Name: ____________________

Teacher: _______________

HURLSTONE AGRICULTURAL HIGH SCHOOL

YEAR 12 ASSESSMENT TASK 1

7th December, 2006

Mathematics Extension 2

Examiners ~ Z Pethers, R Yen.

GENERAL INSTRUCTIONS

- Reading time – 3 minutes.
- Working time – 40 minutes.
- This examination has 2 questions worth 20 marks each. Total: 40 marks.
- Answers are to be written in separate answer booklets. Start each question in a new booklet.
- All necessary working should be shown in every question.
- This paper must be handed in at the end of the examination.

Question 1 (20 marks) Use a SEPARATE writing booklet. **Marks**

(a) Suppose that $z = c - i$, where c is a real number.

Express the following in the form $x + iy$, where x and y are real numbers.

(i) $\overline{(iz)}$ **1**

(ii) $\dfrac{1}{z}$ **1**

(b) (i) Simplify $(18+4i)(3+i)$. **2**

(ii) Express $z = \dfrac{18+4i}{3-i}$ in the form $a + ib$, where a and b are real numbers. **1**

(c) Find real values of x and y such that: **3**

$$(3+2i)(x+iy) = 7-4i.$$

(d) Solve $z^2 - 2z + 15 = 0$ over the complex numbers. **2**

(e) Find the complex square roots of $-39+80i$, **3**
giving your answer in the form $a+ib$, where a and b are real.

(f) (i) Express $1+i\sqrt{3}$ in modulus-argument form. **2**

(ii) Hence evaluate $(1+i\sqrt{3})^{10}$ in the form $x+iy$. **2**

(iii) By using the modulus-argument form or otherwise, **3**
express $\dfrac{1+i\sqrt{3}}{(1+i)^2}$ in the form $x+iy$.

Question 2 (20 marks) Use a SEPARATE writing booklet. . **Marks**

(a) Draw P representing the complex number $z = 4 - 2i$ on an Argand diagram. Then clearly indicate on the same diagram the points Q, R and S representing the complex numbers $\overline{z}, -z$ and $\dfrac{3z}{2}$ respectively. **3**

(b) (i) Solve the equation $z^4 = -1$, expressing all values of z in the form $z = x + iy$. **5**

(ii) If $\omega = \dfrac{1}{\sqrt{2}} + \dfrac{i}{\sqrt{2}}$ is a solution to $z^4 = -1$, express ω^9 in the form $z = x + iy$. **2**

(c) Prove that for any complex number z: **2**

$$\frac{1}{z} = \frac{\overline{z}}{|z|^2}.$$

(d) (i) Prove by expansion that $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$. **2**

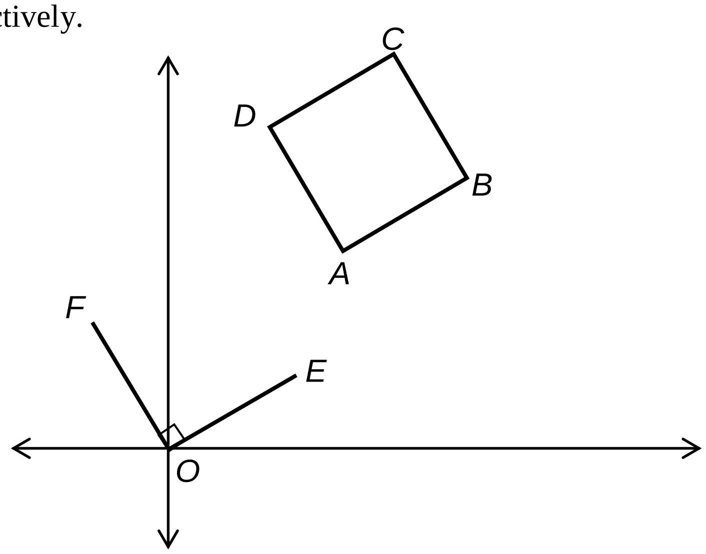
(ii) Hence show that: **2**

$$(\cos\theta + i\sin\theta)^3 = \cos^3\theta + 3i\cos^2\theta\sin\theta - 3\cos\theta\sin^2\theta - i\sin^3\theta.$$

(iii) Hence use De Moivre's theorem to show that: **2**

$$\cos 3\theta = 4\cos^3\theta - 3\cos\theta.$$

(e) In the Argand diagram, $ABCD$ is a square. OE and OF are parallel and equal to AB and AD respectively. The points A and B correspond to the complex numbers z and ω respectively.



(i) Explain why point E corresponds to $\omega - z$. **1**

(ii) What complex number corresponds to point F? **1**

END OF EXAMINATION.