

SYDNEY TECHNICAL HIGH SCHOOL

MATHEMATICS EXTENSION 1

HSC ASSESSMENT TASK 1 - 2008

Students' name: \_\_\_\_\_ Teacher's name: \_\_\_\_\_

Q1	Q2	Q3	Q4	Q5	Q6	Total
						/ 50

**Time allowed: 70 minutes**

- Attempt all questions
- Show all necessary working.
- Marks may not be awarded for insufficient working or poorly set out solutions.

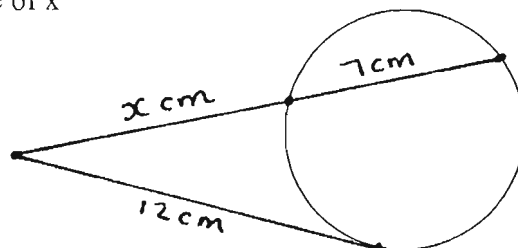
**START EACH QUESTION ON A NEW PAGE**

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**Question 1 ( 8 marks )**

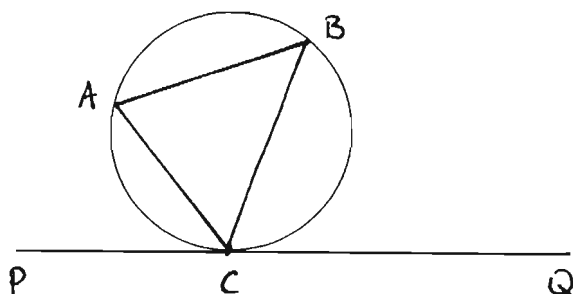
a) Find the value of  $x$

2



b) Explain why angle PCA equals angle CBA.

1



c) i. Insert 6 terms between -10 and 165 so that the series is arithmetic. 2

ii. If this pattern continues find the sum of the first 20 terms. 2

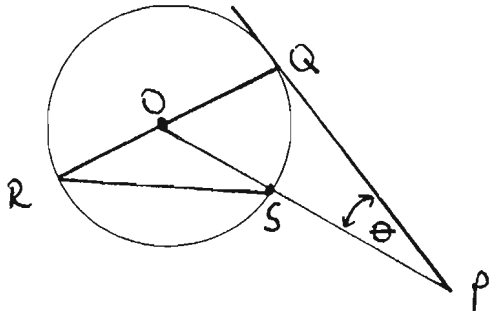
d) Evaluate  $\sum_{n=3}^7 n^3$  1

**Question 2 (start a new page) (8 marks)**

a) Given  $S_n = 17n - 3n^2$ , find an expression for the nth term. 3

b) Which term of the series,  $100 + 20 + 4 + \dots$ ,  
is the first term with a value of less than  $10^{-4}$  3

c) Given QP is a tangent to a the circle centre O,  
Find, the size of angle ORS in terms of  $\theta$ . Give reasons. 2



**Question 3 (start a new page) (8 marks)**

a) Given  $\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \dots + \frac{1}{k(k+1)} = \frac{k}{k+1}$  3

Show that  $\sum_{n=1}^{k+1} \frac{1}{n(n+1)} = \frac{k+1}{k+2}$

b)  $P(2ap, ap^2)$  is any point on the parabola  $x^2 = 4ay$ .  
The line  $l$  passes through the focus,  $S$ , and is parallel to the tangent at  $P$ .

- i. Find the equation of the line  $l$  2
- ii. The line  $l$  intersects the  $x$  – axis at  $Q$ .  
Write down the coordinates of  $Q$ . 1
- iii. Find the equation of the locus of the midpoint of  $QS$ . 2

**Question 4 ( start a new page ) ( 9 marks )**

a) Consider the series,  $x + 4x^2 + 16x^3 + \dots$

- i. For what value of  $x$  does the series have a limiting sum 1
- ii. Find the value of  $x$  given  $S_\infty = \frac{3}{2}$  2

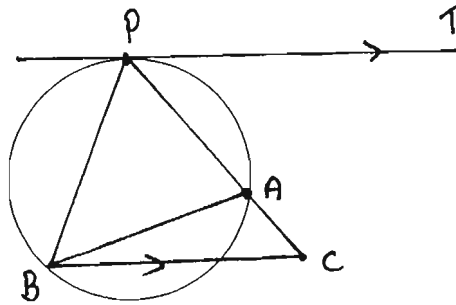
b) Tom borrows \$250 000 to buy a unit in Cronulla.  
The rates are 6% p.a. monthly reducible interest and equal monthly repayments of \$1900 are payable at the end of each month.

- i. How much does Tom owe immediately after the second repayment. 1
- ii. Show that after  $n$  months Tom owes  
 $\$ ( 380\,000 - 130\,000(1.005)^n )$  2
- iii. Find the balance owing at the end of 5 years 1
- iv. If after 5 years the interest rate increases to 7.2% p.a.,  
find the minimum monthly repayment needed to repay  
the loan in a further 20 years. 2

**Question 5 ( start a new page ) ( 8 marks )**

a) Prove by mathematical induction that  $13 \times 6^n + 2$  is divisible by 5 for every positive integer  $n$ . 4

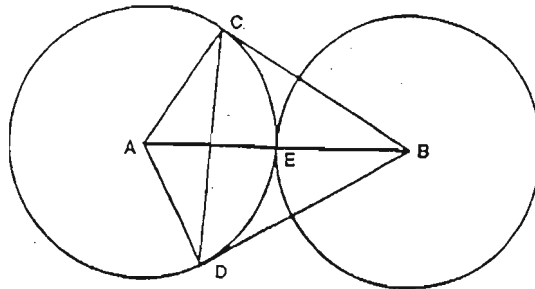
b) In the diagram A, P and B are points on the circle. The chord PA is produced to C and BC is parallel to the tangent at P.



i. Show that Angle PBA = Angle PCB 2

ii. Deduce that  $PB^2 = PA \times PC$  2

**Question 6 ( start a new page ) ( 9 marks )**



Two circles of equal radius and with centres at A and B respectively touch each other externally at E. BC and BD are tangents from B to the circle with centre A.

- i. Copy the diagram 1
- ii. Show that BCAD is a cyclic quadrilateral. 2
- iii. Show that E is the centre of the circle which passes through B, C, A and D. 2
- iv. Show that angle CBA = angle DBA =  $30^\circ$  2
- v. Show that triangle BCD is equilateral 2

# 2008 HSC task I Extension I.

## Question 1.

a)

$$12^2 = x(x+7) \quad \checkmark$$

$$144 = x^2 + 7x$$

$$0 = x^2 + 7x - 144$$

$$0 = (x+16)(x-9)$$

$$x > 0 \quad \therefore x = 9. \quad \checkmark$$

b) The angle between a tangent and a chord is equal to the angle in the alternate segment.  $\checkmark$

c)

i.  $-10, -, -, -, -, -, -, 165$   $\frac{AP}{\checkmark}$

$$165 = -10 + 7d \quad \checkmark$$

$$175 = 7d$$

$$d = 25 \quad \checkmark$$

$$\therefore 15, 40, 65, 90, 115, 140$$

ii.  $S_{20} = \frac{20}{2}(-20 + 19 \times 25)$

$$= 4550 \quad \checkmark$$

d)  $\sum = 3^3 + 4^3 + 5^3 + 6^3 + 7^3$

$$= 775. \quad \checkmark$$

## Question 2.

a)  $T_n = S_n - S_{n-1} \quad \checkmark$

$$= 17n - 3n^2 - [17(n-1) - 3(n-1)^2]$$

$$= 17n - 3n^2 - [17n - 17 - 3n^2 + 6n - 3]$$

$$= 17n - 3n^2 - [-3n^2 + 17n + 6n - 20]$$

$$= 20 - 6n \quad \checkmark$$

b)  $100 + 20 + 4 + \dots \quad a = 100$

$$r = \frac{1}{5}$$

$$ar^{n-1} < 10^{-4} \quad \checkmark \quad T_n = 10^{-4}$$

$$100 \times \left(\frac{1}{5}\right)^{n-1} < 10^{-4}$$

$$0.2^{n-1} < 0.000001 \quad \checkmark$$

LOGS  $\textcircled{OR}$  Trial & error

$$(n-1) \log 0.2 < \log 10^{-6}$$

$$n-1 > 8.584 \dots$$

$$n > 9.584 \dots$$

$$n = 10$$

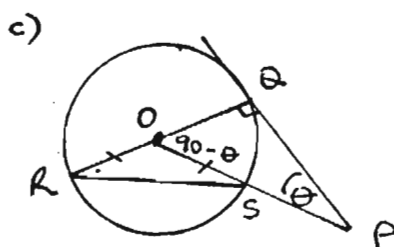
$$0.2^9 = 5.12 \times 10^{-7}$$

$$0.2^8 = 2.56 \times 10^{-4}$$

$$\therefore n-1 = 9$$

$$n = 10$$

$\downarrow \checkmark$



$\angle OQP = 90^\circ$  (radius to tangent is  $90^\circ$ )

$\angle QOS = 90 - \theta$  (angle sum  $\triangle OQP$ )

$\angle ROS = 90 + \theta$  (angles on a straight line)

$RO = OS$  radii

$\therefore \angle ORS = \angle OSR$  (equal angles opp equal sides)

$$\angle ORS = \frac{90 - \theta}{2} \quad \checkmark$$

## Question 3

a)  $\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \dots + \frac{1}{k(k+1)} + \frac{1}{(k+1)(k+2)}$

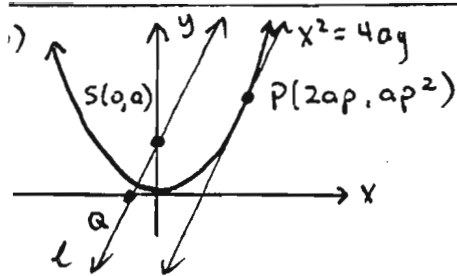
$$= \frac{k}{k+1} + \frac{1}{(k+1)(k+2)} \quad \checkmark$$

$$= \frac{k(k+2) + 1}{(k+1)(k+2)} \quad \checkmark$$

$$= \frac{k^2 + 2k + 1}{(k+1)(k+2)}$$

$$= \frac{(k+1)^2}{(k+1)(k+2)}$$

$$= \frac{k+1}{k+2} \quad \text{as required} \quad \checkmark$$



$$1. y = \frac{x^2}{4a}$$

$$y' = \frac{2x}{4a} \text{ at } x = 2ap$$

$m_T = p$  as  $l$  is // to tangent

$m_n = p$  thru  $(0, a)$  ✓

$$\therefore l: y - a = p(x - 0)$$

$$y = px + a \quad \checkmark$$

$$II. Q: y = 0 \quad 0 = px + a$$

$$\frac{-a}{p} = x$$

$$\therefore \left(-\frac{a}{p}, 0\right)$$

$$III. \text{Midpt} \left[ \frac{-\frac{a}{p} + 0}{2}, \frac{0 + a}{2} \right] \quad \checkmark$$

$$\left[ -\frac{a}{2p}, \frac{a}{2} \right]$$

$$\text{Locus of midpt} \Rightarrow y = \frac{1}{2}a \quad \checkmark$$

Question 4.

$$a) x + 4x^2 + 16x^3 + \dots$$

$$1. -1 < r < 1$$

$$-1 < 4x < 1$$

$$-\frac{1}{4} < x < \frac{1}{4} \quad \checkmark$$

$$II. \frac{x}{1-4x} = \frac{3}{2} \quad \checkmark$$

$$2x = 3 - 12x$$

$$14x = 3$$

$$x = \frac{3}{14} \quad \checkmark$$

$$b) 6\% p.a = 0.005 \text{ a month}$$

$$I. A_1 = 250\,000(1.005) - 1900$$

$$A_2 = 250\,000(1.005)^2 - 1900(1.005) - 1900$$

$$II. A_3 = 250\,000(1.005)^3 - 1900[1.005^2 + 1.005 + 1]$$

$$A_n = 250\,000(1.005)^n - 1900[1.005^{n-1} + 1.005^{n-2} + \dots + 1]$$

$$A_n = 250\,000(1.005)^n - 1900 \left[ \frac{1(1.005^n - 1)}{0.005} \right]$$

$$= 250\,000(1.005)^n - 380\,000(1.005^n - 1)$$

$$= 380\,000 - 130\,000(1.005)^n$$

$$III. n = 5 \times 12 = 60$$

$$A_{60} = 380\,000 - 130\,000(1.005)^{60}$$

$$= 204649.48 \quad \checkmark$$

$$IV. 0 = 204649.48(1.006)^{240} - m \left[ \frac{1.006^{240} - 1}{0.006} \right]$$

$$m \left[ \frac{1.006^{240} - 1}{0.006} \right] = 204649.48(1.006)^{240}$$

$$m = \$1611.32$$

### Question 5

a) Test  $n=1$

$$13 \times 6^1 + 2 = 80 \\ = 5 \times 16 \therefore \text{divisible by } 5$$

Assume true for  $n=k$

$$\text{ie } 13 \times 6^k + 2 = 5M \quad \text{where } M \text{ is a +ve integer}$$

Prove true for  $n=k+1$

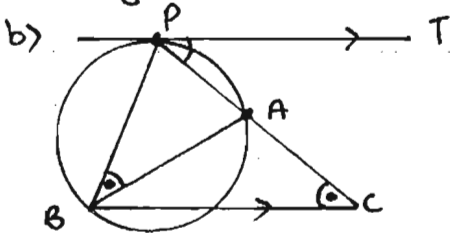
$$13 \times 6^{k+1} + 2 \\ = 6 \times 13 \times 6^k + 2 \\ = 6[5M - 2] + 2 \\ = 30M - 12 + 2 \\ = 30M - 10 \\ = 5[6M - 2]$$

which is  $\div$  by 5  $\therefore$  true  $n=k+1$ .

If true  $n=k$  also true  $n=k+1$

As true  $n=1$  also true  $n=2, 3, 4, \dots$

Hence by M.I true all positive integer  $n$ .



angle between a tangent

- i.  $\angle PBA = \angle LTPC$  (and a chord equals angle in alternate segment)
- $\therefore \angle BCP = \angle LTPC$  (alternate angles  $PT \parallel BC$ )
- $\therefore \angle PBA = \angle PCB$ .

ii. In  $\triangle PBA$  and  $\triangle PCB$

$\angle P$  is common

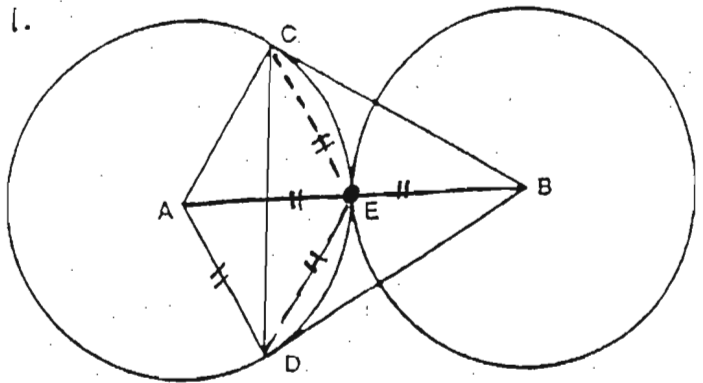
$$\angle PBA = \angle PCB \text{ (part i)}$$

$$\therefore \triangle PBA \sim \triangle PCB \text{ (equiangular)}$$

$$\therefore \frac{PB}{PC} = \frac{PA}{PB} \text{ (ratio of corresp sides)}$$

$$\therefore PB^2 = PA \times PC$$

### Question 6



- ii.  $\angle ACB = 90^\circ$  (radius to tangent)
- $\angle ADB = 90^\circ$  (radius to tangent)
- Now  $\angle ACB + \angle ADB = 180^\circ$
- and  $BCAD$  is cyclic (opposite angles supplementary)

iii. As  $\angle ACB = 90^\circ$

$AB$  is a diameter (angle in semi circle)

As  $AE = EB$

radii of equal circles

$E$  is the midpoint of  $AB$

and the centre of circle  $B, C, A, D$ .

iv. As  $E$  is the centre of circle

$$\therefore CE = EB = ED$$

$$AE = AD \text{ radii centre } A$$

$$\therefore AE = ED = AD$$

and  $\angle DAE = 60^\circ$

$$\angle ADB = 90^\circ \text{ (radii to tangent)}$$

$$\angle DBA = 30^\circ \text{ (angle sum } \triangle ABD)$$

Likewise  $\angle ABC = 30^\circ$

$$\therefore \angle DBA = \angle ABC = 30^\circ$$

v.

$CB = DB$  tangents from external point equal

$$\therefore \angle BCD = \angle BDC \text{ (equal angles opposite equal sides)}$$

$$\angle CBD = 60^\circ \text{ (part iv)}$$

$$\therefore \angle BCD = \angle BDC = 60^\circ \text{ (angle sum } \triangle BCD)$$

$\therefore \triangle BCD$  is equilateral.