HURLSTONE AGRICULTURAL HIGH SCHOOL

MATHEMATICS – EXTENSION TWO

2008 HSC

ASSESSMENT TASK 1

Examiners ~ G Rawson, J Dillon

GENERAL INSTRUCTIONS

- Reading Time – 3 minutes.
- Working Time – 40 MINUTES.
- Attempt **all** questions.
- **All** necessary working should be shown in every question.
- This paper contains two (2) questions.
- Marks may not be awarded for careless or badly arranged work.
- Board approved calculators may be used.
- **Each question is to be started on a new piece of paper.**
- This examination paper must **NOT** be removed from the examination room.

STUDENT NAME: _____

TEACHER: _____

QUESTION ONE 18 marks *Start a SEPARATE sheet*

(a) Evaluate i^{2007} **1**

(b) Simplify $\dfrac{14+3i}{i(4-5i)}$ **2**

(c) Let z be a non-zero complex number.
If $z^2 = (\overline{z})^2$, show that z is either real or purely imaginary. **2**

(d) Find the complex square roots of $15 + 8i$,
giving your answer in the form $a + ib$, where a, b are real. **3**

(e) Let $z = \dfrac{\sqrt{2}}{2} - \dfrac{\sqrt{2}}{2}i$. Find

(i) $|z|$ **1**

(ii) Arg (z) **1**

(f) Simplify $\dfrac{\left(\cos\dfrac{\pi}{3}+i\sin\dfrac{\pi}{3}\right)\left(\cos\dfrac{\pi}{4}+i\sin\dfrac{\pi}{4}\right)}{\left(\cos\dfrac{\pi}{12}+i\sin\dfrac{\pi}{12}\right)}$ **3**

(g) Let $z = 1+\cos\theta+i\sin\theta$ for $0<\theta<\pi$.

(i) Show that $z = 2\cos\dfrac{\theta}{2}\left(\cos\dfrac{\theta}{2}+i\sin\dfrac{\theta}{2}\right)$ **2**

(ii) Find $|z|$ and arg (z) in terms of θ. **2**

(h) It is given that $2+i$ is a root of $P(z) = z^3 + rz^2 + sz + k$.

Under what conditions would $2 - i$ also be a root of $P(z)$? **1**

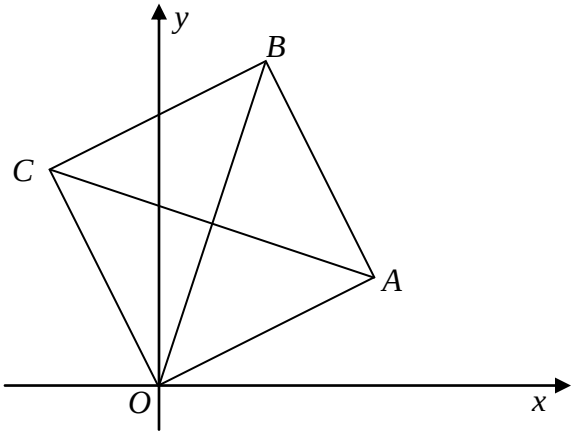
QUESTION TWO 18 marks *Start a SEPARATE sheet*

(a) (i) Find the equation of the locus of the point representing the complex number z which satisfies the equation $|z-2+i|=2$. **2**

(ii) Describe this locus geometrically. **1**

(b) Find the locus of the point representing the complex number z which satisfies the equation $|z-2+i|=|z+1+3i|$. **2**

(c) The points A, B and C represent the complex numbers z_1, z_2 and z_3 respectively



Suppose $OABC$ is a square, and $z_1 = a+ib$,

(i) Which vectors would represent z_1+z_3 and z_3-z_1? **2**

(ii) Express z_3 and z_2 in terms of a and b. **2**

(ii) Find $\dfrac{z_3-z_1}{z_2}$ **2**

(d) Sketch the region in the complex plane where the inequalities

$$|z-1-i|<2 \text{ and } 0<\arg(z-1-i)<\frac{\pi}{4}$$

hold simultaneously. **3**

(e) Given $z = x + iy$, sketch the locus of z if $\arg\left(\dfrac{z-1}{z+1}\right)=\dfrac{\pi}{4}$.

Show all important features . **4**