2017 | HSC Assessment Task 1

# Mathematics Extension 2

**General Instructions**

- Reading time – 5 minutes
- Working time – 50 minutes
- Write using black pen
- NESA approved calculators may be used
- In Questions 6 – 8, show relevant mathematical reasoning and/or calculations.

**Total marks: 39**

**Section I – 5 marks** (pages 2 – 3)

- Attempt Questions 1 – 5
- Allow about 10 minutes for this section

**Section II – 34 marks** (pages 4 – 6)

- Attempt Questions 6 – 8
- Allow about 40 minutes for this section

## Section I

**5 marks**
**Attempt Questions 1 – 5**
**Allow about 10 minutes for this section**

Use the multiple-choice answer page in the writing booklet for Questions 1 – 5.

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**1** What is the value of $i^{2017}$ ?

(A) $i$ (B) $-i$ (C) $1$ (D) $-1$

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**2** Given $z = 3 - i$, $\overline{iz} =$

(A) $-1+3i$ (B) $-1-3i$ (C) $1+3i$ (D) $1-3i$

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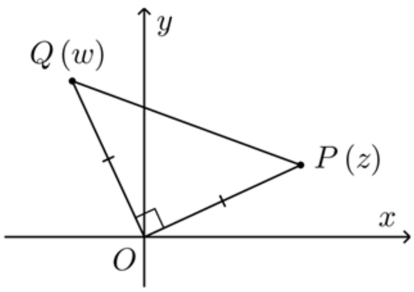
**3** On the diagram below, $P$ and $Q$ represent complex numbers $z$ and $w$ respectively. Triangle $OPQ$ is isosceles and right-angled at the origin.




Which of the following statements is <u>**incorrect**</u>?

(A) $|z|^2 + |w|^2 = |z+w|^2$

(B) $z^2 + w^2 = 0$

(C) $z^2 - w^2 = 0$

(D) $w = iz$

**4** Which of the following statements is **incorrect**?

(A) $z\overline{z}$ is purely real, where $z$ is a complex number.

(B) $z+\overline{z}$ is purely real, where $z$ is a complex number.

(C) $iz$ is purely real, where $z$ is a purely imaginary number.

(D) $\frac{1}{z}$ is purely real, where $z$ is a purely imaginary number.

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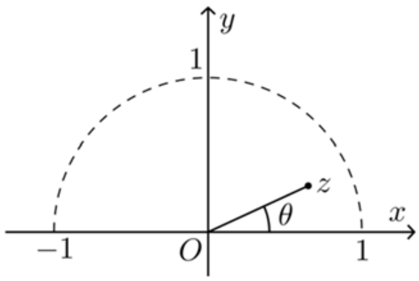
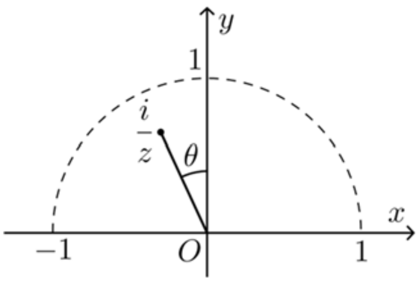
**5** The Argand diagram below shows the complex number z.

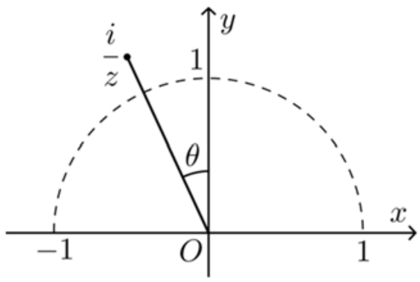



Which of the following diagram best represents $\frac{i}{z}$?

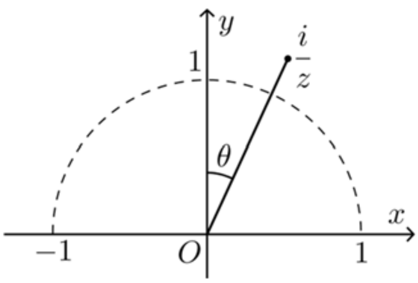
(A)



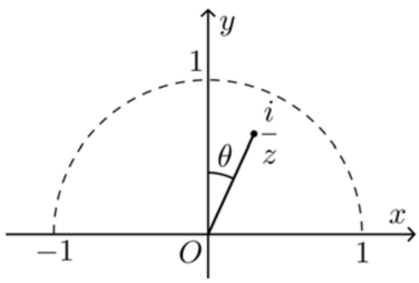

(B)




(C)




(D)




**End of Section I**

## Section II

**34 marks**
**Attempt Questions 6 – 8**
**Allow about 40 minutes for this section**

Answer each question in the appropriate section of the writing booklet. Extra writing paper is available.

In Questions 6 – 8, your responses should include relevant mathematical reasoning and/or calculations.

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**Question 6** (11 marks) Use the Question 6 section of the writing booklet.

(a) Given $z = 7 + 4i$ and $w = 3 - 2i$. Express each of the following in $x + yi$ form, where $x$ and $y$ are real numbers.

(i) $\overline{z} - w$ **1**

(ii) $\dfrac{zw}{2}$ **1**

(iii) $\dfrac{z}{w}$ **2**

(b) On an Argand diagram, sketch the locus of the point z such that $|z+1| = |z-2i|$. **2**

(c) (i) Shade in the region on the Argand diagram where the following inequalities both hold. **2**

$$1 \le \text{Im}(z) \le 4 \quad \text{and} \quad \frac{\pi}{4} \le \arg(z) \le \frac{\pi}{3}$$

(ii) Let $\omega$ be the complex number of maximum modulus satisfying the above inequalities. Express $\omega$ in the form $x + yi$, where $x$ and $y$ are real number. **1**

(d) Find, in modulus-argument form, all solutions of $z^3 + i = 0$. **2**

**End of Question 6**

**Question 7** (12 marks) Use the Question 7 section of the writing booklet.

(a) (i) Find all pairs of integers $x$ and $y$ such that $(x+yi)^2 = 7+24i$. **2**

(ii) Hence, or otherwise, find all the solutions to the following quadratic equation: **2**

$$z^2 + 3iz - (4+6i) = 0 .$$

(b) (i) Express $z = \dfrac{1+\sqrt{3}\,i}{1+i}$ in modulus-argument form. **2**

(ii) Hence, find the smallest positive integer $n$ such that $z^n$ is a real number. **2**

(c) (i) On the Argand diagram, sketch the locus of a point $z$ such that: **2**

$$\arg\left(\frac{z-2\sqrt{3}}{z}\right) = \frac{2\pi}{3}$$

(ii) Hence, find the Cartesian equation of the locus, stating any restrictions. **2**

**End of Question 7**

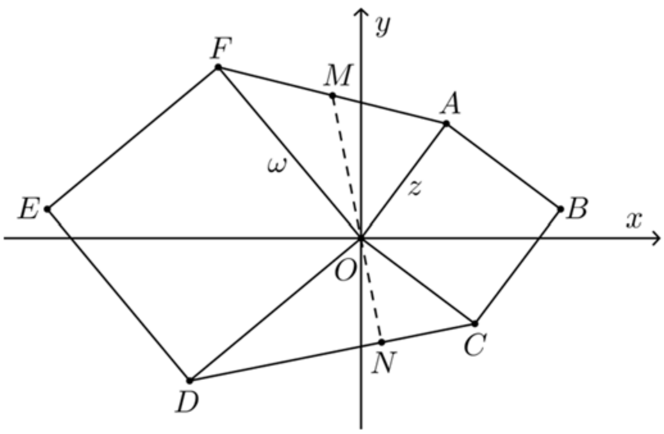
**Question 8** (11 marks) Use the Question 8 section of the writing booklet.

(a) If $\omega$ is a non-real cube root of unity, evaluate $\left(1-3\omega+\omega^2\right)\left(1+\omega-8\omega^2\right)$. **2**

(b) (i) By expanding $(\cos\theta + i\sin\theta)^3$ and using de Moivre's theorem, find expressions for $\sin 3\theta$ and $\cos 3\theta$. **2**

(ii) Hence show that $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$. **1**

(c) In the following diagram, $OABC$ and $ODEF$ are squares. $\overrightarrow{OA}$ and $\overrightarrow{OF}$ represent the complex numbers $z$ and $\omega$ respectively. $M$ is the midpoint on the interval $FA$. $MO$ produced meets $DC$ at $N$.




(i) Show that $\overrightarrow{OM} = \dfrac{z+\omega}{2}$. **1**

(ii) Express $\overrightarrow{CD}$ in terms of $z$ and $\omega$. **1**

(iii) Hence, or otherwise, justify why $NM \perp CD$. **2**

(d) It is known that $\omega = \dfrac{az+b}{bz+a}$, where $a$ and $b$ are non-zero real numbers. **2**
Given $|z|=1$, show that $|\omega|=1$.

**End of Paper**