GIRRAWEEN HIGH SCHOOL

MATHEMATICS EXTENSION 2 TASK 1

December 2008

Instructions:

*Write all solutions on your own paper.

*Show all necessary working.

*Marks may be deducted for careless or badly arranged work.

*Approved scientific calculators may be used.

Time allowed: 90 minutes. **MARKS**

Question 1 7 Marks

Given $z = 1 - 4i$ and $w = -2 + 5i$ find in Cartesian ($x + iy$) form:

(a) $z + w$ 1

(b) $\bar{z}$ 1

(c) $\frac{z}{w}$ 2

(d) w^2 1

(e) $\overline{zw}$ 2

Question 2 9 Marks

Given $z = 1 + i\sqrt{3}$ and $w = 1 + i$

(a) Find zw in $x + iy$ form. 1

(b) Convert z and w to modulus-argument form 4

(c) Hence find zw in modulus-argument form and use this to find the exact value of $\sin\frac{7\pi}{12}$. 4

Question 3 5 Marks

(a) Convert $1 - i$ to mod/arg form. 2

(b) Hence find $(1 - i)^{11}$ in $x + iy$ form. 3

Question 4 7 Marks

(a) Find all real numbers x and y such that $\sqrt{-5+12i} = x+iy$ 5

(b) Hence (or otherwise) solve for z : $z^2+(2+i)z+(2-2i)=0$. 2

Question 5 10 Marks

Sketch these regions on separate Argand diagrams:

(a) $|z-2|<3$ 2

(b) $\frac{-\pi}{3} < Arg(z+1) \le \frac{\pi}{2}$ 3

(c) $3<|z|<4$ and $Arg(z) > \frac{\pi}{2}$ 2

(d) $|z-\bar{z}|<3$ and $z\bar{z} \le 5$ 3

Question 6 10 marks

(a) Find the Cartesian equations for the following loci:

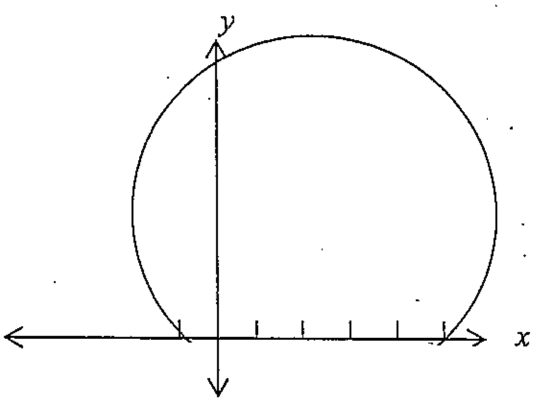
(i) $|z+2-5i|=3$ 1

(ii) $|z-1+2i|=|z+1+i|$ 2

(iii) $\text{Re}\left(\frac{z-8i}{z+6}\right)=0$ 3

(b) The locus $Arg\left(\frac{z-5}{z+1}\right)=\frac{\pi}{3}$ represents part of a circle 4

(as shown). Find the centre and radius of the circle. Hence write the locus of z in Cartesian form.



Question 7   13 Marks

(a) z is an arbitrary complex number such that $|z| = 1$

and $0 < Arg(z) < \frac{\pi}{2}$. Copy the diagram and sketch on it

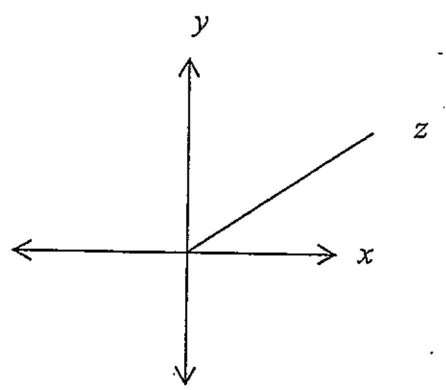
(labelling clearly)

(i) iz **1**

(ii) $\bar{z}$ **1**

(iii) $-z$ **1**

(iv) $z+1$ **1**



(b) For z in the diagram above (so $|z| = 1$)show that if $Arg(z) = \theta$ **2**

then $Arg(z+1) = \frac{\theta}{2}$ (Hint: Use basic geometry!)

(c) z_1 is an arbitrary point on the complex plane.

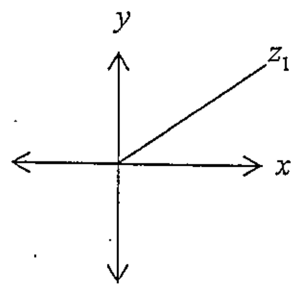
z_2 is a point on the complex plane such that $|z_1| = |z_2|$

and $Arg(z_2) = \frac{2\pi}{3} + Arg(z_1)$.

(i) Copy out the diagram below marking the points **3**

$z_1, z_2, z_1 + z_2$ and $z_1 - z_2$.

(ii) Prove that $|z_1 + z_2| = |z_1|$. **4**



Question 8 4 Marks

Prove DeMoivre's theorem i.e. $\left[(\cos\theta + i\sin\theta)\right]^n = (\cos n\theta + i\sin n\theta)$ for all positive integers n using the method of mathematical induction. In this proof you may NOT assume that $(\cos\alpha + i\sin\alpha)\times(\cos\beta + i\sin\beta) = \left[(\cos(\alpha+\beta) + i\sin(\alpha+\beta)\right]$.

Question 9 11 Marks

(a) If $z = \cos\theta + i\sin\theta$ prove

(i) $z^n + \dfrac{1}{z^n} = 2\cos n\theta$ **1**

(ii) $z^n - \dfrac{1}{z^n} = 2i\sin n\theta$ **1**

(b) By letting $z = \cos\theta + i\sin\theta$ and expanding z^5 prove that

(i) $\sin 5\theta = 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta$ **3**

(ii) $\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$ **1**

(iii) Using your *workings* for parts (i) and (ii) find an expression for $\tan 5\theta$. **2**

(c) Prove that $\sin^5\theta = \dfrac{1}{16}\left[\sin 5\theta - 5\sin 3\theta + 10\sin\theta\right]$ **3**

Question 10 22 Marks

(a)(i) Solve the equation $z^9 - 1 = 0$ over the complex field leaving your solutions in mod/arg form. **2**

(ii) Find the area of the nonagon formed by joining the roots of $z^9 - 1 = 0$ on the complex plane. Answer correct to 2 decimal places. **1**

(b) Let w be the non real root of $z^9 - 1 = 0$ with the smallest positive argument.

(i) Show that $w^2, w^3, w^4, w^5, w^6, w^7, w^8$ are the other non real roots of $z^9 - 1 = 0$. **2**

(ii) Show that $w + w^2 + w^3 + w^4 + w^5 + w^6 + w^7 + w^8 = -1$. **2**

(iii) Show that $(w + w^8)(w^2 + w^7)(w^4 + w^5) = -1$ **2**

(iv) Hence show that $\cos\dfrac{\pi}{9}\cos\dfrac{2\pi}{9}\cos\dfrac{4\pi}{9} = \dfrac{1}{8}$ **4**

PTO →

2008 Q.(10) (continued).

(c)(i) Resolve $z^5 - 1$ into real linear and quadratic factors. **2**

(Leave your quadratic factors in terms of $\cos\theta$)

(ii) Hence show that $\cos\frac{2\pi}{5} + \cos\frac{4\pi}{5} = -\frac{1}{2}$ **3**

and $\cos\frac{2\pi}{5}\cos\frac{4\pi}{5} = -\frac{1}{4}$

(iii) Hence find the exact value of $\cos\frac{2\pi}{5}$. **4**

Solutions 2 Extension 2

Assessment Task 1 '08 Complex Numbers

MARKS

Q.(1)(a) $z+w=-1+i$ (1)

(b) $\bar{z}=1+4i$ (1)

(c) $\frac{z}{w}=\frac{1-4i}{-2+5i}\times\frac{(-2-5i)}{(-2-5i)}$ (2)

$=\frac{-22+3i}{29}$

(d) $w^2=(-2+5i)^2$ (1)

$=-21-20i$

(e) $\overline{zw}=\overline{(1-4i)(-2+5i)}$ 1 (2)

$=\overline{18+13i}$ 1

$=18-13i$

Q.(2)(a) $zw=(1+i\sqrt{3})(1+i)$

$=(1-\sqrt{3})+(1+\sqrt{3})i$ (1)

(b) $z=2\text{cis}\frac{\pi}{3}$, $w=\sqrt{2}\text{ cis}\frac{\pi}{4}$ (4)

(c) In mod/arg form,

$zw=2\text{cis}\frac{\pi}{3}\times\sqrt{2}\text{ cis}\frac{\pi}{4}$

$=2\sqrt{2}\text{ cis}\frac{7\pi}{12}$ 1

$=2\sqrt{2}\cos\frac{7\pi}{12}+2i\sqrt{2}\sin\frac{7\pi}{12}$.

Hence equating imaginary part with (a): (4)

$2\sqrt{2}\sin\frac{7\pi}{12}=1+\sqrt{3}$ 1

$\sin\frac{7\pi}{12}=\frac{1+\sqrt{3}}{2\sqrt{2}}$. 1

(3)(a) $1-i=\sqrt{2}\text{ cis}\left(-\frac{\pi}{4}\right)$ (2)

(b) Hence $(1-i)^{11}=(\sqrt{2})^{11}\text{cis}\left(-\frac{11\pi}{4}\right)$

$=32\sqrt{2}\left[\cos\frac{5\pi}{4}+i\sin\frac{5\pi}{4}\right]$

$=32\sqrt{2}\left[-\frac{1}{\sqrt{2}}-\frac{i}{\sqrt{2}}\right]$ 1 (3)

$=-32-32i$

(4)(a) $(x+iy)^2=-5+12i$

$\therefore x^2+2ixy-y^2=-5+12i$.

Equating real parts: $x^2-y^2=-5$ (1) 1

Equating imaginary parts: $2xy=12$ 1

$\Rightarrow y=\frac{6}{x}$ (2)

Substituting (2) in (1):

$x^2-\left(\frac{6}{x}\right)^2=-5$

$x^2-\frac{36}{x^2}=-5$.

$\times$BS by x^2 & getting all to one side:

$x^4+5x^2-36=0$

$(x^2+9)(x^2-4)=0$ 1 (5)

As x is real, $x=\pm2$. 1

As $y=\frac{6}{x}$, $y=\pm3$. 1

$\therefore$ Square roots of $-5+12i$ are $2+3i$ and $-2-3i$.

(b) Solving $z^2+(2+i)z+(2-2i)=0$

$z=\frac{-(2+i)\pm\sqrt{(2+i)^2-4\times1\times(2-2i)}}{2\times1}$

$=\frac{-2-i\pm\sqrt{-5+12i}}{2}$ 1

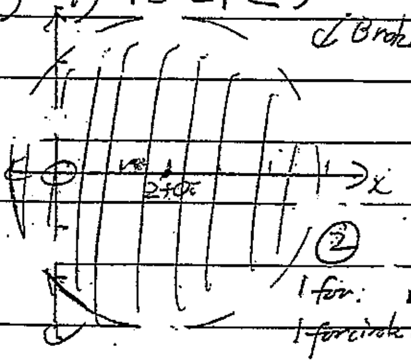
$=\frac{-2-i+2+3i}{2}$ or $=\frac{-2-i-2-3i}{2}$ (2)

[using (a)]

$=i$ or $=-2-2i$. 1

$\therefore z=i$ or $z=-2-2i$.

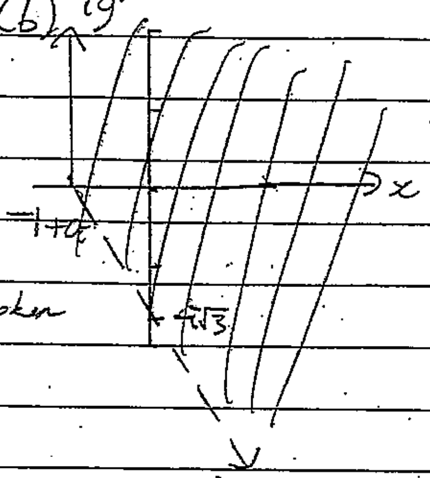
Q.(5)(a) $|z-2|<3$



✓ Broken

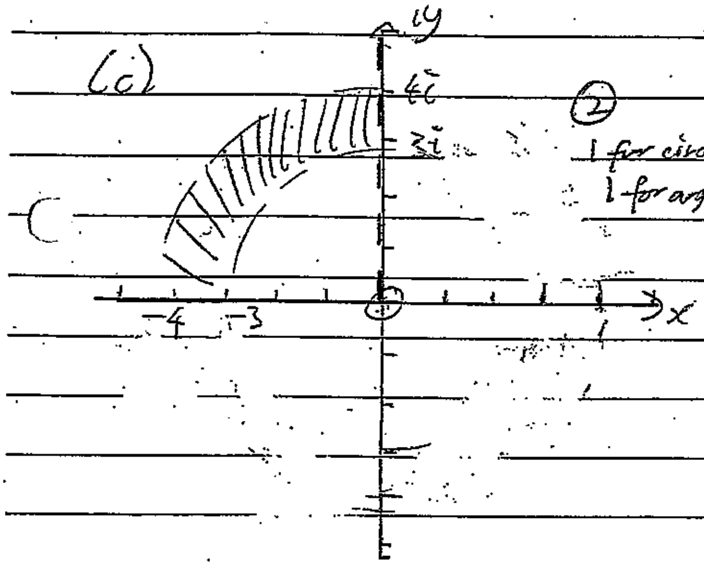
② 1 for: broken
1 for circle.

(b)



③ 1 for position = -1
1 for lines
1 for shading.

(c)



② 1 for circles
1 for args.

(d) $|z-\bar{z}|<3$ ③

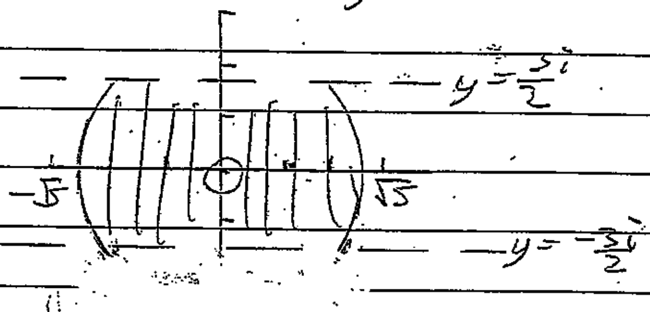
$z-\bar{z}=(x+iy)-(x-iy)$

$=2iy$

$\therefore |2iy|=|2y|$ So $|2y|<3$ 1

$z\bar{z}\le 5 \Rightarrow x^2+y^2\le 5$ 1

& 1 for drawing



Q.(6)(a)(i) $(x+2)^2+(y-5)^2=9$ ①

(ii) $\sqrt{(x-1)^2+(y+2)^2}=\sqrt{(x+1)^2+(y+1)^2}$

$x^2-2x+y^2+4y+5=x^2+2x+y^2+2y+2$ 1

$2y=4x-3$ ②

$4x-2y-3=0$ 1

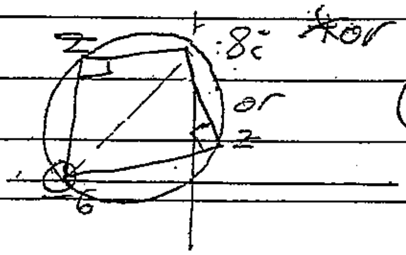
(iii) If $Re\left(\frac{z-8i}{z+6}\right)=0$

then $Arg\left(\frac{z-8i}{z+6}\right)=\frac{\pi}{2}$ or $\frac{3\pi}{2}$.

So $8i$ & 6 would be the endpoints of the diameter of a circle [∠ in semicircle is a right angle]. 1

(iii) [continued] So circle is

$(x+3)^2+(y-4)^2=25$ NOT INCLUD -6. 1



*or

③

[Note: $8i$ can be included.

Q.(6)(a)(iii) Alternatively

$\text{Re}\left(\dfrac{z-8i}{z+6}\right)=0$

or (3)*

As $\dfrac{z-8i}{z+6} = \dfrac{x+(y-8)i}{(x+6)+iy} \times \dfrac{(x+6)-iy}{(x+6)-iy}$

$= \dfrac{x^2+6x+y^2-8y+[(x+6)(y-8) - xy]i}{(x+6)^2+y^2}$

If $\text{Re}(z)=0$, $\dfrac{x^2+6x+y^2-8y}{(x+6)^2+y^2} = 0$

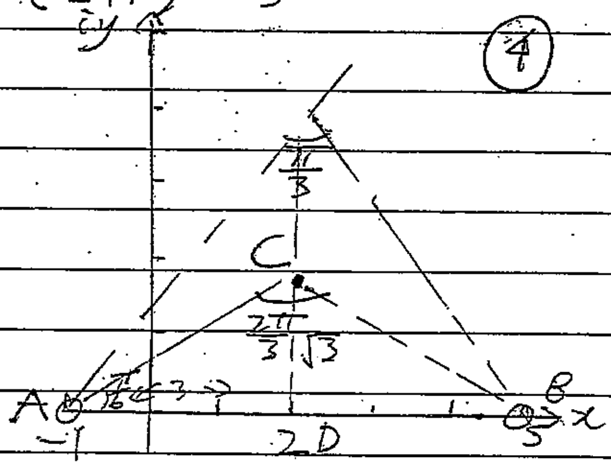
$\times$ BS by $(x+6)^2+y^2$

& completing the square:

$(x+3)^2+(y-4)^2=25$ → But the point $-6+0i$ is NOT included.

(b) $\text{Arg}\left(\dfrac{z-5}{z+1}\right)=\dfrac{\pi}{3}$

(4)



→ Centre is on $x=2$

→ We are looking for C where $\angle ACB = \dfrac{2\pi}{3}$

[∠ at centre of circle $= 2\times$∠ at edge on same arc].

Let $D = 2+0i$

$CA = CB$ [circle radii].

$\angle CAD = \dfrac{\pi}{6}$ [∠s opposite = sides of $\triangle CAB$].

$\therefore DC = 3\tan\dfrac{\pi}{6}$

$= \sqrt{3}$.

So $C = 2+i\sqrt{3}$

$AC = 2\sqrt{3}$ [By Pythagoras' theorem]

Locus is part of circle

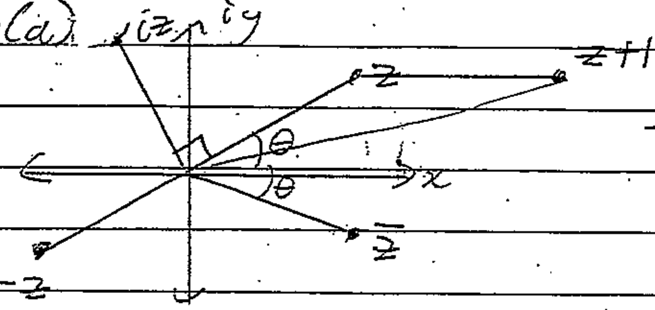
$(x-2)^2+(y-\sqrt{3})^2=12$

where $y>0$

Centre $= 2+i\sqrt{3}$

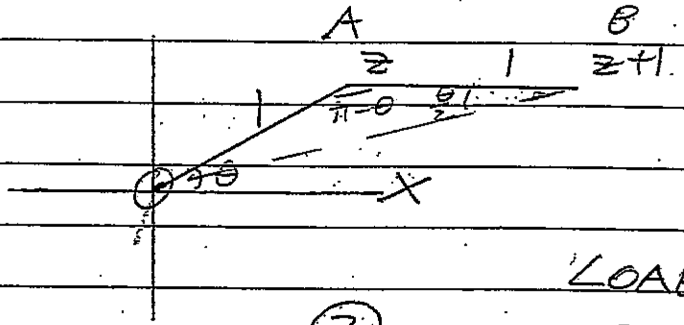
Radius $= 2\sqrt{3}$

Q.(7)(a)



→ ④ 1 each.

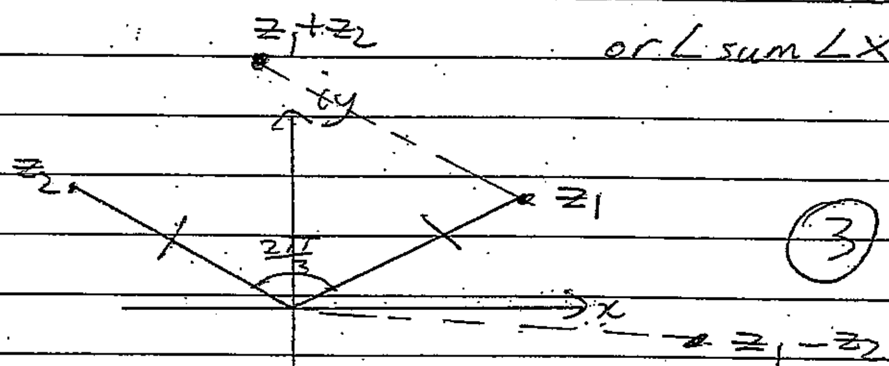
(b)



②

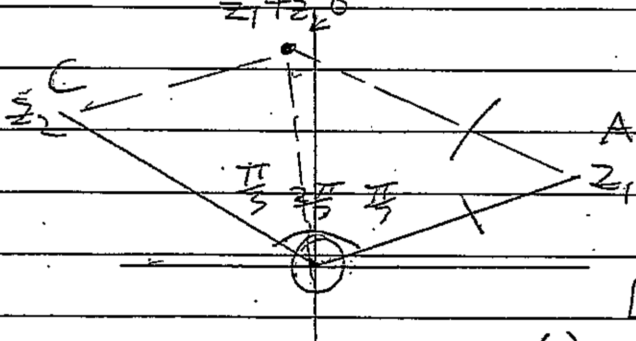
Let z be at A
& $z+1$ be at B
Let $\operatorname{Arg} z = \theta$.
$\angle OAB = \pi - \theta$ [co-interior ∠'s, $OX \parallel AB$].
$\angle ABO = \angle AOB = \frac{\theta}{2}$ [∠'s opposite = sides of isosceles $\triangle OAB$].
∴ $\angle XOB = \frac{\theta}{2}$ (alternate ∠'s, $OX \parallel AB$ or ∠ sum $\angle XOA$].

(c)
(i)



③

(ii)



Let $A = z_1$, $B = z_1+z_2$, $C = z_2$
OABC is a rhombus (all sides =). 1
OB bisects $\angle AOC$
[diagonals of rhombus bisect ∠'s they pass through]. 1
∴ $\angle AOB = \frac{\pi}{3}$. 1 ④
∴ $\triangle AOB$ is equilateral 1
∴ $|z_1+z_2| = |z_1|$ [sides of equilateral △]

Q.(8) Step 1: Show true for $n=1$: (4)

$(\cos\theta + i\sin\theta)^1 = \cos\theta + i\sin\theta$

$\therefore$ True for $n=1$

Step 2: Assume true for $n=k$

i.e. $(\cos\theta + i\sin\theta)^k = \cos k\theta + i\sin k\theta$

Step 3: Prove true for $n=k+1$

i.e. $(\cos\theta + i\sin\theta)^{k+1} = \cos(k+1)\theta + i\sin(k+1)\theta$

$= (\cos k\theta\cos\theta - \sin k\theta\sin\theta) + i(\sin k\theta\cos\theta + \cos k\theta\sin\theta)$

LHS:

$(\cos\theta + i\sin\theta)^{k+1}$

$= (\cos\theta + i\sin\theta)^k \times (\cos\theta + i\sin\theta)$

$= (\cos k\theta + i\sin k\theta) \times (\cos\theta + i\sin\theta)$

$= \cos k\theta\cos\theta + i\cos k\theta\sin\theta + i\sin k\theta\cos\theta - \sin k\theta\sin\theta$

$= (\cos k\theta\cos\theta - \sin k\theta\sin\theta) + i(\sin k\theta\cos\theta + \cos k\theta\sin\theta)$

$=$ RHS

$\therefore$ As it is true for $n=1$ & $n=k+1$ it must be true for all positive integers n by the principle of mathematical induction.

Q. (9)(a)(i) $z^n + \frac{1}{z^n}$

$= (\cos n\theta + i\sin n\theta) + (\cos n\theta - i\sin n\theta)$ (1)

$= 2\cos n\theta.$

(ii) $z^n - \frac{1}{z^n}$ (1)

$= \cos n\theta + i\sin n\theta - (\cos n\theta - i\sin n\theta)$

$= 2i\sin n\theta$

(b)(i) $(\cos\theta + i\sin\theta)^5 = \cos 5\theta + i\sin 5\theta$

$\cos^5\theta + 5i\cos^4\theta\sin\theta - 10\cos^3\theta\sin^2\theta - 10i\cos^2\theta\sin^3\theta + 5\cos\theta\sin^4\theta + i\sin^5\theta$ (2)

Hence equating imaginary parts: $\sin 5\theta = 5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta$

$= 5(1-\sin^2\theta)^2\sin\theta - 10(1-\sin^2\theta)\sin^3\theta + \sin^5\theta$

$= 5\sin\theta(1 - 2\sin^2\theta + \sin^4\theta) - 10(1-\sin^2\theta)\sin^3\theta + \sin^5\theta$

$= 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta$ 1 (3)

(ii) Equating real parts of (i):

$\cos 5\theta = \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$ (3)

$= \cos^5\theta - 10\cos^3\theta(1-\cos^2\theta) + 5\cos\theta(1 - 2\cos^2\theta + \cos^4\theta)$

$= 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta.$ (1)

(iii) Using equations (2) & (3) from parts (i) & (ii) above:

$\tan 5\theta = \frac{\sin 5\theta}{\cos 5\theta} = \frac{5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta}{\cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta}$ 1

Dividing numerator & denominator by $\cos^5\theta$

$= \frac{5\frac{\sin\theta}{\cos\theta} - 10\frac{\sin^3\theta}{\cos^3\theta} + \frac{\sin^5\theta}{\cos^5\theta}}{\frac{\cos^5\theta}{\cos^5\theta} - 10\frac{\sin^2\theta}{\cos^2\theta} + 5\frac{\sin^4\theta}{\cos^4\theta}}$ 1 (2)

$= \frac{5\tan\theta - 10\tan^3\theta + \tan^5\theta}{1 - 10\tan^2\theta + 5\tan^4\theta}$

$=$ RHS.

Ext. 2 Complex '08 p 7 Solutions:

Q.(9)(i) If $z = \cos\theta + i\sin\theta$

$$z - \frac{1}{z} = 2i\sin\theta.$$

$$\left(z - \frac{1}{z}\right)^5 = 32i\sin^5\theta \quad [i^5 = i]. \qquad 1$$

$$\therefore z^5 - 5z^3 + 10z - \frac{10}{z} + \frac{5}{z^3} - \frac{1}{z^5} = 32i\sin^5\theta. \qquad 3$$

$$\left(z^5 - \frac{1}{z^5}\right) - 5\left(z^3 - \frac{1}{z^3}\right) + 10\left(z - \frac{1}{z}\right) = 32i\sin^5\theta \qquad 1$$

As $z^n - \frac{1}{z^n} = 2i\sin n\theta$:

$$2i\sin 5\theta - 10i\sin 3\theta + 20i\sin\theta = 32i\sin^5\theta$$

$$\frac{1}{16}\left[\sin 5\theta - 5\sin 3\theta + 10\sin\theta\right] = \sin^5\theta \qquad 1$$

Q.(10)(a)(i) $z^9 - 1 = 0$

$z = 1, \text{cis}\frac{2\pi}{9}, \text{cis}\frac{4\pi}{9}, \text{cis}\frac{2\pi}{3}, \text{cis}\frac{8\pi}{9}, \text{cis}\frac{10\pi}{9}, \text{cis}\frac{4\pi}{3}, \text{cis}\frac{14\pi}{9}, \text{cis}\frac{16\pi}{9}$. 2

$\to 1$ for $\text{cis}\frac{2\pi}{9}$

$\to 1$ for rest.

(ii) Area $= 9 \times \frac{1}{2} \times \sin\frac{2\pi}{9}$

$= 2{\cdot}89$ square units (2DP). 1

(10) (b)(i) $w = \text{cis}\left(\frac{2\pi}{9}\right) \Rightarrow w^2 = \text{cis}\left(\frac{4\pi}{9}\right), w^3 = \text{cis}\frac{6\pi}{9}$ $\left[= \text{cis}\frac{2\pi}{3}\right]$, $w^4 = \text{cis}\left(\frac{8\pi}{9}\right)$

$w^5 = \text{cis}\frac{10\pi}{9}$ $\left[\text{or cis}\,-\frac{8\pi}{9}\right]$, $w^6 = \text{cis}\frac{12\pi}{9}$ $\left[= \text{cis}\frac{4\pi}{3} \text{ or cis}\,-\frac{2\pi}{3}\right]$, $w^7 = \text{cis}\frac{14\pi}{9} = \text{cis}\,-\frac{4\pi}{9}$, $w^8 = \text{cis}\frac{16\pi}{9}$ $\left[= \text{cis}\,-\frac{2\pi}{9}\right]$.

$\therefore w^2, w^3, w^4, w^5, w^6, w^7, w^8$ are other non-real roots.

(ii) Roots of $z^9 - 1 = 0$ are $1, w, w^2, w^3, w^4, w^5, w^6, w^7, w^8$.

By sum of roots: $1 + w + w^2 + w^3 + w^4 + w^5 + w^6 + w^7 + w^8 = 0$

$$w + w^2 + w^3 + w^4 + w^5 + w^6 + w^7 + w^8 = -1.$$

Ext. 2 Complex '08 p. 8 Solutions:

Q. (10)(b)(iv) As $w = cis\frac{2\pi}{9}$.

$$w + w^8 = \left(\cos\frac{2\pi}{9} + i\sin\frac{2\pi}{9} + \cos\frac{16\pi}{9} + i\sin\frac{16\pi}{9}\right)$$
$$= \left(\cos\frac{2\pi}{9} + i\sin\frac{2\pi}{9} + \cos\frac{2\pi}{9} - i\sin\frac{2\pi}{9}\right)$$
$$= 2\cos\frac{2\pi}{9}.$$

Similarly $w^2 + w^7 = 2\cos\frac{4\pi}{9}$, $w^4 + w^5 = 2\cos\frac{8\pi}{9}$
$$= -2\cos\frac{\pi}{9}.$$

$\therefore$ As $(w + w^8)(w^2 + w^7)(w^4 + w^5) = -1$
$$2\cos\frac{2\pi}{9} \times 2\cos\frac{4\pi}{9} \times -2\cos\frac{\pi}{9} = -1.$$
$$\cos\frac{2\pi}{9} \times \cos\frac{4\pi}{9} \times \cos\frac{\pi}{9} = \frac{1}{8}.$$

4

(c)(i) $z^5 - 1 = (z-1)\left(z - cis\frac{2\pi}{5}\right)\left(z - cis\frac{8\pi}{5}\right)\left(z - cis\frac{4\pi}{5}\right)\left(z - cis\frac{6\pi}{5}\right)$
$$= (z-1)\left(z^2 - 2z\cos\frac{2\pi}{5} + 1\right)\left(z^2 - 2z\cos\frac{4\pi}{5} + 1\right)$$

(ii) $\therefore$ As $z^5 - 1 = (z-1)(z^4 + z^3 + z^2 + z + 1)$

$$z^4 + z^3 + z^2 + z + 1 = \left(z^2 - 2z\cos\frac{2\pi}{5} + 1\right)\left(z^2 - 2z\cos\frac{4\pi}{5} + 1\right)$$
$$= z^4 - 2z^3\left(\cos\frac{2\pi}{5} + \cos\frac{4\pi}{5}\right) + \left(2 + 4\cos\frac{2\pi}{5}\cos\frac{4\pi}{5}\right)z^2$$
$$- 2z\left(\cos\frac{2\pi}{5} + \cos\frac{4\pi}{5}\right) + 1.$$

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Equating co-efficients of z^3 [or z]
$$-2z^3\left(\cos\frac{2\pi}{5} + \cos\frac{4\pi}{5}\right) = 1.$$
$$\cos\frac{2\pi}{5} + \cos\frac{4\pi}{5} = -\frac{1}{2}$$

Equating co-efficients of z^2:
$$2 + 4\cos\frac{2\pi}{5}\cos\frac{4\pi}{5} = 1.$$
$$\cos\frac{2\pi}{5}\cos\frac{4\pi}{5} = -\frac{1}{4}$$

→ Notes This one can be done using sum of roots of $z^5 - 1 = 0$ but a lot of explanation needed.

(iii) Hence letting $x = \cos\frac{2\pi}{5}$, $y = \cos\frac{4\pi}{5}$:
$$x + y = -\frac{1}{2} \quad (1)$$
$$xy = -\frac{1}{4} \quad (2)$$
$$y = -\frac{1}{4x}.$$

Sub. (2) in (1):
$$x - \frac{1}{4x} = -\frac{1}{2}$$
$$4x^2 + 2x - 1 = 0$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4 \times 4 \times -1}}{2 \times 4}$$
$$= \frac{-2 \pm \sqrt{20}}{8}$$
$$= \frac{-2 \pm 2\sqrt{5}}{8}$$
$$x = \frac{-1 \pm \sqrt{5}}{4}$$

As $x = \cos\frac{2\pi}{5}$ & is in Q_1, cos is positive;
$$\cos\frac{2\pi}{5} = \frac{-1 + \sqrt{5}}{4}.$$

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