QUESTION 1 If $z = 1 + i\sqrt{3}$ express z in the following form 2

$$z = r(cos\theta + isin\theta)$$

and hence plot on the Argand diagram:

i) $\bar{z}$ 1

ii) $\frac{12}{z}$ 1

iii) z^2 1

iv) $\sqrt{z}$ 1

QUESTION 2 Solve for x and y: 3

$(x + iy)(2 + 3i) = 18i - 1$

QUESTION 3 Sketch on the Argand diagram where 3

$|z| < 4$ and $\frac{-\pi}{3} \le \arg z \le \frac{\pi}{4}$ both hold.

QUESTION 4 Prove that $(6 + i\sqrt{108})^{12}$ is real. 2

QUESTION 5 Find the modulus of the product of the roots of the equation: 2

$$(2+3i)z^2 + (5-2i)z + (18+i) = 0$$

QUESTION 6 Find $\sqrt{-8-6i}$ and hence solve the equation: 4

$$z^2 - (5-i)z + 8 - i = 0$$

QUESTION 7 P, Q and R represent the complex numbers **4**

z_1 , z_2 *and* z_3 respectively on the Argand diagram.

Given that $z_1 - z_2 = i(z_2 - z_3)$ show this information

on a diagram and hence fully describe ΔPQR.

QUESTION 8 The point P on the Argand diagram represents the complex **3**

number z where z satisfies the equation:

$$|z-2i| = \text{Im}(z+2i)$$

Find the Cartesian equation of the locus of P and give a geometric description of the locus of P.

QUESTION 9 By applying De Moivres Theorem, and by also expanding **4**

$(cos\theta + isin\theta)^4$ express $tan4\theta$ as a polynomial in terms of $tan\theta$.

QUESTION 10 i) If ω is a complex solution of the equation $z^3 = 1$ show that ω^2 **2**

is also a solution of the equation and that:

$1 + \omega + \omega^2 = 0$

(ii) Given that ω is also a root of $z^3 + bz^2 + cz - 12 = 0$, **2**

find the values of b and c given they are both real.

END OF EXAMINATION

EXT 2 DEC

Question 1

$z = 1 + i\sqrt{3}$

$= 2\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$ — 1

$= 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$ — 1

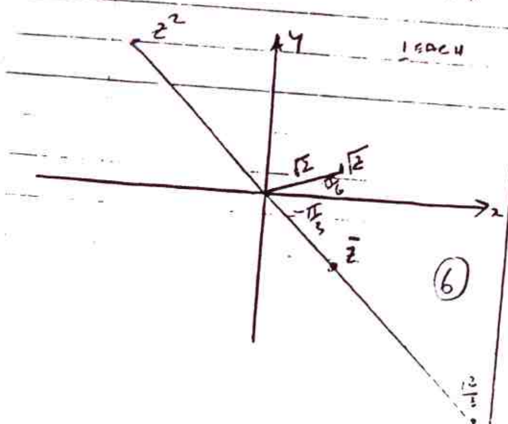
$\bar{z} = 2\left(\cos -\frac{\pi}{3} + i\sin -\frac{\pi}{3}\right)$

$\frac{12}{z} = 6\left(\cos -\frac{\pi}{3} + i\sin -\frac{\pi}{3}\right)$

$z^2 = 4\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

$\sqrt{z} = \sqrt{2}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

1 EACH



(6)

Question 2

$(x+iy)(2+3i) = 18i - 1$

$(2x - 3y) + i(2y + 3x) = 18i - 1$

$\therefore 2x - 3y = -1$ — (1) — 1

$2y + 3x = 18$ — (2)

(1) × 2 ⇒ $4x - 6y = -2$ — (3)

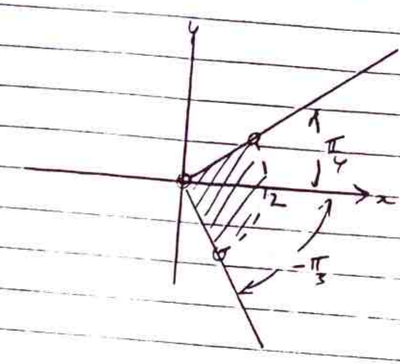
(2) × 3 ⇒ $6y + 9x = 54$ — (4)

(3)+(4) ⇒ $13x = 52$ — 1 — 1

$x = 4,\ y = 3$

(3)

Question 3



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(3)

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Question 4

$(6 + i\sqrt{108})^{12} = (6 + 6\sqrt{3}i)^{12}$

$= 12^{12}\left(\frac{6}{12} + \frac{6\sqrt{3}i}{12}\right)^{12}$

$= 12^{12}\left(\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)^{12}$

$= 12^{12}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)^{12}$ — 1

$= 12^{12}(\cos 4\pi + i\sin 4\pi)$

$= 12^{12}(\cos 0 + i\sin 0)$

$= 12^{12}$ — 1

WHICH IS REAL

(2)

Question 5

$(2+3i)z^2 + (5-2i)z + (18+i) = 0$

$\alpha\beta = \frac{c}{a} = \frac{18+i}{2+3i} \times \frac{2-3i}{2-3i}$ — 1

$= \frac{(36+3) + i(2-54)}{13}$

$= \frac{39 - 52i}{13}$

$= 3 - 4i$

$|\alpha\beta| = \sqrt{3^2 + 4^2} = 5$ — 1

(2)

Question 6

let $z = x + iy$

$\therefore x + iy = \sqrt{-8-6i}$

$\therefore x^2 - y^2 + 2xyi = -8 - 6i$

$\therefore x^2 - y^2 = -8$

$xy = -3$ — 1

subst $y = \frac{-3}{x}$

$\therefore x^2 - \frac{9}{x^2} = -8$

$\therefore x^4 + 8x^2 - 9 = 0$

$(x^2+9)(x^2-1) = 0$ — 1

$\therefore x = \mp 1 \quad y = \pm 3$

$\therefore z = \mp(1 - 3i)$

$z^2 - (5-i)z + 8 - i = 0$

$z = \frac{5 - i \mp \sqrt{(5-i)^2 - 4(8-i)}}{2}$

(4)

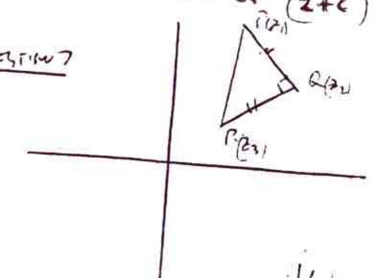
$= \frac{5 - i \mp \sqrt{25 - 10i - 1 - 32 + 4i}}{2}$

$= \frac{5 - i \mp \sqrt{-8 - 6i}}{2}$ — 1

$= \frac{5 - i + 1 - 3i}{2}$ or $\frac{5 - i - 1 + 3i}{2}$

$= 3 - 2i$ or $(2 + i)$ — 1

Question 7



2

(4)

Q7 (cont) $\triangle PQR$ IS AN ISOSCELES RIGHT ANGLED TRIANGLE WITH RT ∠ AT Q(z_2). — 1

1

Question 8.

$|z-2i| = \text{Im}(z+2i)$

let $z = x+iy$

$\therefore$ ~~$\sqrt{\ }$~~

$|(x+iy-2i)| = \text{Im}(x+iy+2i)$

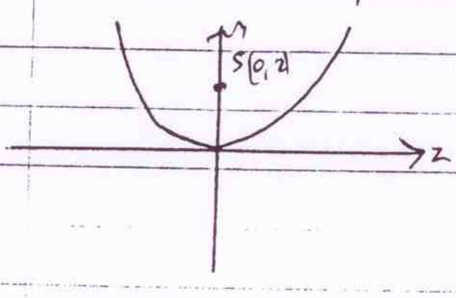
$|x+i(y-2)| = y+2$ 1

$\therefore \sqrt{x^2+(y-2)^2} = y+2$

$x^2+\cancel{y^2}-4y+4 = \cancel{y^2}+4y+4$

$x^2 = 8y$ 1

$\therefore x^2 = 4\cdot 2y.$



(3)

1

A PARABOLA VERTEX O AND FOCUS (0,2)

Question 9

$(\cos 4\theta + i\sin 4\theta) = (\cos\theta + i\sin\theta)^4$ let $c=\cos\theta$, $s=\sin\theta$

$\cos 4\theta + i\sin 4\theta = c^4 + 4c^3 s i + 6c^2 s^2 i^2 + 4cs^3 i^3 + i^4 s^4$ 1

$= c^4 + 4c^3 s i - 6c^2 s^2 - 4cs^3 i + s^4$ 1

$\cos 4\theta = c^4 - 6c^2 s^2 + 1$ $\quad \sin 4\theta = 4c^3 s - 4cs^3$

so $\tan 4\theta = \dfrac{\sin 4\theta}{\cos 4\theta} = \dfrac{4c^3 s - 4cs^3}{c^4 - 6c^2 s^2 + s^4}$ 1

$\div$ top + bottom by c^4

$\therefore \tan 4\theta = \dfrac{\sin 4\theta}{\cos 4\theta} = \dfrac{\frac{4s}{c} - \frac{4s^3}{c^3}}{1 - \frac{6s^2}{c^2} + \frac{s^4}{c^4}}$ (4)

$= \dfrac{4\tan\theta - 4\tan^3\theta}{1 - 6\tan^2\theta + \tan^4\theta}$ 1

Question 10

i) $z^3 = 1$

$\therefore$ ~~$(z-1)(z^2+z$~~

$(z-1)(z^2+z+1) = 0$

if ω is a complex soln of the eqn

$\omega \neq 1$ 1

$\therefore$ ω must satisfy $z^2+z+1=0$

$\therefore \omega^2+\omega+1 = 0$ 1

OR if ω is a soln

then $\omega^3 = 1$

but $(\omega^2)^3 = (\omega^3)^2 = 1$ 1

$\therefore$ ω^2 is also a soln

as is $z=1$

$\therefore$ 3 solns are $1, \omega, \omega^2$

now $\Sigma\alpha$ for $z^3-1=0$

$\Sigma\alpha = -\frac{b}{a} = 0$ 1

$\therefore 1+\omega+\omega^2 = 0$

ii) if ω is a soln so is ω^2

now $\alpha\beta\gamma = -\frac{d}{a} = 12$ (4)

$\omega\cdot\omega^2\cdot\gamma = 12$

$\gamma = 12$

$\Sigma\alpha = 12+\omega+\omega^2 = -b$

$\therefore 11 + (1+\omega+\omega^2) = -b$ 1

$\therefore b = -11$ *

$\alpha\beta+\alpha\gamma+\beta\gamma = \omega^3+\omega\gamma+\omega^2\gamma = c$

$= 1+\gamma(\omega+\omega^2)$

$= 1+12(\omega+\omega^2) = c$

$= 1-11(1+\omega+\omega^2)-12 = c$

$\therefore c = -11$ * 1