GIRRAWEEN HIGH SCHOOL

MATHEMATICS EXTENSION 2

TASK 1 2017 December 2016: COMPLEX NUMBERS

ANSWERS COVER SHEET

Name:__________________________________

Teacher:_________________________________

FINAL MARK

	MARK	E2	E3	E4	E5	E6	E7	E8
1 -5 Multiple Choice	**/5**	√	√					√
6	**/21**	√	√					√
7	**/14**	√	√					√
8	**/13**	√	√					√
9	**/16**	√	√					√
10	**/13**	√	√					√
11	**/13**	√	√					√
TOTAL	**/95**	/95	/95					/95

HSC Outcomes **Mathematics Extension 2**

E2 chooses appropriate strategies to construct arguments and proofs in both concrete and abstract settings.

E3 uses the relationship between algebraic and geometric representations of complex numbers and of conic sections.

E4 uses efficient techniques for the algebraic manipulation required in dealing with questions such as those involving conic sections and polynomials.

E5 uses ideas and techniques from calculus to solve problems in mechanics involving resolution of forces, resisted motion and circular motion

E6 combines the ideas of algebra and calculus to determine the important features of the graphs of a wide variety of functions.

E7 uses the techniques of slicing and cylindrical shells to determine volumes.

E8 applies further techniques of integration, including partial fractions, integration by parts and recurrence formulae, to problems.

E9 communicates abstract ideas and relationships using appropriate notation and logical argument.

GIRRAWEEN HIGH SCHOOL

TASK 1 2017 (December 2016)

MATHEMATICS

EXTENSION 2

Complex Numbers

Time allowed – 100 Minutes

DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Board-approved calculators may be used.
- Each question attempted is to be marked clearly Question 1, Question 2 etc. Each question is to be returned on a separate page in your answer booklet.
- You may ask for spare answer booklets if you need them.
- For Multiple choice: Fill in the circle corresponding to the correct answer on the multiple choice answer sheet in your answer booklet.

Questions 1-5 (Multiple Choice) Circle the correct answer on the Examination Paper

(1) The value of i^{-6} is:

(A) i (B) -i (C) 1 (D)-1

(2) 1If $z = 4 + 3i$ and $w = 2 - i$ then $\bar{z}w =$

(A)$11 - 2i$ (B) $5 + 10i$ (C) $5 - 10i$ (D) $11 + 2i$

(3) When expressed in modulus/ argument form, $\sqrt{3} - 3i =$

(A) $2\sqrt{3}(cos\frac{\pi}{3} + isin\frac{\pi}{3})$ (B) $2\sqrt{3}(cos\frac{-\pi}{3} + isin\frac{-\pi}{3})$ (C) $2\sqrt{3}(cos\frac{2\pi}{3} + isin\frac{2\pi}{3})$

(D) $2\sqrt{3}(cos\frac{-2\pi}{3} + isin\frac{-2\pi}{3})$

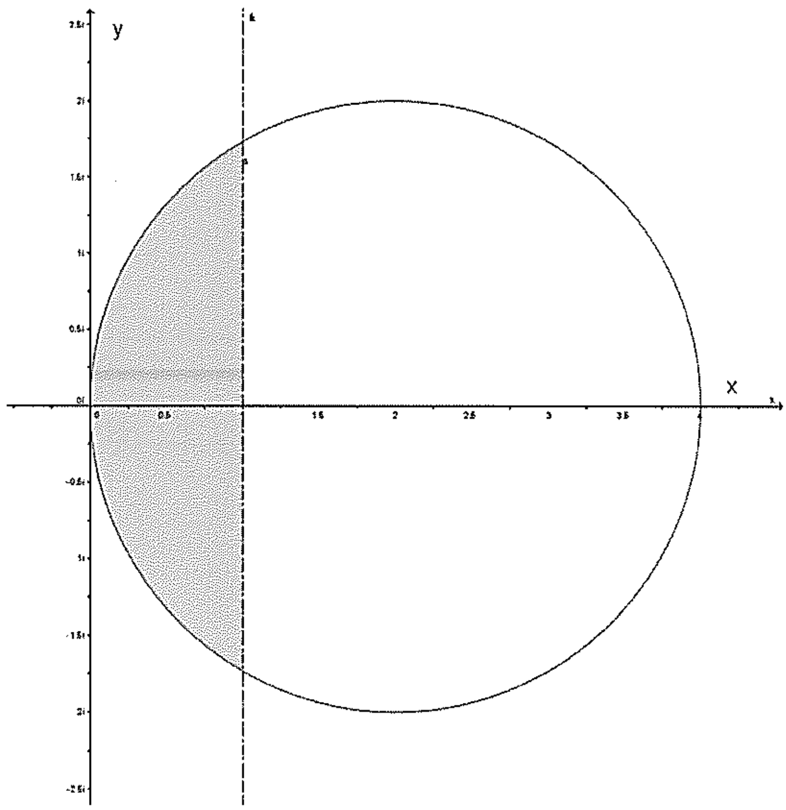
(4) In Cartesian form, $\frac{6+8i}{4-3i} =$

(A)$2i$ (B) $-2i$ (C) $\frac{48+14i}{25}$ (D) $\frac{28+96i}{25}$

(5) The region drawn in the complex plane below is::

(A) $|z| \leq 2 \; and \; Re(z) < 1$ (B) $|z| \leq 2 \; and \; Im(z) < 1$

(C) $|z - 2| \leq 2 \; and \; Re(z) < 1$ (D) $|z - 2| \leq 2 \; and \; Im(z) < 1$



Examination continues on the following page

For Question 6 onward show all workings on the blank paper provided:

Question 6 *(21 Marks)* **Marks**

(a) (i) Find $\frac{-1+i}{\sqrt{3}-i}$ in Cartesian form. **2**

(ii) Convert both $-1+i$ and $\sqrt{3}-i$ to Modulus/argument form. **4**

(iii) Using the answers to (i) and (ii) find the exact value of $\cos\frac{11\pi}{12}$. **2**

(b) (i) If $x+iy=\sqrt{12+16i}$ find the exact value of x and y. **5**

(ii) Hence solve the equation $z^2+(2+6i)z+(-11+2i)=0$ **3**

(c) Use DeMoivre's theorem to find $(-1+i\sqrt{3})^{11}$ in Cartesian form. **3**

(d) Find all three cube roots of $4\sqrt{2}+4i\sqrt{2}$. Leave your answers in modulus/ argument form. **2**

Question 7 *(14 Marks)*

(a) Sketch each of the following loci on separate Argand diagrams:

(i) $|z+2-i|=2$ **2**

(ii) $Arg(z-i)=\frac{2\pi}{3}$ **2**

(iii) $|z-2+i|=|z+4-5i|$ **2**

(iv) $\left|\frac{z+5+6i}{z-4-6i}\right|=2$ **3**

(b) Sketch and shade the region satisfied by $|z-2i|\le 3$ and $\frac{\pi}{2}\le Arg\,(z+i)<\frac{3\pi}{4}$ on an Argand diagram. **5**

Examination continues on the following page

Question 8 *(13 Marks)* **Marks**

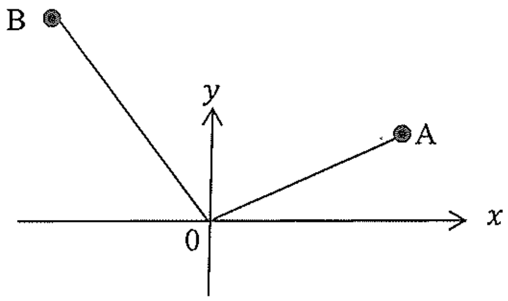
(a) z and w are arbitrary points on the Argand diagram such that $\overrightarrow{OA} = z$ and $\overrightarrow{OB} = w$. (see diagram below). Copy the diagram on to your answer paper and draw in:

(i) $\overrightarrow{OC}$ so that $\overrightarrow{OC} = z+w$ **1**

(ii) $\overrightarrow{OD}$ so that $\overrightarrow{OD} = z - w$ **1**

(iii) $\overrightarrow{OE}$ so that $\overrightarrow{OE} = iw$ **1**

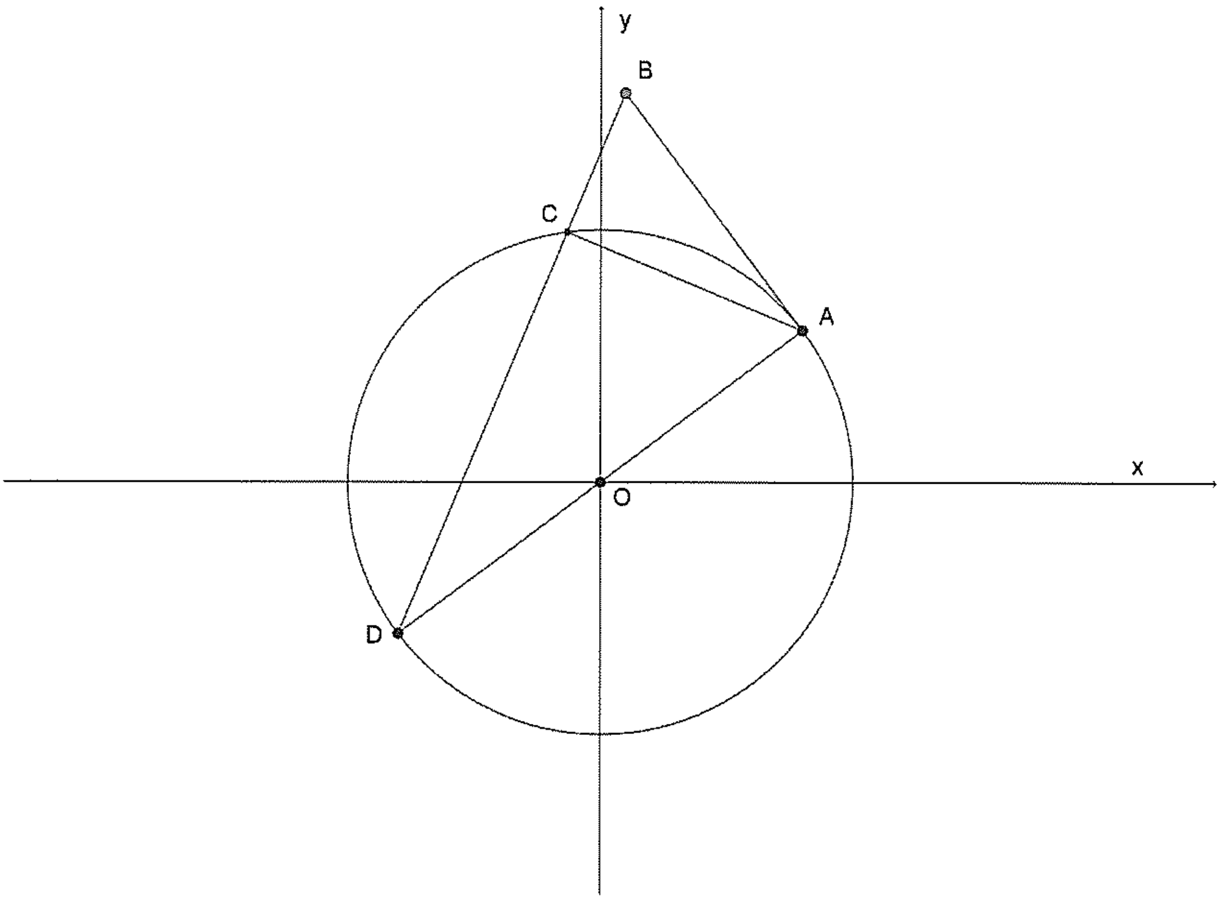
(iv) $\overrightarrow{OF}$ so that $\overrightarrow{OF} = w \times (cos\frac{\pi}{3} + isin\frac{\pi}{3})$ **1**



Examination continues on the following page

Question 8 (continued) **Marks**

(b) In the Argand diagram below, $\overrightarrow{OA} = z_1, \overrightarrow{OB} = z_2$ and $\overrightarrow{OC} = z_3$.
DA and BD are straight lines through O and C respectively and AB is a tangent to the circle ACD with centre O.



(i) Write $\overrightarrow{OD}$ in terms of z_1. 1

(ii) State why $\frac{z_2 - z_1}{z_1}$ is entirely imaginary. **2**

(iii) Prove $Arg\left(\frac{z_3 - z_1}{z_2 - z_1}\right) = Arg\left(\frac{z_1 + z_2}{z_1}\right)$ **3**

(iv) Prove $|z_2 - z_1|^2 = |(z_2 + z_1)(z_3 - z_2)|$ **3**

Examination continues on the following page

Question 9 ***(16 Marks)***

(a) The graph of $Arg\left(\frac{z-5}{z+3}\right) = \frac{\pi}{4}$ represents part of a circle. .Draw this **5**

circle part and find its centre, radius and equation.

(b)(i) If $z = cos\theta + isin\theta$, prove that $z^n \quad + \frac{1}{z^n} = 2\cos n\theta$ and **5**

$z^n \quad - \frac{1}{z^n} = 2isin\, n\theta.$

(ii) Hence express $sin^8\theta$ in terms of $\cos 8\theta,\ \cos 6\theta,\ \cos 4\theta$ and $\cos 2\theta$. **3**

(iii) Hence find the exact value of $sin\,\frac{\pi}{8}$. *(Leave your answer as an 8th root.)* **3**

Question 10 ***(13 Marks)***

(a) Bu using DeMoivre's Theorem and the expansion of $(cos\theta + isin\theta)^6$, find **3**

formulas for $\sin 6\theta$ and $\cos 6\theta$. *(You may leave your answers in terms of both* $\sin\theta$ *and* $\cos\theta$*).*

(b) Hence find the formula for $\cot 6\theta$ in terms of $\cot\theta$. **2**

(c) Hence show that $cot\frac{\pi}{12}$ is a root of the equation $z^4 - 14z^2 + 1 = 0$ and **8**

find the exact value of $tan\,\frac{\pi}{12}$.

Question 11 ***(13 Marks)***

(a)(i) If w is the complex root of $z^5 - 1 = 0$ with the smallest positive **2**

argument, show that w^2, w^3 *and* w^4 are also roots of $z^5 - 1 = 0$.

(ii) Show that $w + w^2 + w^3 + w^4 = -1$. **2**

(b)(i) By matching pairs of conjugate roots, resolve $z^5 - 1$ into real linear **2**

and quadratic factors (you may use your answer to (a) here).

(ii) Hence resolve $z^4 + z^3 + z^2 + z + 1$ into real quadratic factors. **1**

(iii) Hence show that **3**

$$2cos2\theta + 2cos\theta + 1 = (2cos\theta - 2cos\frac{2\pi}{5})\ (2cos\theta + 2cos\frac{\pi}{5})$$

(iv) Hence by substituting in an appropriate value for θ show that **3**

$$(1\ - 2cos\frac{2\pi}{5}\ )\ (1 + 2cos\frac{\pi}{5}) = 1$$

End of examination