FINAL MARK

GIRRAWEEN HIGH SCHOOL
MATHEMATICS EXTENSION 2
HSC ASSESSMENT TASK 1
ANSWERS COVER SHEET

Name: ____

QUESTION	MARK	E2	E3	E4	E5	E6	E7	E8	E9
1-5	/5		✓						✓
6	/16		✓						✓
7	/18		✓						✓
8	/17		✓						✓
9	/16		✓						✓
10	/13		✓						✓
11	/13		✓						✓
TOTAL									
	/98		/98						/98

HSC Outcomes **Mathematics Extension 2**

E2 chooses appropriate strategies to construct arguments and proofs in both concrete and abstract settings.

E3 uses the relationship between algebraic and geometric representations of complex numbers and of conic sections.

E4 uses efficient techniques for the algebraic manipulation required in dealing with questions such as those involving conic sections and polynomials.

E5 uses ideas and techniques from calculus to solve problems in mechanics involving resolution of forces, resisted motion and circular motion

E6 combines the ideas of algebra and calculus to determine the important features of the graphs of a wide variety of functions.

E7 uses the techniques of slicing and cylindrical shells to determine volumes.

E8 applies further techniques of integration, including partial fractions, integration by parts and recurrence formulae, to problems.

E9 communicates abstract ideas and relationships using appropriate notation and logical argument.

GIRRAWEEN HIGH SCHOOL

HSC Task 1

YEAR 11 Mathematics Extension 2
2013

Time allowed – 90 minutes

DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- All necessary working should be shown in Questions 6 - 10.
  Marks may be deducted for careless or badly arranged work.
- Board-approved calculators may be used.
- For Questions 1 - 5, write the letter corresponding to the correct answer in your answer booklet. For Questions 6 – 10, each question is to be returned on a *separate* piece of paper clearly marked Question 6, Question 7, etc. Each piece of paper must show your name.
- You may ask for extra pieces of paper if you need them.

Multiple Choice(5 marks)

Write the letter corresponding to the correct answer in your answer booklet.

1 Let $z=1+i$ and $w=1-2i$. What is the value of zw?

(A) $-1-i$

(B) $-1+i$

(C) $3-i$

(D) $3+i$

2 Let $z=3-4i$ and $w=\sqrt{3}+i$. What is the value of $\dfrac{z}{w}$?

(A) $\dfrac{3\sqrt{3}+4}{4}+\dfrac{(-4\sqrt{3}-3)i}{4}$

(B) $\dfrac{3\sqrt{3}-4}{4}+\dfrac{(-4\sqrt{3}-3)i}{4}$

(C) $\dfrac{3\sqrt{3}+4}{2}+\dfrac{(-4\sqrt{3}-3)i}{2}$

(D) $\dfrac{3\sqrt{3}-4}{2}+\dfrac{(-4\sqrt{3}-3)i}{2}$

3 Let $z = 3 - i$. What is the value of $\overline{iz}$?

(A) $-1-3i$

(B) $-1+3i$

(C) $1-3i$

(D) $1+3i$

4 What is $-2+2\sqrt{3}i$ expressed in modulus-argument form?

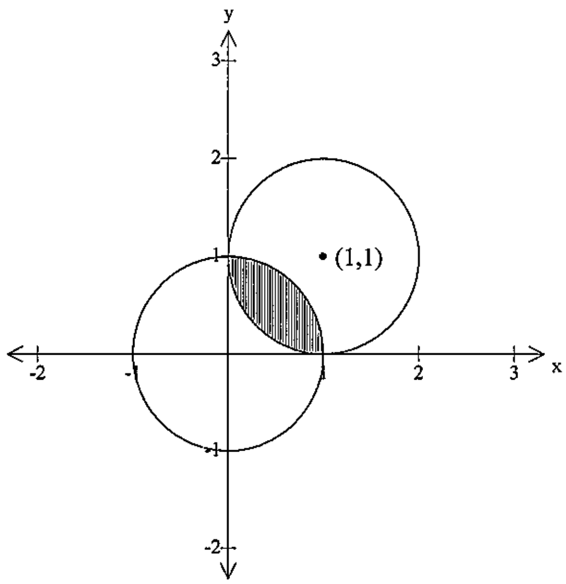
(A) $2(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3})$

(B) $4(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3})$

(C) $2(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3})$ The

(D) $4(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3})$

5 Consider the Argand diagram below.



Which inequality could define the shaded area?

(A) $|z| \le 1$ and $|z-(1-i)| \ge 1$

(B) $|z| \le 1$ and $|z-(1+i)| \ge 1$

(C) $|z| \le 1$ and $|z-(1-i)| \le 1$

(D) $|z| \le 1$ and $|z-(1+i)| \le 1$

Question 6(16 marks)

a. (i) Express $\sqrt{3}+i$ and $1-i$ in modulus-argument form. **4**

(ii) Express $\frac{(\sqrt{3}+i)^6}{(1-i)^8}$ in the form $x+iy$. **3**

b. Find the five fifth roots of $1+\sqrt{3}i$. Show these on an Argand Diagram and find the area of the pentagon formed by the five points representing these roots. **5**

c. Let $w=\frac{3+4i}{5}$ and $z=\frac{5+12i}{13}$,

(i) Find wz and $w\bar{z}$ in the form $x+iy$. **2**

(ii)Hence, find two distinct ways of writing 65^2 as a sum of a^2+b^2, where a and b are integers and $0<a<b$ and $|w|=|z|=1$. **2**

Question 7(18 marks)

a (i) Find all real numbers x and y such that $(x+iy)^2=-3+4i$ **4**

(ii) Hence solve $z^2-3z+(3-i)=0$ **3**

b. By using De Moivre's theorem and the Binomial theorem, obtain an expression for

(i) $\sin 4\theta$ **2**

(ii) $\cos 4\theta$ **2**

(iii) $\tan 4\theta$ **2**

c. Given that ω is a complex root of $z^3=1$,

(i) Show that ω^2 is also a root of the equation. **1**

(ii) Show that $1+\omega+\omega^2=0$. **1**

(iii) Show that $(1-\omega)(1-\omega^2)(1-\omega^4)(1-\omega^5)(1-\omega^7)(1-\omega^8)=27$ **3**

Question 8(17 marks)

a. (i) Show that if $z=\cos\theta+i\sin\theta$ then $z^n+\frac{1}{z^n}=2\cos n\theta$. **3**

(ii) Hence express $\cos^5\theta$ in terms of $\cos 5\theta, \cos 3\theta, \cos\theta$. **4**

b. (i) Find the six sixth roots of unity. Write your answer in mod-arg form. **4**

(ii) Resolve z^6-1 into real quadratic factors. **3**

(iii) Hence show that $\cos\frac{\pi}{3}\cos\frac{2\pi}{3}=-\frac{1}{4}$ **3**

Question 9(16 marks)

a. Sketch the following on separate Argand Diagrams. Find the Cartesian equation and describe the locus.

(i) $\arg(z+2)=\frac{\pi}{4}$ (ii) $|z+2-i|=4$ (iii) $|z-2i|=|z-4|$ **9**

b. (i) On the same diagram, draw a neat sketch of the locus described by: **4**

I. $|z-(3+2i)|=2$

II. $|z+3|=|z-5|$

(ii) Hence, write down all the values of z which satisfy simultaneously **1**

$|z-(3+2i)|=2$ and $|z+3|=|z-5|$

(iii)Use your diagram in (i) to determine the values of k for which the simultaneous equations for $|z-(3+2i)|=2$ and $|z-2i|=k$ have exactly one solution for z. **2**

Question 10(13 marks)

a. Let $z=a+ib$ where $a^2+b^2\neq 0$

(i) Show that if $\operatorname{Im}(z)>0$, then $\operatorname{Im}\left(\frac{1}{z}\right)<0$. **4**

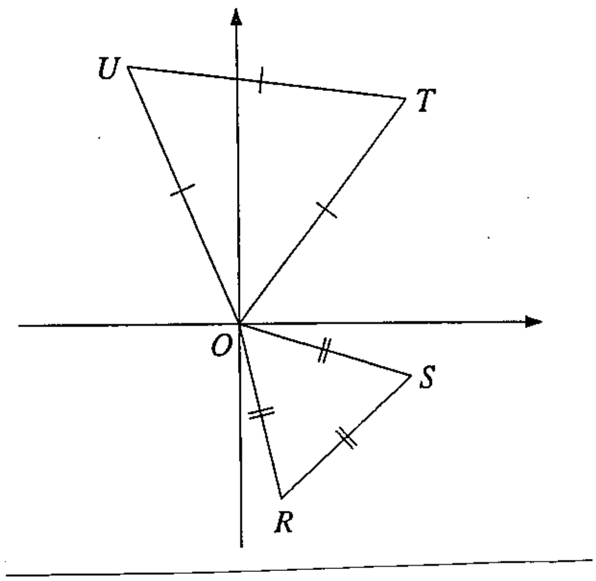
(ii) Prove that $\left|\frac{1}{z}\right|=\frac{1}{|z|}$ **3**

b. Show that the roots of $(z-1)^4+(z+1)^4=0$ are $\pm i\cot\left(\frac{\pi}{8}\right)$ *and* $\pm i\cot\left(\frac{3\pi}{8}\right)$. **6**

Question 11(13 marks)

a. Sketch $\arg\left(\frac{z-3}{z+1}\right)=\frac{\pi}{3}$. Describe the locus and find its Cartesian equation. **6**

b.



The diagram shows points O, R, S, T and U in the complex plane. These points correspond to the complex numbers O, r, s, t and u respectively. The triangles ORS and OTU are equilateral.

Let $\omega=\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}$.

(i) Explain why $u=\omega t$. **1**

(ii) Find the complex number r in terms of s and $\overline{\omega}$. **2**

(iii)Using the complex numbers, show that the lengths of RT and SU are equal. **4**

END OF EXAMINATION

HSC Task 1 Mathematics Extension 2

SOLUTIONS 2013

Multiple Choice

1. C 2. B 3. C 4. B 5. D /5

Question 6 (16 marks)

a) (i) $\sqrt{3}+i = 2\text{cis}\frac{\pi}{6}$ (2)

$1-i = \sqrt{2}\text{cis}\frac{-\pi}{4}$ (2)

ii) $\frac{(\sqrt{3}+i)^6}{(1-i)^8} = \frac{\left(2\text{cis}\frac{\pi}{6}\right)^6}{\left(\sqrt{2}\text{cis}\frac{-\pi}{4}\right)^8}$

$= \frac{64\text{cis}\,\pi}{16\text{cis}\,{-2\pi}}$

$= \frac{4(\cos\pi + i\sin\pi)}{\cos(-2\pi)+i\sin(-2\pi)}$

$= -4$ (3)

b) Let $z^5 = 1+\sqrt{3}i$

$= 2\text{cis}\frac{\pi}{3} = 2\text{cis}\left(\frac{\pi}{3}+2k\pi\right)$

$\therefore z = 2^{1/5}\left(\frac{\frac{\pi}{3}+2k\pi}{5}\right)$; $k=0,1,\dots 4$

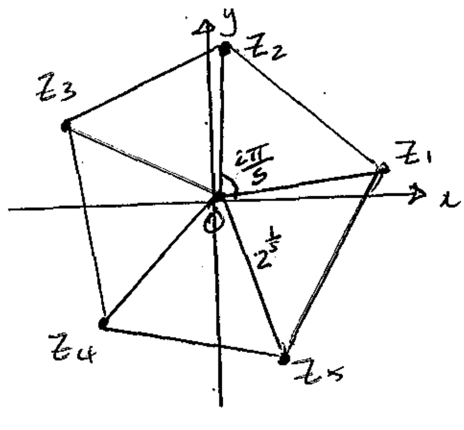
$z_1 = 2^{1/5}\text{cis}\frac{\pi}{15}$

$z_2 = 2^{1/5}\text{cis}\frac{7\pi}{15}$

$z_3 = 2^{1/5}\text{cis}\frac{13\pi}{15}$

$z_4 = 2^{1/5}\text{cis}\frac{19\pi}{15} = 2^{1/5}\text{cis}\frac{-11\pi}{15}$

$z_5 = 2^{1/5}\text{cis}\frac{25\pi}{15} = 2^{1/5}\text{cis}\frac{-\pi}{3}$



Area $= 5\times\frac{1}{2}\times 2^{1/5}\times 2^{1/5}\sin\frac{2\pi}{5}$

$= 3.14$ sq. units (5)

c) $\omega = \frac{3+4i}{5}$; $z = \frac{5+12i}{13}$

i) $\omega z = \frac{(3+4i)(5+12i)}{65}$

$= \frac{15+36i+20i-48}{65}$ (1)

$= \frac{-33}{65}+\frac{56}{65}i$

ii) $\omega\bar{z} = \frac{(3+4i)(5-12i)}{65}$

$= \frac{63}{65} - \frac{16i}{65}$ (1)

iii) $|\omega z| = |\omega||z| = \sqrt{\left(\frac{33}{65}\right)^2+\left(\frac{56}{65}\right)^2}$

$|\omega| = |z| = 1$

$\therefore \left(\frac{33}{65}\right)^2+\left(\frac{56}{65}\right)^2 = 1$

$\therefore 65^2 = 33^2+56^2$ (1)

Also $|\omega\bar{z}| = |\omega||\bar{z}| = \sqrt{\left(\frac{63}{65}\right)^2+\left(\frac{16}{65}\right)^2}$

$|\omega| = |\bar{z}| = 1$ & $|\bar{z}| = |z|$

$\therefore \left(\frac{63}{65}\right)^2+\left(\frac{16}{65}\right)^2 = 1$

$\therefore 65^2 = 63^2+16^2$ (1)

Question 7 (18 marks)

a) i) $(x+iy)^2 = -3+4i$

$x^2+2xiy-y^2 = -3+4i$

Equating real & imaginary parts,

$x^2-y^2=-3$ — ① ; $2xy=4$

$y = \frac{2}{x}$ — ②

Substitute ② into ①

$x^2 - \left(\frac{2}{x}\right)^2 + 3 = 0$

$x^2 - \frac{4}{x^2} + 3 = 0$

$x^4 + 3x^2 - 4 = 0$

$(x^2+4)(x^2-1) = 0$

x^2+4: no real solutions; $x^2-1=0$, $x = \pm 1$

when $x=1$, $y=2$ ④
$x=-1$, $y=-2$

ii) $z^2-3z+(3-i)=0$

$z = \frac{3 \pm \sqrt{9-4(1)(3-i)}}{2}$

$= \frac{3 \pm \sqrt{-3+4i}}{2}$

$= \frac{3 \pm (1+2i)}{2}$

$z = 2+i,\ 1-i$ ③

b) $(\cos\theta + i\sin\theta)^4 = \cos 4\theta + i\sin 4\theta$ — ①
(de Moivre's Thm)

$(\cos\theta+i\sin\theta)^4 = \cos^4\theta + 4i\cos^3\theta\sin\theta - 6\cos^2\theta\sin^2\theta - 4i\cos\theta\sin^3\theta + \sin^4\theta$ — ②

Equating real and imaginary parts in ① and ②

i) $\sin 4\theta = 4\cos^3\theta\sin\theta - 4\cos\theta\sin^3\theta$

ii) $\cos 4\theta = \cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta$

iii) $\tan 4\theta = \frac{\sin 4\theta}{\cos 4\theta}$

$= \frac{4\cos^3\theta\sin\theta - 4\cos\theta\sin^3\theta}{\cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta}$

$\tan 4\theta = \frac{4\tan\theta - 4\tan^3\theta}{1-6\tan^2\theta+\tan^4\theta}$

(dividing numerator & denominator by $\cos^4\theta$.) ⑥

c) ω is a solution of $z^3=1$

i) If ω is a solution, then

$\omega^3 = 1$

$(\omega^2)^3 = (\omega^3)^2 = 1^2 = 1$

$\therefore \omega^2$ is also a root. ①

ii) roots of $z^3=1$ are $1, \omega$ & ω^2.

Sum of the roots $= \frac{-b}{a} = 0$ ①

$\therefore 1+\omega+\omega^2 = 0$

iii) $(1-\omega)(1-\omega^2)(1-\omega^4)(1-\omega^5)(1-\omega^7)(1-\omega^8)$

$= (1-\omega)(1-\omega^2)(1-\omega)(1-\omega^2)(1-\omega)(1-\omega^2)$

$= (1-\omega)^3(1-\omega^2)^3$

$= [(1-\omega)(1-\omega^2)]^3$

$= (1-\omega^2-\omega+\omega^3)^3$

$= (1-\omega^2-\omega+1)^3$

$= (2-(\omega+\omega^2))^3$

$= (2-(-1))^3$

$= 27$ ③

Question 8 (17 marks)

a) i) $z = \cos\theta + i\sin\theta$

$z^n = \cos n\theta + i\sin n\theta$

$\frac{1}{z^n} = z^{-n} = \cos(-n\theta) + i\sin(-n\theta)$

$z^n + \frac{1}{z^n} = \cos n\theta + i\sin n\theta + \cos(-n\theta) + i\sin(-n\theta)$

$= \cos n\theta + i\sin n\theta + \cos n\theta - i\sin n\theta$

$= 2\cos n\theta$ (3)

ii) $z^5 + \frac{1}{z^5} = 2\cos 5\theta$

$\left(z + \frac{1}{z}\right)^5 = z^5 + 5z^3 + 10z + \frac{10}{z} + \frac{5}{z^3} + \frac{1}{z^5}$

$(2\cos\theta)^5 = \left(z^5 + \frac{1}{z^5}\right) + 5\left(z^3 + \frac{1}{z^3}\right) + 10\left(z + \frac{1}{z}\right)$

$32\cos^5\theta = 2\cos 5\theta + 10\cos 3\theta + 20\cos\theta$

$\cos^5\theta = \frac{1}{16}\cos 5\theta + \frac{5}{16}\cos 3\theta + \frac{5}{8}\cos\theta$

$\cos^5\theta = \frac{1}{16}\left(\cos 5\theta + 5\cos 3\theta + 10\cos\theta\right)$ (4)

b) i) $z^6 = 1 = \text{cis } 0$

$= \text{cis}(0 + 2k\pi)$

$z = \text{cis}\left(\frac{2k\pi}{6}\right)$

$= \text{cis}\left(\frac{k\pi}{3}\right) \quad k = 0, 1, \ldots 5$

$z_1 = \text{cis } 0 = 1$

$z_2 = \text{cis } \frac{\pi}{3}$

$z_3 = \text{cis } \frac{2\pi}{3}$

$z_4 = \text{cis } \pi = -1$

$z_5 = \text{cis } \frac{4\pi}{3} = \text{cis } \frac{-2\pi}{3}$

$z_6 = \text{cis } \frac{5\pi}{3} = \text{cis } \frac{-\pi}{3}$ (4)

ii) $z^6 - 1 = (z-1)(z+1)\left(z - \text{cis}\frac{\pi}{3}\right)\left(z - \text{cis}\frac{-\pi}{3}\right)\left(z - \text{cis}\frac{2\pi}{3}\right)\left(z - \text{cis}\frac{-2\pi}{3}\right)$

$z^6 - 1 = (z^2 - 1)\left(z^2 - 2z\cos\frac{\pi}{3} + 1\right)\left(z^2 - 2z\cos\frac{2\pi}{3} + 1\right)$ (1) (3)

iii)

$$\begin{array}{r} z^4 + z^2 + 1 \\ z^2 - 1 \enclose{longdiv}{z^6 + 0z^5 + 0z^4 + 0z^3 + 0z^2 + 0z - 1} \\ -\;z^6 - 0z^5 - z^4 \\ z^4 + 0z^3 + 0z^2 \\ -\;z^4 + 0z^3 - z^2 \\ z^2 + 0z - 1 \\ -\;z^2 + 0z - 1 \end{array}$$

(OR)

$z^6 - 1 = (z^2)^3 - 1$

$= (z^2 - 1)(z^4 + z^2 + 1)$

$z^6 - 1 = (z^2 - 1)(z^4 + z^2 + 1)$ — (2)

From (1) & (2)

$(z^2 - 1)(z^4 + z^2 + 1) = (z^2 - 1)\left(z^2 - 2z\cos\frac{\pi}{3} + 1\right)\left(z^2 - 2z\cos\frac{2\pi}{3} + 1\right)$

$\therefore z^4 + z^2 + 1 = \left(z^2 - 2z\cos\frac{\pi}{3} + 1\right)\left(z^2 - 2z\cos\frac{2\pi}{3} + 1\right)$

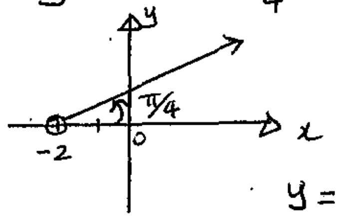
Equating coefficients of x^2

$2 + 4\cos\frac{\pi}{3}\cos\frac{2\pi}{3} = 1$

$\cos\frac{\pi}{3}\cos\frac{2\pi}{3} = -\frac{1}{4}$ (3)

Question 9 (16 marks)

a) i) $\arg(z+2) = \frac{\pi}{4}$



(3)

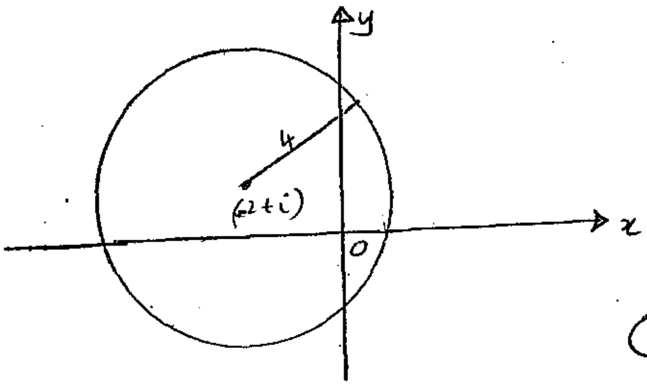
$y = x+2, \; y>0$

ii) $|z+2-i| = 4$

$|z-(-2+i)| = 4$

circle, centre $(-2, 1)$

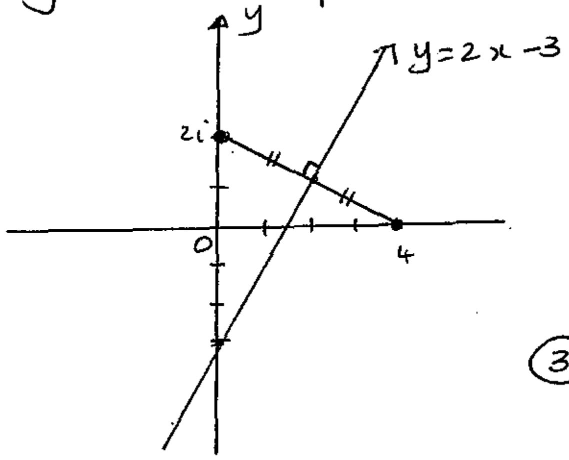
radius 4



(3)

iii) $|z-2i| = |z-4|$

locus is the perpendicular bisector of the line segment joining $2i$ and 4.

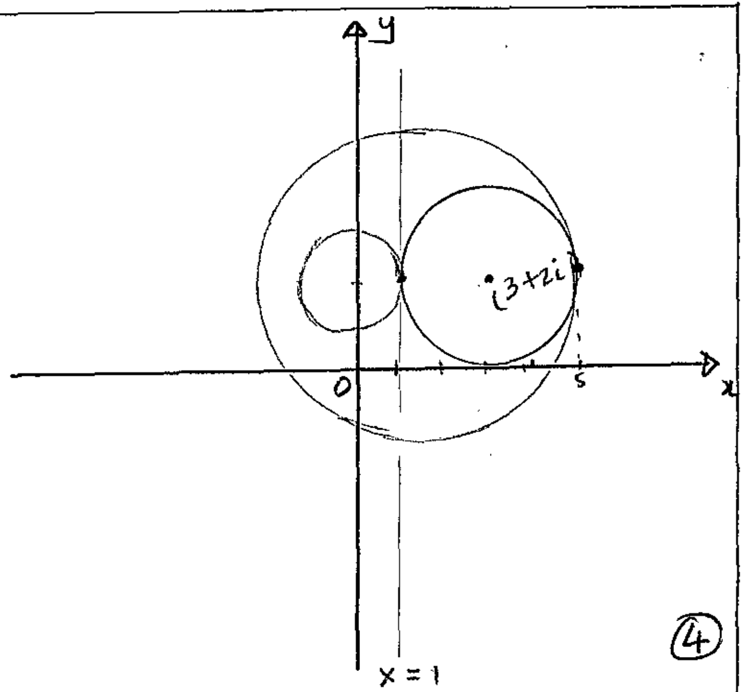


(3)

b) i) I. $|z-(3+2i)| = 2$ is a circle centre $(3+2i)$, $r=2$

II. $|z+3| = |z-5|$ is a perpendicular bisector of interval joining -3 & 5

ie line $x=1$



(4)

ii) The line $x=1$ is a tangent to the circle.

$\therefore$ $z = 1+2i$ is the only point of intersection. (1)

iii) $|z-2i| = k$ represents a circle, centre $2i$, radius k.

Hence the equations

$|z-(3+2i)| = 2$ and

$|z-2i| = k$ have exactly one solution when

$k=1$ or $k=5$.

(see diagram in part (i)) (2)

Question 10 (13 marks)

a) Let $z = a + ib$ where $a^2+b^2 \neq 0$

i) $\frac{1}{z} = \frac{1}{a+ib}$

$= \frac{a-ib}{(a+ib)(a-ib)}$

$= \frac{a-ib}{a^2+b^2}$

$\text{Im}\left(\frac{1}{z}\right) = \frac{-b}{a^2+b^2}$

If $\text{Im}(z) > 0$, ie $b > 0$,

then $\text{Im}\left(\frac{1}{z}\right) = \frac{-b}{a^2+b^2} < 0$

(since $a^2+b^2 > 0$)

(4)

ii) $\left|\frac{1}{z}\right| = \left|\frac{a-ib}{a^2+b^2}\right|$

$= \sqrt{\left(\frac{a}{a^2+b^2}\right)^2 + \left(\frac{-b}{a^2+b^2}\right)^2}$

$= \sqrt{\frac{a^2+b^2}{(a^2+b^2)^2}}$

$= \sqrt{\frac{1}{a^2+b^2}}$

$= \frac{1}{\sqrt{a^2+b^2}}$

$= \frac{1}{|z|}$

(3)

b) $(z-1)^4 + (z+1)^4 = 0$

$(z-1)^4 = -(z+1)^4$

$\left(\frac{z-1}{z+1}\right)^4 = -1 = \text{cis}(\pi + 2k\pi)$

Let $y = \frac{z-1}{z+1} \Rightarrow y^4 = \text{cis}(\pi + 2k\pi)$

$\therefore y = \text{cis}\left(\frac{\pi + 2k\pi}{4}\right)$

let $\theta = \frac{\pi + 2k\pi}{4}$ $\quad k = 0, 1, 2, 3$

then $y = \frac{z-1}{z+1} = \cos\theta + i\sin\theta$

$z - 1 = (z+1)(\cos\theta + i\sin\theta)$

$z - 1 = z\cos\theta + iz\sin\theta + \cos\theta + i\sin\theta$

$z = z\cos\theta + iz\sin\theta + \cos\theta + i\sin\theta + 1$

$z - z\cos\theta - iz\sin\theta = \cos\theta + i\sin\theta + 1$

$z(1 - \cos\theta - i\sin\theta) = \cos\theta + i\sin\theta + 1$

$z = \frac{\cos\theta + i\sin\theta + 1}{1 - \cos\theta - i\sin\theta}$

$= \frac{2\cos^2\frac{\theta}{2} + 2i\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2} - 2i\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$

$= \frac{2\cos\frac{\theta}{2}\left(\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\right)}{2\sin\frac{\theta}{2}\left(\sin\frac{\theta}{2} - i\cos\frac{\theta}{2}\right)} \times \frac{i}{i}$

$= i\cot\frac{\theta}{2}\left(\frac{\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}}{\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}}\right)$

$= i\cot\frac{\theta}{2}$; $\theta = \frac{\pi + 2k\pi}{4}$, $k = 0, 1, 2, 3$

$\therefore z_1 = i\cot\frac{\pi}{8}$

$z_2 = i\cot\frac{3\pi}{8}$

$z_3 = i\cot\frac{5\pi}{8} = i\cot\frac{-3\pi}{8}$

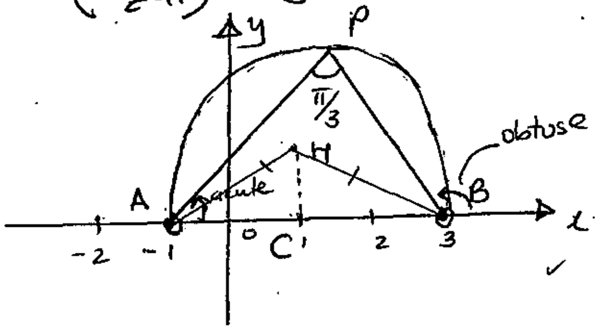
$z_4 = i\cot\frac{7\pi}{8} = i\cot\frac{-\pi}{8}$

Roots: $\pm i\cot\frac{\pi}{8}$, $\pm i\cot\frac{3\pi}{8}$

(6)

Question 11 (13 marks)

a) $\arg\left(\frac{z-3}{z+1}\right) = \frac{\pi}{3}$



centre of circle (H) on line $x=1$

$AH = BH$ (radii)

$\angle APB = \frac{\pi}{3}$

$\therefore \angle AHB = \frac{2\pi}{3}$ ($\angle$ at centre is twice $\angle$ at circumference on same arc)

$\therefore \angle HAB = \angle HBA$ ($\angle$ sum of Isosceles Δ)

$= \frac{\pi}{6}$

$CH = 2\tan\frac{\pi}{6}$

$= \frac{2}{\sqrt{3}}$

$\therefore H = \left(1, \frac{2}{\sqrt{3}}\right)$

$AH = \sqrt{2^2 + \left(\frac{2}{\sqrt{3}}\right)^2}$

$= \sqrt{4 + \frac{4}{3}}$

$= \sqrt{\frac{16}{3}} = \frac{4}{\sqrt{3}}$

$\therefore r = \frac{4}{\sqrt{3}}$

$\therefore$ Locus is the major arc of circle with centre $\left(1, \frac{2}{\sqrt{3}}\right)$, $r = \frac{4}{\sqrt{3}}$

Equation: $(x-1)^2 + \left(y - \frac{2}{\sqrt{3}}\right)^2 = \frac{16}{3}$, $y > 0$

(6)

b) i) $\angle UOT = \frac{\pi}{3}$ (ΔUOT is equilateral)

$\omega = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$

multiplying t by ω will rotate t about O through $\frac{\pi}{3}$ in an anticlockwise direction.

$\therefore u = \omega t$

(1)

ii) $\angle ROS = \frac{\pi}{3}$ (ΔORS is equilateral)

To get from R to S, there is a rotation of $\frac{\pi}{3}$ in a clockwise direction.

$\therefore r = s\left[\cos\frac{-\pi}{3} + i\sin\frac{-\pi}{3}\right]$

$= s\left[\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right]$

$r = s\bar{\omega}$

(2)

iii) $\overrightarrow{RT} = t - r$

$\overrightarrow{SU} = u - s$

$\omega\overrightarrow{RT} = \omega t - \omega r$

$= u - s = \overrightarrow{SU}$

$|\overrightarrow{SU}| = |\omega\overrightarrow{RT}|$

$= |\omega|\,|\overrightarrow{RT}|$

$= |\overrightarrow{RT}|$ (since $|\omega| = 1$)

ie. $SU = RT$

(4)