Sydney Girls High School

2016

YEAR 12 HSC ASSESSMENT TASK 1

MATHEMATICS EXTENSION 1

Time Allowed: 60 minutes + 5 minutes reading time

Topic: Parametrics, Trigonometry II and Integration

Total: 56 marks

General Instructions:

- There are Seven questions which are of equal value.
- Attempt all questions.
- Show all necessary working. Marks may be deducted for badly arranged work or incomplete working.
- Start each Question on a new page.
- Write on one side of the paper only.
- Diagrams are NOT to scale.
- Board-approved calculators may be used.
- Write your name clearly at the top of each question and clearly number each question.
- A reference sheet is provided with this paper.

Student Name :_____ **Teacher Name :**_____

Question One (8 marks)

a) Find

i) $\int (3x^3 - 5\ )dx$ (1)

ii) $\int \frac{4x^4 - 5}{x^2}\, dx$ (2)

iii) $\int (7 - 2x)^9\ dx$ (1)

b) If $\cos 2x = \frac{1}{5}$, find the exact value of $\sin x$ (where x is acute). (2)

c) Find the acute angle between the lines $3x - y - 2 = 0$ and $y = 5 - 7x$ to the nearest degree. (2)

Question Two (8 marks)

a) Find $\int (3 + 2x^{-2})^2\ dx$. (2)

b) Solve $4\sin\theta\cos\theta = -1$ where $0 \le \theta \le 360^\circ$. (3)

c) By eliminating t find the Cartesian equation for the following. (3)
Express the equation in a simplified form.

$x = \frac{1}{t+1}$, $y = \frac{2t}{t+1}$

Question Three (8 marks)

a) Evaluate $\int_2^4 2x\sqrt[3]{x}\,dx$. (3)

b) A chord of contact to the parabola $x^2 = 4y$ has the equation $y = x + 3$. Determine the external point from which the tangents are drawn. (2)

c) If $\sec A = \sin B + \cos B$ show that $\tan^2 A = \sin 2B$. (3)

Question Four (8 marks)

a) Given $7\cos x + 24\sin x = R\cos(x - \alpha)$, where $R > 0$ and $0^\circ \le \alpha \le 90^\circ$.

i) Find the value of R. (1)

ii) Find the value of α correct to the nearest degree. (1)

iii) Hence, solve $7\cos x + 24\sin x = 10$ for $0^\circ \le x \le 360^\circ$. (2)

b) The points $A(6p, 3p^2)$ and $B(6q, 3q^2)$ lie on the parabola $x^2 = 12y$.

i) Find the equation of chord AB. (1)

ii) Write down the co-ordinates of M, the midpoint of AB. (1)

iii) Find the equation of the locus of M, given AB is a focal chord. (2)

Question Five (8 marks)

a) $P(2ap, ap^2)$ is a point on the parabola $x^2 = 4ay$ with focus S. The tangent and normal at P meet the $y-$axis at T and N respectively.

i) State the equation of the tangent at P. (1)

ii) State the equation of the normal at P. (1)

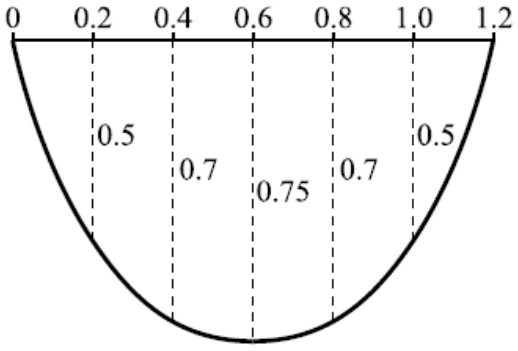
iii) Show that P lies on a circle with diameter TN. (3)

b) i) Sketch the graph of $y = 2 - |x|$. (1)

ii) Hence find the value of $\int_1^5 2 - |x| \, dx$. (2)

Question Six (8 marks)

a) Use Simpson's rule to approximate the area of the ditch shown below correct to 2 decimal places. (2)



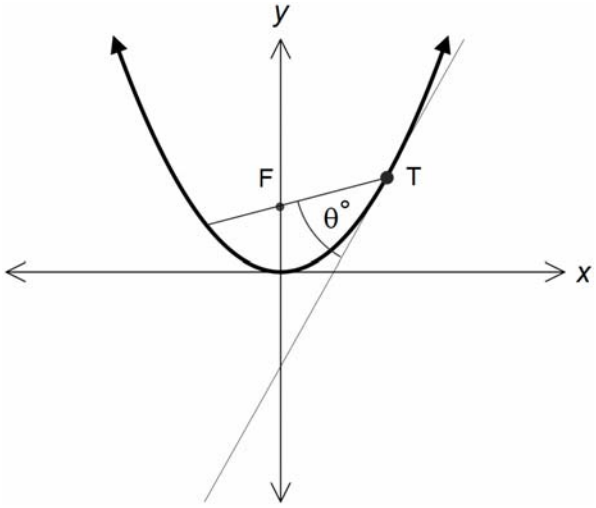
b) Solve $\sin x - 2\cos x = 2$, using t formulas for $0 \le x \le 360^\circ$. (3)
(where $t = \tan\frac{\theta}{2}$)

c) Find the area of the region bounded by the curve $y = (x-2)^2$, the line $y = 2x - 1$ and the $x-$axis. (3)

Question Seven (8 marks)

a) The area bounded by the curve $y = x^2 + 4$ and the line $y = 8$, is rotated around the $x-$axis. Find the volume of the solid of revolution. (3)

b) $T(2t, t^2)$ is a point on the parabola $x^2 = 4y$ with focus F. The tangent to the parabola at T makes an acute angle θ with line FT.



i) Find the gradient of the tangent at T. (1)

ii) Find $\tan\theta$ in simplest form in terms of t. (2)

c) Show that if $2\cos\alpha = k + \frac{1}{k}$ then $2\cos 3\alpha = k^3 + \frac{1}{k^3}$. (2)

(You may use the identity $\cos 3x = 4\cos^3 x - 3\cos x$.)

The End