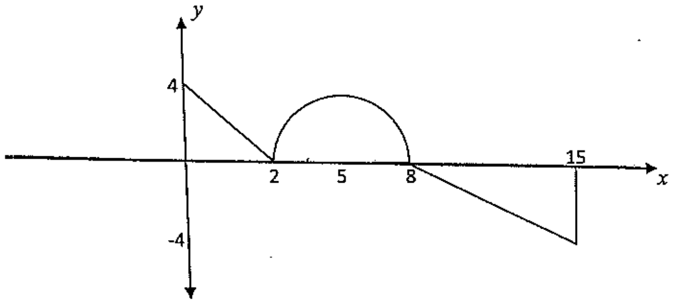
d) Evaluate

$$\int_1^4 \left(x-\frac{2}{x}\right)^2 dx$$ (3)

QUESTION THREEE (12 marks)

a) Find $\int \frac{x^3}{\sqrt{x}}\,dx$ (2)

b) The graph below shows $y = f(x)$. Evaluate $\int_0^{15} f(x)\,dx$ (2)



c) The point $P(x, y)$ moves so that $\frac{PO}{PA} = \frac{2}{1}$, where O is the origin and $A(3,0)$.

i) Find the locus of P (3)

ii) Describe this locus (1)

d) A vase is designed to hold $50\pi\ cm^3$ of water. It's shape is determined by rotating the parabola $x = \frac{y^2}{30}$ in the first quadrant about the y axis. If the height of the vase is $k\ cm$,

i) Show that $\int_0^k y^4 dy = 45000$ (2)

ii) Find the value of k (correct to the nearest whole number) (2)

QUESTION FOUR (12 marks)

a) For the parabola $y = x^2 - 6x + 4$ find

i) The coordinates of the vertex (2)

ii) The coordinates of the focus (1)

iii) The equation of the directrix (1)

b) Find the area between the curve $y = x^2 - x - 6$ the x axis, and the lines $x = 0$ and $x = 4$. (3)

c) Evaluate $\int_0^4 -\sqrt{16 - x^2}\,dx$ (2)

d) Sketch the parabola $(x+1)^2 = 6(6 - 2y)$ showing the co-ordinates of the focus and the equation of the directrix. (3)

QUESTION FIVE (12 marks)

a)

i) Find the equation of the tangent to the parabola $x^2 = 4y$ at $x = 4$. (2)

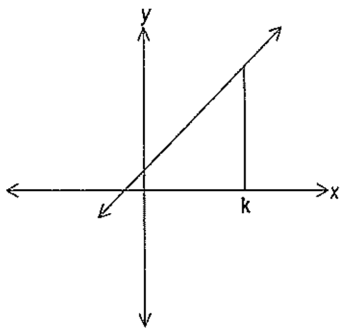
ii) Find the area enclosed by the curve , the tangent and the x axis. (3)

b) Find the volume of the solid of revolution if the area bounded by the curves $y = x^2$ and $y = 4 - x^2$ rotates around the y axis. (3)

c) The shaded area in the diagram is bounded by the x axis , the y axis , the line $y = 5x + 1$ and the $x = k$. This area equals to $44\,u^2$.

i) Find the value of k (2)

ii) This area is rotated about the y axis. Find the volume of the solid generated. (2)



THE END

Year 12, Ext 1 2011

1) a)

$f(x) = \frac{3x^3}{3} - \frac{3x^2}{2} + x + C$

$0 = -1 - \frac{3}{2} - 1 + C$

$0 = -\frac{7}{2} + C$

$C = \frac{7}{2}$

$\boxed{f(x) = x^3 - \frac{3x^2}{2} + x + \frac{7}{2}}$

b) i) $\int 1 + \frac{2}{x^2}\, dx$

$\int 1 + 2x^{-2}\, dx$

$= x + \frac{2x^{-1}}{-1} + C$

$= \boxed{x - \frac{2}{x} + C}$

ii) $\int x(x^2 - 2x + 1)\, dx$

$\int x^3 - 2x^2 + x$

$= \boxed{\frac{x^4}{4} - \frac{2x^3}{3} + \frac{x^2}{2} + C}$

c) $\frac{y-3}{x-1} \times \frac{y-9}{x-5} = -1$

$\frac{y^2 - 9y - 3y + 27}{x^2 - 5x - x + 5} = -1$

$y^2 - 12y + 27 = -x^2 + 6x - 5$

$x^2 - 6x + y^2 - 12y + 32 = 0$

$x^2 - 6x + 9 + y^2 - 12y + 36 = -32 + 45$

$\boxed{(x-3)^2 + (y-6)^2 = 13}$

d) $x^2 = ky$

$36 = k \times 3$

$k = 12$

$4a = 12$

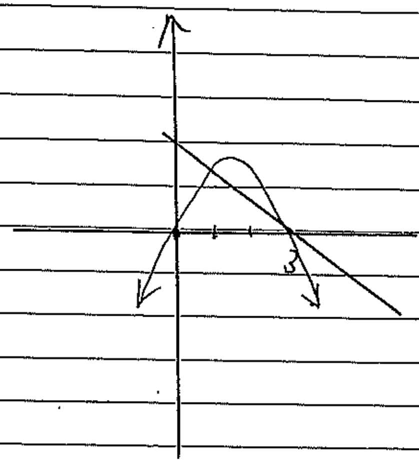
$a = 3$

i) $F\ (0, 3)$

ii) $y = -3$

Q2

a)



$y = x(3-x)$

$y = 3 - x$

$3 - x = 3x - x^2$

$x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

$x = 3$ or 1

$A = \int_1^3 3x - x^2 - (3-x)\, dx$

$= \int_1^3 3x - x^2 - 3 + x\, dx$

$= \int 4x - x^2 - 3\, dx$

$= \left[\frac{4x^2}{2} - \frac{x^3}{3} - 3x\right]_1^3$

$= \left[18 - 9 - 9\right] - \left[2 - \frac{1}{3} \ldots\right.$

$= -\left(-\frac{4}{3}\right)$

$= \frac{4}{3}\ u^2$

b) $\int (5x+3)^{-2}$

$= \frac{(5x+3)^{-1}}{-1 \times 5} + C$

$= -\frac{1}{5(5x+3)} + C$

c) $A \doteq \frac{9}{3} \times 122$

$\doteq 366\ u\ldots$

x	$f(x)$	w	$f(x)w$
0	8.5	1	8.5
9	12.2	4	48.8
18	6.2	2	12.4
27	10.8	4	43.2
36	9.1	1	9.1

$\Sigma = 122$

d) $\int_1^4 x^2 - 4 + 4x^{-2}\, dx$

$= \left[\frac{x^3}{3} - 4x - \frac{4x^{-1}}{1}\right]$

$= \left[\frac{x^3}{3} - 4x - \frac{4}{x}\right]_1^4$

Q3

a) $\int x^3 \times x^{-\frac{1}{2}}\, dx$

$= \int x^{\frac{5}{2}}\, dx$

$= \left[\frac{2x^{\frac{7}{2}}}{7} + C\right]$

b) $\int_0^{15} f(x)\, dx = \frac{1}{2}4\times2 + \frac{1}{2}\pi\times3^2 - \frac{1}{2}4\times7$

$= 4 + \frac{9\pi}{2} - 14$

$= \frac{9\pi}{2} - 10$

c) i) $PO = 2PA$

$\sqrt{(x-0)^2+(y-0)^2} = 2\sqrt{(x-3)^2+(y-0)^2}$

$\sqrt{x^2+y^2} = 2\sqrt{(x-3)^2+y^2}$

$x^2+y^2 = 4\left[(x^2-6x+9)+y^2\right]$

$x^2+y^2 = 4x^2-24x+36+4y^2$

$3x^2+3y^2-24x+36=0$

$x^2+y^2-8x=-12$

$x^2-8x+16+y^2=-12+16$

$(x-4)^2+y^2=4$

ii) circle

$C(4,0)$

$r=2$

d)

$V = \pi\int\left(\frac{y^2}{30}\right)^2 dy$

$50\pi = \pi\int_0^k \frac{y^4}{900}\, dy$

$\therefore$

$\int_0^k y^4\, dy = 900\times50$

$\int_0^k y^4\, dy = 45000$

$\left[\frac{y^5}{5}\right]_0^k = 45000$

$\frac{k^5}{5} = 45000$

$k^5 = 225000$

$k = 12$

Q4) a)

$y = x^2-6x+4$

$y-4 = x^2-6x$

$x^2-6x+9 = y-4+9$

$(x-3)^2 = y+5$

$V(3,-5)$

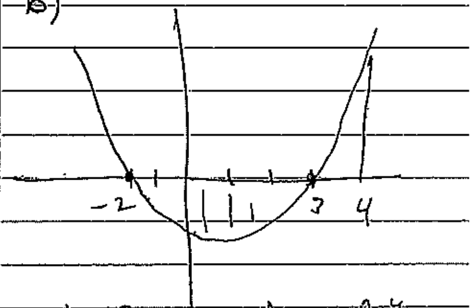
$4a=1$

$a=\frac{1}{4}$

$F(3,-4\frac{3}{4})$

$y=-5\frac{1}{4}$

b) $(x-3)(x+2)$



$A = \left|\int_0^3 x^2-x-6\right| + \int_3^4 x^2-x-$

$\left|\left[\frac{x^3}{3}-\frac{x^2}{2}-6x\right]_0^3\right| + \left[\frac{x^3}{3}-\frac{x^2}{2}\right.$

$= \left|\left(\frac{27}{3}-\frac{9}{2}-18\right)\right| + \left[\left(\frac{64}{3}-8-24\right)-\right.$ $\left(9-\frac{9}{2}-18\right)$

$= \frac{27}{2} + \frac{17}{6}$

$= \frac{49}{3}$

$= 16\frac{1}{3}\ u^2$

c) $\int_0^4 -\sqrt{16-x^2}\,dx$

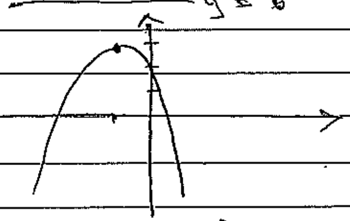
$= -\frac{1}{4}\pi \times 4^2$

$= -4\pi$

d) $(x+1)^2 = -12(y-3)$

$4a = 12$

$a = 3$



$V = (-1, 3)$

$F = (-1, 0)$

directrix

$y = 6$

5) a)

$y = \frac{x^2}{4}$

$y' = \frac{2x}{4} = \frac{x}{2}$

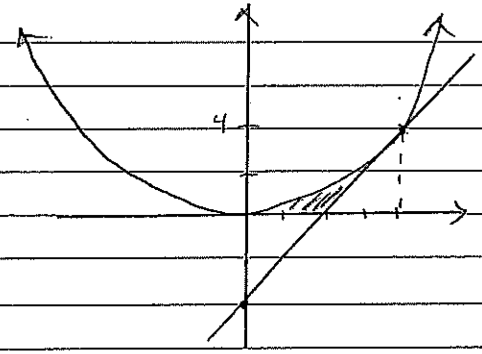
at $x = 4$ $m = 2$

at $x = 4$ $y = 4$

$y - 4 = 2(x - 4)$

$y = 2x - 8 + 4$

$\boxed{y = 2x - 4}$



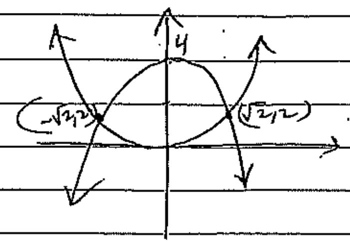
$A = \int_0^4 \frac{x^2}{4}\,dx - \frac{1}{2}\times 4 \times 2$

$= \left[\frac{x^3}{12}\right]_0^4 - 4$

$= 5\frac{1}{3} - 4$

$= 1\frac{1}{3}$

b)



$x^2 = 4 - x^2$

$2x^2 = 4$

$x^2 = 2$

$x = \pm\sqrt{2}$

$x = \sqrt{2} \therefore y = 2$

$V = \pi\int_0^2 y\,dy + \pi\int_2^4 4 - y\,dy$

$= \pi\left[\frac{y^2}{2}\right]_0^2 + \pi\left[4y - \frac{y^2}{2}\right]_2^4$

$= 2\pi + \pi\left(16 - 8 - (8 - 2)\right)$

$= 2\pi + \pi(2)$

$= 4\pi\ u^3$

c)

i) $A = \int_0^k 5x + 1\,dx$

$44 = \left[\frac{5x^2}{2} + x\right]_0^k$

$44 = \frac{5k^2}{2} + k$

$88 = 5k^2 + 2k$

$5k^2 + 2k - 88 = 0$

$5k^2 + 22k - 20k - 88 = $ [illegible]

$k(5k + 22) - 4(5k + 22)$

$(k - 4)(5k + 22) = 0$

$k = 4$ or $\frac{-22}{5}$

$k = 4 \rightarrow y$ [illegible]

ii)

$V = \pi \times 4^2 \times 21$

$\pi\int_1^{21}\left(\frac{y-1}{5}\right)^2$ [illegible]

$= 336\pi - \pi\left[\frac{(y-}{7}\right.$ [illegible]

$= 336\pi - 106\frac{2}{3}$

$= 229\frac{1}{3}\pi$