Name:

Teacher:

Sydney Technical High School

Year 11 Extension 1 Mathematics

HSC Assessment Task 1

December 2004

Time Allowed : 70 Minutes

Directions:

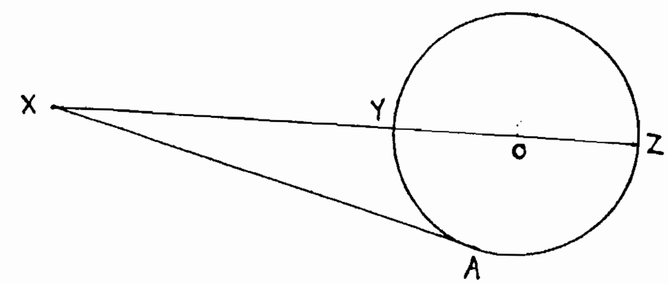
- Attempt all questions
- Marks indicated for each question are a guide and may be changed slightly if necessary.
- Start each question on a new side of the sheet.
- Do not work in two columns.
- Marks may not be given for careless or badly arranged work

Q1	Q2	Q3	Q4	Q5	Q6	Total

Question 1

(a) If the 8^{th} term of an AP is 52 and the 14^{th} term is 88, find the value of the 1^{st} term and the common difference.

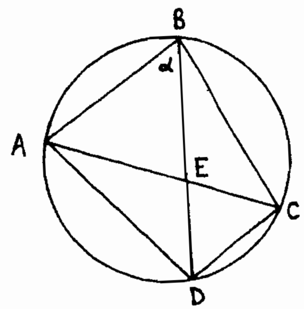
(b) O is the centre of the circle, XA is a tangent. If XA = 40 and the diameter of the circle is 16, find XY correct to one decimal place.



c) Using the fact that the positive multiples of 7 form an Arithmetic Progression, find how may multiples of 7 lie between 700 and 7000.

Question 2 (start a new page)

(a) In the diagram below, A,B,C and D are concyclic points. Diagonals BD and AC intersect at E. Copy the diagram onto your page. If DB bisects $\angle ABC$, prove that $AD=DC$ without adding any constructions to the diagram. **3**



(b) The sum of the first n terms of a certain series is given by

$$S_n = \frac{n}{3}(n+1)(n+2)$$

(i) Show that the nth term is given by $T_n = n(n+1)$ **3**

(ii) Find $\sum_{n=51}^{100} n(n+1)$ **2**

(c) In the diagram below, ΔABC is isosceles, M is the midpoint of the line NY and $XM \| AB$. Copy the diagram onto your page. By using similar triangles, or otherwise, show that $\frac{MX}{NB} = \frac{1}{2}$. **2**

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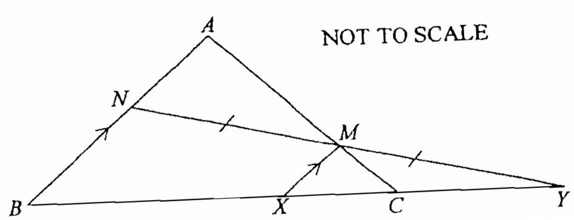


Question 3 (start a new page)

a) (i) Prove by Mathematical Induction that

$$\frac{1}{1\times 3} + \frac{1}{3\times 5} + \frac{1}{5\times 7} + \dots\dots\dots\dots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}.$$

(ii) Hence evaluate $\lim_{n\to\infty}\left[\frac{1}{1\times 3} + \frac{1}{3\times 5} + \frac{1}{5\times 7} + \dots\dots + \frac{1}{(2n-1)(2n+1)}\right]$.

(b) By using the limiting sum formula for a geometric progression, express $0.3\dot{1}\dot{7}$ as a fraction.

Question 4 (start a new page)

(a) An amount of money is invested for 6 years. The interest rate is 8% p.a. and is compounded quarterly. After 6 years the original amount has compounded to \$12,000. Find the amount of interest earned.

(b) A parent makes a one off investment of \$P to set up a fund which will provide their child with a payment of \$500 every year for 15 years. The investment earns interest at the rate of 8% p.a. compound annually. The first payment is made one year after the fund is set up and payments are always to be made each year after interest had been added.

(i) If the amount remaining in the fund after n years, is given by \$$A_n$, show that **2**

$$A_2 = P\times 1.08^2 - 500(1+1.08)$$

(ii) Hence show that $A_n = 6250 - 1.08^n(6250 - P)$ **4**

(iii) Hence find \$P, the amount of the original investment if the fund runs out with the 15^{th} payment. **2**

Question 5 (start a new page)

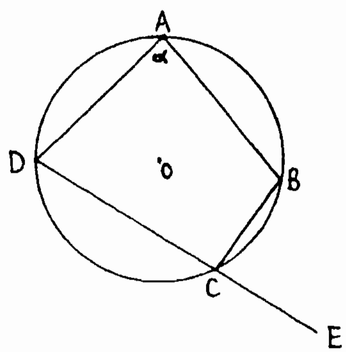
(a) How many terms of the series **3**

$45+47+49+\ldots$ are needed to give a sum of 1365?

(b) How many terms of the series below are needed to give a sum greater than 1000? **2**

$\frac{3}{4}+2\frac{1}{4}+6\frac{3}{4}+\ldots\ldots\ldots$

(c)



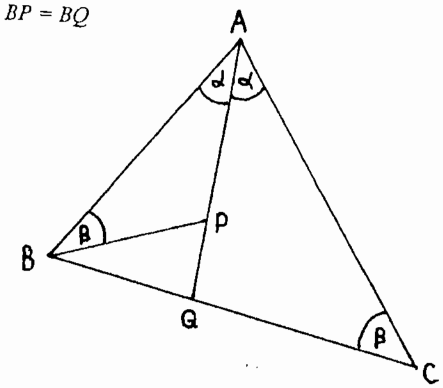
(i) Copy the above diagram onto your page and prove that the opposite angles of a cyclic quadrilateral are supplementary. **4**

(ii) Hence show why the exterior angle of a cyclic quadrilateral is equal to the interior opposite angle. **1**

Question 6 (start a new page)

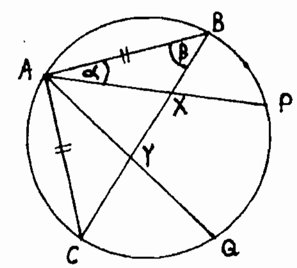
(a) In the diagram below, $\angle BAQ = \angle QAC = \alpha$ and $\angle ABP = \angle QCA = \beta$

Prove that $BP = BQ$



(b) From a piece of wire 75m long, 30 pieces are cut. Each piece is 10cm longer than the previous one. The wire is used up exactly by the 30 pieces. How long are the shortest and longest pieces?

(c)



Copy the diagram and

(i) Prove that $\angle BQP = \alpha$ 1

(ii) Prove that $\angle BQA = \beta$ 1

(iii) Prove that PQYX is a cyclic quadrilateral 2

End of Questions

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Solution to HSC Ass. Task 1 - Dec. 2004

Question 1

(a) $T_8 = a + 7d = 52$ (1)

$T_{14} = a + 13d = 88$ (2)

(2) − (1) $\Rightarrow$

$6d = 36$

$d = 6$ (1)

$\therefore$ $a = 10$ (1)

(1)

(b) $XA^2 = XY \times XZ$ (1)

$40^2 = x(x+16)$

$x^2 + 16x - 1600 = 0$

$x = \frac{-16 \pm \sqrt{256 - 4\times1\times-1600}}{2}$ (1)

$= \frac{-16 \pm \sqrt{6656}}{2}$

$= \frac{-16 \pm 81.6}{2}$ (1)

$XY = 32.8$ as $x > 0$

need to menti for 3rd mark

(c) 7, 14, 21 ...

$T_n = 7 + (n-1)7$

$T_n = 7n$

$700 < 7n < 7000$ (1)

$100 < n < 1000$

$\therefore$ from 101 to 999 (1)

$\therefore$ 999 − 100 terms

equals 899 terms (1)

Question 2

(a) $\angle DBC = \alpha$ (BD bisects $\angle ABC$ given)

$\angle ACD = \alpha$ (angle in same segment) (1)

$\angle DAC = \alpha$ (angle in same segment) (1)

$\therefore$ $AD = DC$ (opposite sides equal opposite equal angles in ΔADC) (1)

(b)(i) $S_n = \frac{n}{3}(n+1)(n+2)$

(1) $T_n = S_n - S_{n-1}$

$= \frac{n}{3}(n+1)(n+2) - \frac{n-1}{3}(n)(n+1)$

$= \frac{n+1}{3}[n(n+2) - n(n-1$

(1) $= \frac{n+1}{3}[n^2 + 2n - n^2 + n$

$= \frac{n+1}{3}(3n)$

(1) $= n(n+1)$ as req

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(ii) $\sum_{n=51}^{100} n(n+1)$

$= S_{100} - S_{50}$ (1)

$= \frac{100}{3}(101)(102) - \frac{50}{3}(51)(52)$

$= 299\,200$ (1)

(c) $\Delta MXY \ ||| \ \Delta NBY$

$\frac{MY}{NY} = \frac{1}{2}$ (M

$\therefore$ $\frac{MX}{NB} = \frac{1}{2}$ (corre

in

Question 3

(a)(i) Step 1

Show result is true for $n = 1$

$\frac{1}{(2\times1-1)(2\times1+1)} = \frac{1}{2\times1+1}$

$\frac{1}{1\times3} = \frac{1}{2+1}$

Step 2

Assume result is true for $n = k$

$\frac{1}{1\times3} + \frac{1}{3\times5} + \frac{1}{5\times7} + \ldots\ldots + \frac{1}{(2k-1)(2k+1)} = \frac{\ }{2k}$

Step 3

Show result is true for $n = k+1$

ie: $S_k + T_{k+1} = S_{k+1}$ (1)

$\frac{k}{2k+1} + \frac{1}{(2k+1)(2k+3)} = \frac{k+1}{2(k+1)+1}$

$\frac{k(2k+3)+1}{(2k+1)(2k+3)}$ (1)

$\frac{2k^2+3k+1}{(2k+1)(2k+3)}$

$\frac{\cancel{(2k+1)}(k+1)}{(2k+3)\cancel{(2k+1)}}$

$= \frac{k+1}{2(\ )}$ $=$ RHS (1)

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Step 4

Since result is true for $n=1$, it must also be true for $n=1+1=2$, $n=2+1=3$ and hence for all positive integral values of n

(ii) $\lim_{n\to\infty}\left[\frac{1}{1\times3}+\frac{1}{3\times5}+\frac{1}{5\times7}+\cdots\cdots+\frac{1}{(2n-1)(2n+1)}\right]$

$= \lim_{n\to\infty}\left[\frac{n}{2n+1}\right]$ from (i) (1)

$= \lim_{n\to\infty}\frac{1}{2+\frac{1}{n}}$

$= \frac{1}{2}$ (1)

b) $0\cdot3171717\ldots$

$= 0\cdot3 + 0\cdot017 + 0\cdot00017 + 0\cdot0000017 + \ldots$

$= \frac{3}{10} + \frac{17}{1000} + \frac{17}{100000} + \frac{17}{10000000} + \ldots$ (1)

$= \frac{3}{10} + \frac{a}{1-r}$

$= \frac{3}{10} + \frac{\frac{17}{1000}}{1-\frac{1}{100}}$ (1)

$= \frac{157}{495}$ (1)

Question 4

a) $A = P(1+\frac{r}{100})^n$

$12000 = P(1+\frac{3}{100})^{24}$

$P = \$7460\cdot66$ (1)

$\therefore$ Interest $= 12000 - 7460\cdot66$

$= \$4539\cdot34$ (1)

b) (i) $A_1 = P\times1\cdot08 - 500$ (1)

$A_2 = A_1\times1\cdot08 - 500$

$= [P\times1\cdot08 - 500]\times1\cdot08 - 500$

$= P\times1\cdot08^2 - 500\times1\cdot08 - 500$

$A_2 = P\times1\cdot08^2 - 500(1+1\cdot08)$ as req'd (1)

(ii) $A_2 = P\times1\cdot08^2 - 500(1+1\cdot08)$

$\vdots$

$A_n = P\times1\cdot08^n - 500(1+1\cdot08+1\cdot08^2+\ldots 1\cdot08^{n-1})$ (1)

$= P\times1\cdot08^n - 500\times\frac{1\cdot08^n-1}{1\cdot08-1}$ (1)

$= P\times1\cdot08^n - 500\times\frac{1\cdot08^n-1}{0\cdot08}$

$= P\times1\cdot08^n - 6250(1\cdot08^n-1)$ (1)

$= P\times1\cdot08^n - 6250\times1\cdot08^n + 6250$

$= 6250 - 1\cdot08^n(6250-P)$ as req'd (1)

(iii) When $n=15$, $A_n=0$

$0 = 6250 - 1\cdot08^n(6250-P)$

$1\cdot08^{15}(6250-P) = 6250$ (1)

$6250 - P = 1970\cdot26$

$P = \$4279\cdot74$ (1)

Question 5

(a) $45+47+49+\ldots\ldots = 1365$

LHS is an A.P.

$a=45$, $d=2$, $n=?$

$S_n = \frac{n}{2}\{2a+(n-1)d\} = 1365$ (1)

$\frac{n}{2}\{2\times45+(n-1)\times2\} = 1365$

$n[90+2(n-1)] = 2730$

$n[90+2n-2] = 2730$

$2n^2+88n-2730=0$

$n^2+44n-1365=0$ (1)

$n = \frac{-44\pm\sqrt{44^2-4\times1\times-1365}}{2}$

$= \frac{-44\pm86}{2}$

$n = 21$ as $n>0$ (1)

should mention to get 3rd mark.

b) $\frac{3}{4} + 2\frac{1}{4} + 6\frac{3}{4}$

$a=\frac{3}{4}$, $r=3$

$S_n = \frac{\frac{3}{4}(3^n-1)}{3-1}$

$\frac{3}{4}(3^n-1) >$

$3^n-1 >$

$3^n > 26$

when $n =$

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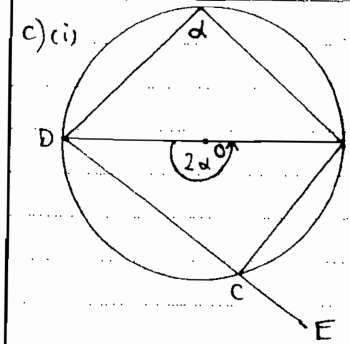
c)(i)



$\angle DAB = \alpha$

$\therefore \angle DOB = 2\alpha$ (Angle at centre is twice angle at circur when standing on same arc) (1)

$\therefore \angle DOB\,(\text{reflex}) = 360 - 2\alpha$ (1)

$\therefore \angle DCB = \frac{1}{2}(360° - 2\alpha)$ (Angle at centre ......) (1)

$= 180 - \alpha$

Now $\angle DAB + \angle DCB$

$= \alpha + 180 - \alpha$ (1)

$= 180$ $\therefore$ supplementary.

(ii) $\angle BCE = 180 - \angle DCB$

$= 180 - (180 - \alpha)$ from (i)

$= \alpha$ (1)

$\angle BCE = \angle DAB$

Question 6

(a) $\angle BPQ = \alpha + \beta$ (ext. $\angle$ of ΔAPB) (1)

$\angle BQA = \alpha + \beta$ (ext. $\angle$ of ΔAQC) (1)

$\therefore$ $BP = BQ$ as the are opposite equal angles in a triangle

b) $\underbrace{x,\ x+10,\ x+20,\ \ldots}_{30 \text{ pieces}}$

$S_n = \frac{n}{2}\{2a + (n-1)d\}$

$7500 = \frac{30}{2}\{2x + 29 \times 10\}$ (1)

$500 = 2x + 290$

$2x = 210$

$\therefore x = 1\text{st piece} = 105\text{cm}$ (1)

$S_n = \frac{n}{2}(a + l)$

$7500 = \frac{30}{2}(105 + l)$

last piece $= 395\text{cm}$ (1)

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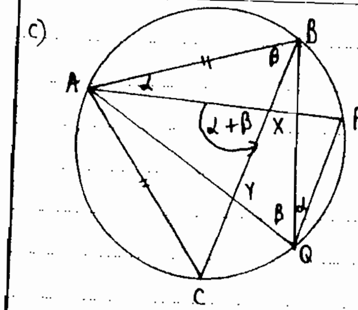
c)



(i) Join BQ and QP

$\angle BQP = \alpha$ (angle in sa

(ii) $\angle ACB = \beta$ (isosceles Δ)

$\therefore \angle BQA = \beta$ (angle in sa

(iii) $\angle PQY = \alpha + \beta$ (1)

$\angle AXC = \alpha + \beta$ (given)

$\therefore \angle YXP = 180 - (\alpha + \beta)$ (1) (str

$\therefore PQYX$ is a cyclic (1) qua

$\angle$'s YXP and YQP are s