Name: .... Maths Class: ....

SYDNEY TECHNICAL HIGH SCHOOL

(Est. 1911)

Year 11

Extension 1 Mathematics

Assessment 1
HSC Course

December, 2014

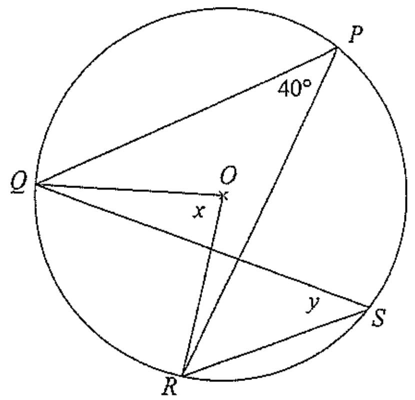
Time allowed: 70 minutes

General Instructions:

- Questions are not of equal value
- Approved calculators may be used
- All necessary working should be shown
- Begin each question on a new page
- Write using black or blue pen
- Full marks may not be awarded for careless work or illegible writing

Question 1

P, *Q*, *R* and *S* are points on a circle with centre *O*. $\angle QPR = 40^\circ$.



Why are the values of x and y?

(A) $x = 40^\circ$ and $y = 20^\circ$

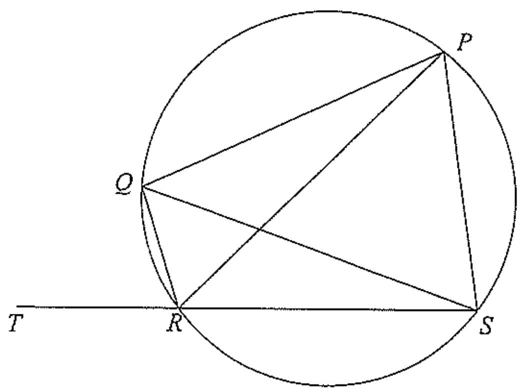
(B) $x = 40^\circ$ and $y = 40^\circ$

(C) $x = 80^\circ$ and $y = 20^\circ$

(D) $x = 80^\circ$ and $y = 40^\circ$

Question 2

PQRS is a cyclic quadrilateral. *SR* is produced to *T* and $\angle PRS = \angle QRT$.



Why is $\angle PQS = \angle PRS$?

(A) Angle at the circumference is equal to the angle in the alternate segment.

(B) Angle between the tangent and a chord is equal to the angle in the alternate segment.

(C) Angle between the two chords in the same segment are equal.

(D) Angles in the same segment standing on the same arc are equal.

Question 3

The third and seventh terms of a geometric series are 1.25 and 20 respectively. What is the first term?

(A) ± 2

(B) ± 4

(C) $\frac{5}{16}$

(D) $\frac{5}{1024}$

Question 4

The number of zeros of the polynomial $f(x) = x^3 - x$:-

(A) 0
(B) 1
(C) 2
(D) 3

Question 5

If α and β are roots of the equation $x^2 + 5x + 7 = 0$, the quadratic equation whose roots are $(\alpha - \beta)^2$ and $(\alpha + \beta)^2$ is:

(A) $x^2 + 22x + 75 = 0$
(B) $x^2 + 22x - 75 = 0$
(C) $x^2 - 22x - 75 + 0$
(D) $x^2 - 22x + 75 = 0$

Question 6

If R is $(2p + 2q, 4p^2 + 4q^2)$ and it is known that $pq = 3$, the Cartesian equation of the locus of R is:

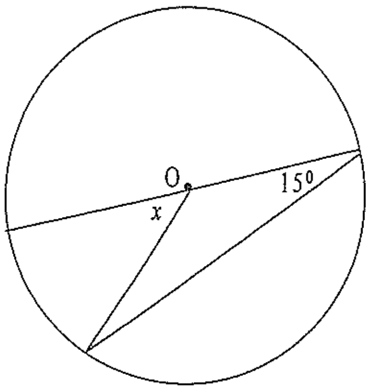
(A) $y = x^2$
(B) $y = x^2 - 24$
(C) $y = x^2 + 24$
(D) $y = 24x^2$

Question 7 **11 marks**

(A) $(x - 2)$ is a factor of the polynomial $P(x) = 2x^3 + x + a$. Find the value of a (1)

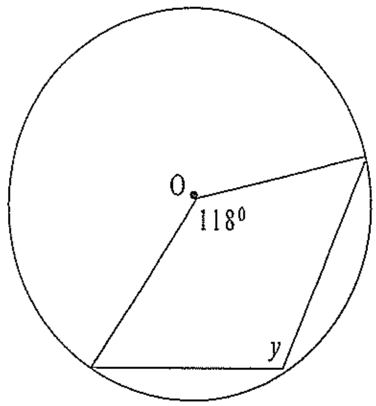
(B) For the following questions, find the value of the unknown's, without giving reasons:- (9)

(i)



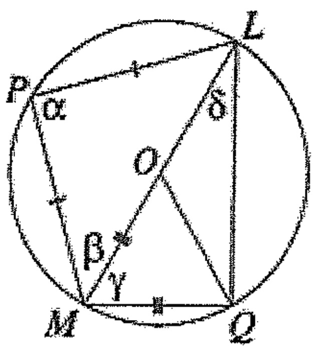
Find x

(ii)



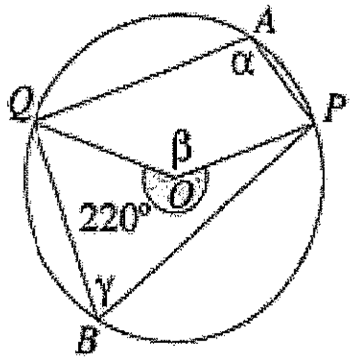
Find y

(iii)



Find $\alpha, \beta, \gamma, and\ \delta$

(iv)



Find α, β, γ

(C) Evaluate $\sum_{r=1}^{5} r^2 + 2r$. (1)

Question 8 **10 marks**

Two points P $(2ap, ap^2)$ and Q $(2aq, aq^2)$ lie on the parabola $x^2 = 4ay$.

(i) Show that the equation of the tangent to the parabola at P is $y = px - ap^2$ (3)

(ii) The tangent at P and the line through Q parallel to the y axis intersect at T.
Find the coordinates of T (2)

(iii) Write down the coordinates of M, the midpoint of PT (2)

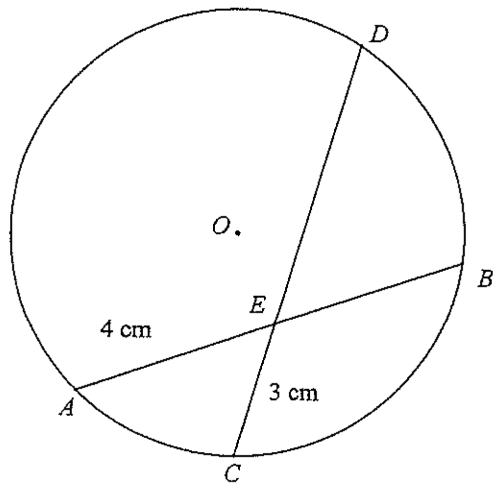
(iv) Determine the locus of M when $pq = -1$ (3)

Question 9 **11 marks**

(A) When the polynomial $Q(x) = x^4 + bx^2 - ax + 4$ is divided by $(x - 1)$ the remainder is 4.
It is also known that $(x - 2)$ is a factor of $Q(x)$. Find a, b (2)

(B) How many terms are in the arithmetic series $27 + 23 + 19 + \dots -85$? (2)
What is the least number of terms in the arithmetic series such that the sum is negative ? (2)

(C) In the circle centered at O, the chords AB and CD intersect at E.
The length of AB is x cm and of CD is y cm. $AE = 4$ cm and $CE = 3$ cm. (3)

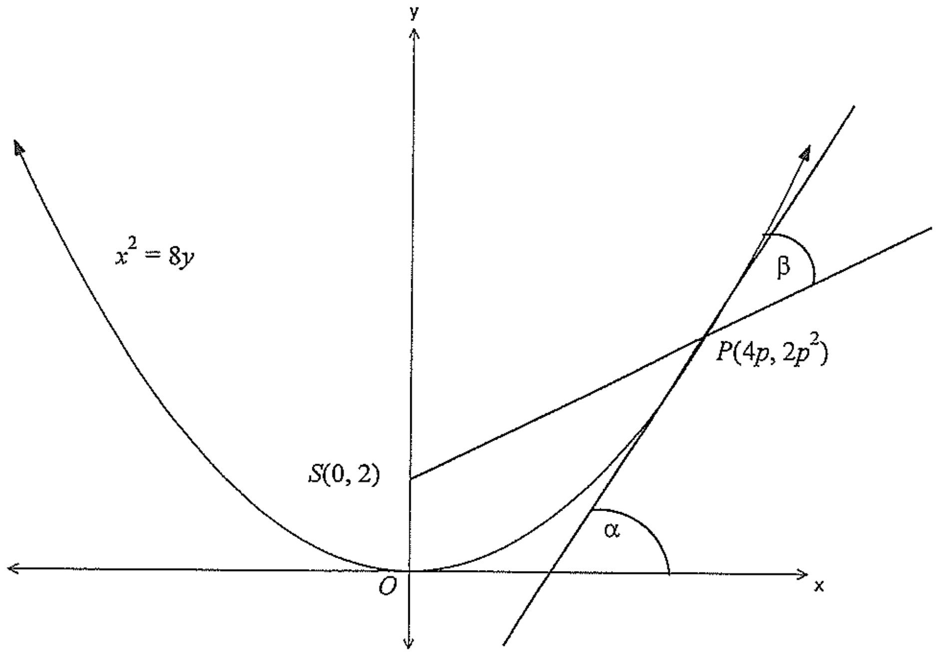


Show that $4x = 3y + 7$

(D) Find the sum of the first 50 terms of an arithmetic progression, given that the 15th term is 34 and the sum of the first 8 terms is 20 (2)

Question 10 **11 marks**

(A) The point $P(4p, 2p^2)$ is a point on the parabola $x^2 = 8y$, with focus $S(0, 2)$.



The tangent at P makes an angle of α with the x axis and an angle of β with the line SP.

(i) Show that the gradient of the tangent is p. (2)

(ii) Show that the gradient of SP is $\frac{p^2 - 1}{2p}$. (2)

(iii) Show that $\tan \beta = \frac{1}{p}$ (2)

(iv) Show that $\alpha + \beta = \frac{\pi}{2}$ (2)

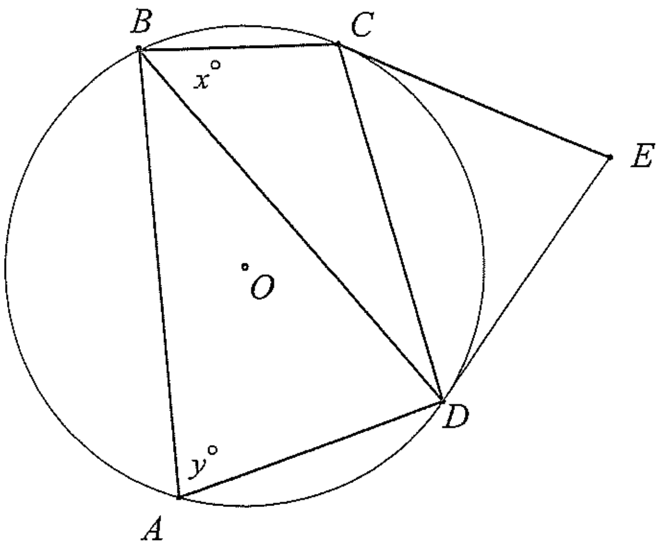
(B) A book seller sell 10 000 books in the first year after opening her shop. The following year she sells 11 000 books and 12 100 in the third year. She notes the 10% increase in per annum sales If this trend continues:

(i) How many books can she expect to sell in the 12th year of sales ? (1)

(ii) Calculate the total number of books sold in these 12 years. (2)

Question 11 **10 marks**

(C) The circle ABCD has centre O. Tangents are drawn from an external point E to contact the circle at C and D. $\angle CBD = x^\circ$ and $\angle BAD = y^\circ$



(i) Show that $\angle CED = (180 - 2x)^\circ$,with reasons (2)

(ii) Show that $\angle BDC = (y - x)^\circ$,with reasons (2)

(D) Factorise $m^3 - 3m + 2$ and solve the equation $(3x - 4)^3 - 9x + 14 = 0$ (3)

(E) The roots of the polynomial equation $x^3 - 4x^2 - 11x + 30 = 0$, are α, β, and γ. Given that $\alpha = \beta + \gamma$,solve the equation $P(x) = 0$ (3)

Question 12 is over the page

Question 12 **10 marks**

(A) For the series 5 + 7 + 9 + 11 + . . . find :

i. the 10^{th} term. 1

ii. the sum of the first 10 terms. 2

(B) For the series $1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\ldots$ find

i. the 10^{th} term 1

ii. the sum of the first 10 terms. 2

(C) Hence or otherwise, find the sum of the series:

$$6+6\frac{1}{2}+9\frac{1}{4}+10\frac{7}{8}+\ldots+22\frac{511}{512}$$ 2

(D) One of the series has a sum to infinity. State the series and find this sum. 2

End of Exam

DD CD CB

Q7 (a) $P(x) = 2x^3 + x + a$

$P(2) = 16 + 2 + a$

$a = -18$

(b) (i) $x = 30°$

(ii) $y = 121°$

(iii) $\alpha = 90°$

$\beta = 45°$

$\gamma = 60°$

$\delta = 30°$

(iv) $\gamma = 70°$

$\alpha = 110°$

$\beta = 140°$

(c) $\Sigma(3+8+15+24+35) = 85$

Q8 (i) $y = \frac{x^2}{4a}$

$y' = x/2a.$

at $x = 2ap$ $m = p$

$y - y_1 = m(x - x_1)$

$y - ap^2 = p(x - 2ap)$

$y - ap^2 = px - 2ap^2$

$y = px - ap^2$

(ii) $y = px - ap^2$ $x = 2aq$

$y = p(2aq) - ap^2$

$(2aq, 2apq - ap^2)$

(iii) $(2ap, ap^2)(2aq, 2apq - ap^2)$

M.P. $\left(\frac{2ap + 2aq}{2}, \frac{ap^2 + 2apq - ap^2}{2}\right)$

$= [a(p+q), apq]$

(iv) $x = a(p+q)$ $y = apq$

$pq = -1$ $y = -a$

$\therefore y = -a$

Q9 $Q(x) = x^4 + bx^2 - ax + 4$

$Q(1) = 1 + b - a + 4$

$5 + b - a = 4$

$b - a = -1$ — ①

$Q(2) = 16 + 4b - 2a + 4$

$= 20 + 4b - 2a$

$4b - 2a = -20$ — ②

$b - a = -1$ — ① ×2.

$-(2b - 2a = -2)$

$2b = -18$

$b = -9$ into ①

$-a = 8$

$a = -8$

(b) $27 + 23 + 19 + \ldots -85$

$a = 27$ $d = -4$

$t_n = a + (n-1)d$

$= 27 - 4n + 4$

$= 31 - 4n$

$-85 = 31 - 4n$

$n = 29$

$S_n = n/2[2a + (n-1)d]$

$= n/2[54 - 4n + 4]$

$S_n = n/2(58 - 4n)$

$n/2(58 - 4n) < 0$

$n(58 - 4n) < 0$

$n = 0$ or $58 - 4n <$ [illegible]

$-4n <$ [illegible]

$n >$ [illegible]

check for $n = 15$

$S_{15} = 7.5(-2)$

$= -15$

$S_{14} = 7(58 - \quad)$

$=$

$\therefore$ 15 terms min.

(c) $\frac{ED}{EB} = \frac{EA}{EC}$

$\frac{y-3}{x-4} = \frac{4}{3}$

$3y - 9 = 4x - 16$

$4x - 3y + 7 = 0$

(D) $t_{15} = 34$ $a + 14d$

$S_8 = 20$ $= 4[2a + 7d$ [illegible]

$= 8a + 28$[illegible]

$a + 14d = 34$ — ×2

$8a + 28d = 20$

$-(2a + 28d = 68)$

$a = -8$

$d = 3$

Q9 (i) $t_1 = 8$

$t_{50} = 139$

$S_{50} = \frac{n}{2}(a+l)$

$= (25)(-8+139)$

$= 3275$

Q10 (A) (i) $x^2 = 4ay$ $a=2$

$y = \frac{x^2}{8y}$

$y' = \frac{x}{4y}$

at $x = 4p$

$m_p = 4p/4$

$= p$

(ii) $m_{sp} = \frac{2p^2-2}{4p-0}$

$= 2(p^2-1)/4p$

$= p^2-1/2p$

(iii) $\tan\beta = \left|\frac{m_1 - m_2}{1+m_1 m_2}\right|$

$= \left|\frac{p - \frac{p^2-1}{2}}{1+p\left(\frac{p^2-1}{2}\right)}\right|$

$= 1/p$

(iv) $\alpha + \beta = \pi/2$

$\tan\alpha = p$ $\tan\beta = 1/p$

$= \frac{1}{\tan\alpha}$

$= \cot\alpha$

$\cot\alpha = \tan(\pi/2 - \alpha)$

$\tan\beta = \tan(\pi/2 - \alpha)$

$\alpha + \beta = \pi/2$

(B) $V = P(1+0.1)^{11}$

$= 10{,}000(1.1)^{11}$

$= \$28{,}531$

(ii) $S_{12} = \frac{a(r^n-1)}{r-1}$

$= \frac{10{,}000(1.1^{12}-1)}{0.1}$

$= \$213{,}842.84$

$= \$213{,}842.84$

Q11

A(i) $\angle EDC = \angle CBD$ $(=x)$ alt. seg. theory.

$\angle ECD = \angle CBD$ $(=x)$ alt. seg theory.

$\angle ECD = \angle EDC$ $= x$.

$\therefore \angle CED = 180 - 2x$ (angle sum of triangle

OR

$EC = ED$ tangents from external point

$\therefore \angle CED = (180 - 2x)$

(ii) $\angle EDB = \angle DAB$ alt. seg. theory $(=y)$

$\therefore \angle BDC = (y-x)°$

(B) $f(m) = m^3 - 3m + 2$ $(3x-4)^3 - 9x + 14 = 0$

$f(1) = 0$

$$
\begin{array}{r}
m^2 + m - 2 \\
m-1 \overline{)\, m^3 + 0m^2 - 3m + 2} \\
-(m^3 - m^2) \\
\hline
m^2 - 3m \\
-(m^2 - m) \\
\hline
-2m + 2 \\
-(-2m + 2) \\
\hline
-
\end{array}
$$

factors: $(m-1)(m^2+m-2)$

$(m-1)(m+2)(m-1)$

$(3x-4)^3 - 9x + 14 = 0 \;:\; (3x-4)^3 - 3(3x-4) + 2$

Q11 (cont'd)

let $m = (3x-4)$

$(3x-4)^3 - 9x + 14 = m^3 - 3m + 2$

$= (m-1)^2 (m+2)$

$= [(3x-4)-1]^2 [(3x-4)+2]$

$(3x-4-1)^2 = 0$
$3x = 5$
$x = 5/3$

$3x-4+2 = 0$
$3x = 2$
$x = 2/3$.

(c) $x^3 - 4x^2 - 11x + 30 = 0$ roots $\alpha, \beta \,\&\, \gamma$

$\alpha = \beta + \gamma$

$\alpha + \beta + \gamma = -b/a$
$= 4$
$\alpha + \beta + \alpha + \beta = 4$
$\alpha + \beta = 2$
$\alpha = 2$

$\therefore x = 2$ is a root.

$$\begin{array}{r} x^2 - 2x - 15 \\ x-2 \overline{) x^3 - 4x^2 - 11x + 30} \\ -(x^3 - 2x^2) \\ \hline -2x^2 - 11x \\ -(-2x^2 + 4x) \\ \hline -15x + 30 \\ -(-15x + 30) \\ \hline - \end{array}$$

$(x^2 - 2x - 15)(x-2)$
$(x-5)(x+3)(x-2)$ roots $x = 5, -3$ and 2.

Q12

(A) $5 + 7 + 9 + 11 \ldots\ldots$

$a = 5 \quad d = 2$
$t_n = 5 + (n-1)2$
$= 5 + 2n - 2$
$= 2n + 3$

(i) $t_{10} = 23$

(ii) $S_{10} = \frac{n}{2}[2a + (n-1)d]$
$= 5[10 + 9d]$

$S_{10} = 140$

(B) $1 - 1/2 + 1/4 - 1/8 + \ldots\ldots$

$a = 1 \quad r = -1/2$

$t_n = ar^{n-1}$
$t_{10} = (-1/2)^9$
$= -1/512$

(ii) $S_{10} = \frac{a(1-r^n)}{1-r}$

$= \frac{(1 - (-1/2)^{10})}{1 + 1/2}$

$= 341/512$

(C) $S_{10} = 140 + \frac{341}{512}$

$= 140\ 341/512$

(D) G.P.
$S_\infty = \frac{a}{1-r}$

$= \frac{1}{3/2}$

$= 2/3$