HURLSTONE AGRICULTURAL HIGH SCHOOL

MATHEMATICS – EXTENSION TWO

2005 HSC

ASSESSMENT TASK 1

Examiners ~ G Rawson, J Dillon

GENERAL INSTRUCTIONS

- Reading Time – 3 minutes.
- Working Time – 40 MINUTES.
- Attempt **all** questions.
- **All** necessary working should be shown in every question.
- This paper contains two (2) questions.
- Marks may not be awarded for careless or badly arranged work.
- Board approved calculators may be used.
- **Each question is to be started on a new piece of paper.**
- This examination paper must **NOT** be removed from the examination room.

STUDENT NAME: _____

TEACHER: _____

QUESTION ONE **20 marks** *Start a SEPARATE sheet*

(a) If $z = 3 - i$ and $w = 1 + 3i$, find

(i) $z + w$ **1**

(ii) $z - w$ **1**

(iii) zw **1**

(iv) $z\overline{w}$ **2**

(v) $\dfrac{z}{w}$ **2**

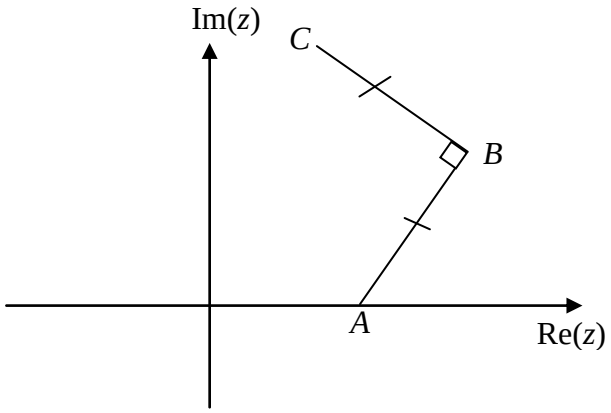
(b) (i) Show that $(1 - 2i)^2 = -3 - 4i$ **1**

(ii) Hence, or otherwise, solve the equation $z^2 - 5z + (7 + i) = 0$ **3**

(c) (i) Find the modulus and argument of $5 + 5\sqrt{3}\, i$ **2**

(ii) Hence, or otherwise, find the two square roots of $5 + 5\sqrt{3}\, i$ **3**

(d)



The diagram above shows the fixed points *A*, *B* and *C* in the Argand plane, where $AB = BC$, $\angle ABC = \dfrac{\pi}{2}$, and *A*, *B* and *C* are in anticlockwise order. The point *A* represents the complex number $z_1 = 2$ and the point *B* represents the complex number $z_2 = 3 + \sqrt{5}\, i$.

(i) Find the complex number z_3 represented by the point *C*. **2**

(ii) *D* is the point on the Argand plane such that *ABCD* is a square. Find the complex number z_4 represented by *D*. **2**

QUESTION TWO 20 marks *Start a SEPARATE sheet*

(a) Given $z = \sqrt{6} - \sqrt{2}\, i$, find

(i) $\mathrm{Re}\left(z^2\right)$ **1**

(ii) $|z|$ **1**

(iii) $\arg z$ **2**

(iv) z^4 in the form $x + iy$ **2**

(b) Find and plot on the Complex Plane, the values of z for which $z^3 - 8i = 0$ **3**

(c) Sketch the locus of z satisfying:

(i) $\arg(z-4) = \dfrac{3\pi}{4}$ **3**

(ii) $\mathrm{Im}\, z = |z|$ **2**

(d) The equations $|z - 8 - 6i| = 2\sqrt{10}$ and $\arg z = \tan^{-1} 2$ both represent loci on the Argand plane.

(i) Write down the Cartesian equations of the loci, and hence show that the points of intersection of the loci are $2 + 4i$ and $6 + 12i$. **3**

(ii) Sketch both loci on the same diagram, showing their points of intersection. (You need not show the intercepts with the axes.) **3**